MECHANICS

1.0 MEASUREMENT

Measurement is the process of assigning numbers to a given physical quantity.

1.1 Physical Quantity

In describing the behavior of objects around us we have to consider to matter, space and time. A moving body covers distance with time and for an object to move energy is required. For the motion to take place, force must be applied. When an object is in the course of motion changes its speed within a given time interval we said that it is undergoing acceleration. In all this we have physical quantities which are measurable and whose values can be used in the mathematical expressions to give numerical description about the object in a question. The physical quantities are divided into two categories which are fundamental / basic quantities and derived quantities.

(a)Fundamental quantities

These are independent physical quantities such as mass, length and time. These quantities have both dimensions and standard units which can be expressed dimensionally.The dimensions of mass, length and time are represented as M, L and T respectively. The term dimension is used to denote the nature of physical quantity.

(b)Derived quantities

The physical quantities which are obtained from fundamental quantities are called derived quantities. An example of derived quantities are such as area, volume, density, speed, and momentum. These quantities can be obtained by combining the fundamental quantities in one way or the other. The following are the few examples:

(i) Area =Length Length

$[A]= L \times L = L^2$

(ii) Volume =Length × Length × Length

$[V] = L \times L \times L$

(iii) Density = Mass/Volume

$[\rho] = M/V$

(iv)Speed = Distance/Time

$[V] = L/T = LT^{-1}$

DIMENSION

Dimension is the way in which the physical quantities are related to fundamental physical quantities.

DIMENSIONAL ANALYSIS

Dimensional analysis is the way of showing how physical quantities are related to each other. The alphabets used to represent particular unit may be called a symbol. There are various systems in use for the same unit , however the symbol M,L and T are dimensionally used for mass, length and time respectively.

The dimensions of a physical quantity refers to a fundamental units contained in it. Any quantity which can be measured in mass unit only, may said to have the dimension of mass.

The derived units are based on the fundamental quantities and in many cases it involves more than one fundamentals in such case the dimension of such quantity is expressed in general as $K(M)^X(L)^Y(T)^Z$ where K is the pure numeral of x, y and z which indicate how many times a particular unit is involved.

The power to which the fundamental units are raised can be obtained and are called the dimension of the derived unit.

For example the area of a square whose sides are in m each.

$1m \times 1m = 1m^2$

The dimension of the area of a square $1m^2$ is;

$(L) \times (L) = L^2$; then area has the dimension of length.

The dimension of velocity can be obtained from the definition of velocity which is;

Velocity is the rate of change in displacement. Its unit is meter per second.

The dimension of the velocity $V = L/T = LT^{-1}$

USES OF DIMENSION

Dimensions of physical quantities can be used in the derivation of formula, checking of homogeneity of the formula etc

Derivation of Formula

Dimensions are sometimes used as a tool in establishing relationship between physical quantities. For example through observation one would like to establish the connection between mass (m), its velocity (v) and the work done(w) on it.

The following are steps to follow:

Form a statement that: Work is proportional to mass and velocity

i.e. W = k$m^x v^y$ ....(i)

Where k is the proportionality constant

Dimensions

Work = Force × Distance

W = F × S

Where F = ma

[W] = [F] [S] = [m] [a] [s]

= $MLT^{-2}L$

= ML^2T^{-2}

[V] = LT^{-1}

Substitute the dimension in equation (i)

$M^1L^2T^{-2} = kM^xL^yT^{-z}$

Compare and equate the indices of corresponding dimensions

For M: x = 1

L: y = 2

T: -y = -2 or = 2

Substitute for x and y in equation (i)

$W = km^1v^2$ or $W = kmv^2$

This is an empirical expression for the work done to move the body and the body acquiring kinetic energy.
Through the experiment or mathematical analysis it can be shown that k = ½

Checking of homogeneity of the formula

Another area which dimensions can be useful is to check consistence of the equation. An equation with several of terms each with number of variables is consistent if every term has dimensions. In the process of proving consistency or homogeneity, we are supposed to show that the left hand side of the equation is dimensionally equal to the right hand side of that equation. As an example let us consider the third Newton´s equation of motion

$$v^2 = u^2 + 2as$$

Where v, u, a and s are final velocity, initial velocity, acceleration and distance respectively. Dimensionally

$$[v^2] = [u^2] + 2[a]\,[s]$$

We get,

$$[LT^{-1}]^2 = [LT^{-1}]^2 + 2[LT^{-2}][L]$$

$$L^2T^{-2} = L^2T^{-2} + 2L^2T^{-2}$$

This shows that each term in the equation above represents dimensions of the square of velocity.

To convert a physical quantity from one system of units to another

The value of a physical quantity can be obtained in some other system, when its value in one system is given by using the method of dimensional analysis.

Measurement of a physical quantity is given by X = nu,

u - Size of unit,

n - Numerical value of physical quantity for the chosen unit.

Let u_1 and u_2 be units for measurement of a physical quantity in two systems and let n_1 and n_2 be the numerical values of physical quantity for two units.

$\therefore n_1u_1 = n_2u_2$

Let a, b and c be the dimensions of physical quantity in mass, length and time

$$n_1\left[M_1^a L_1^b T_1^c\right] = n_2\left[M_2^a L_2^b T_2^c\right]$$

M_1, L1, T1 and M2, L2, T2 are units in two systems of mass, length and time.

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

This equation is used to find the value of a physical quantity in the second or the new system, when its value in the first or the given system is known.

Determination of units

At time when solving problem using an expression involves number of variables raised to some powers, the final unity of quantity being calculated may not be immediately recalled. One way of finding what unit is, is by making use of dimension. For example if unit of gravitation constant g after substituting the numerical values in the formula and evaluate the value, from the basic equation we have;

$G = Fr^2/Mm$

Dimensionally,

$[G] = [F][r^2]/[M][m]$

Where, $[F] = MLT^{-2}$

$[r^2] = L^2$

$[M] = M$

$[m] = M$

Therefore, $[G] = MLT^{-2}L^2/M^2$

$= M^{-1}L^3T^2$

Since $M^{-1} = Kg^{-1}$, $L^3 = m^3$ and $T^{-2} = s^{-2}$, the units of G are $m^3s^{-2}kg^{-1}$ which can also be written as Nm^2kg^{-1}

Limitations of dimensional analysis

There are shortcomings in the use of dimensions for the cases we have considered above. In the case of deriving empirical expressions, dimensional method cannot be used to derive trigonometric, logarithmic and exponential formula. As far as checking homogeneity is concerned, dimension method cannot detect the presence of dimensionless constant in the equation,can not be used to find dimensions of physical quantities with more than three fundamental quantities. It is restricted to only mass, length and time.

Example 1

After being deformed and then let free, a drop of liquid vibrate with a frequency which appears to depend on the surface tension γ of drop, the density ρ of the liquid and radius r of the drop. By means of dimensions, derive an expression for the frequency of vibration.

Solution

Frequency f depends on

(i) Surface tension

(ii) Density

(iii) Radius

These are connected together by the equation

$$f = k\gamma^x \rho^y r^z$$

Dimensions $[f] = T^{-1}$, $[\gamma] = MT^{-2}$, $[\rho] = ML^{-3}$ and $[r] = L$

Substitute these dimensions in the equation above and simplify to get

$$M^0L^0T^{-1} = k[ML^{-2}]^X[ML^{-3}]^Y[L]^Z$$

M^0Lâ• °$T^{-1} = kM^{X+Y}L^{-3Y+Z}T^{-2X}$

Equate the indices of corresponding dimensions

For M: x + y = 0…………….. (i)

L: -3y + z =0……………… (ii)

T: -2x = -1…………………. (ii)

Solving simultaneous equations we have,

x =1/2 , y = -1/2 and z = -1/2

$\therefore f = k\gamma^{1/2}\rho^{-1/2}r^{-3/2} = k\left(\frac{\gamma}{\rho}\right)^{1/2}\left(\frac{1}{r^3}\right)^{1/2} = k(\gamma/\rho r)^{1/2}$

Exercise 1

1. (a) Distinguish between a fundamental physical quantity and derived physical quantity?

(b)What is the dimension of physical quantity?

(c) Write the quantities below in dimension form

(i)The coefficient of viscosity

(ii)The surface tension

(iii)The gravitational constant

2. (a) State the uses and limitations of dimensions

(b)Find out whether or not the equation below is dimensionally homogeneous

V/t = k(Prâ• ´/ηL)

Where k, v, t, p, r, η and L are dimensionless constant, volume, time, pressure, radius, viscosity constant and length respectively

(c) A wave is produced in taut wire by plucking it, the speed of the wave is said to be dependent on tension T of the wire, the mass M and length L. Using this information derive an empirical equation for the speed of the wave in wire.

3. A ball bearing of radius r is released from the surface of the viscous liquid of viscosity constant η in a tall tube. The bearing attains maximum velocity v as it falls through the liquid. Given the friction force opposing the motion of the ball as;

$F = k r^x \eta^y v^z$

Find numerical value of x, y, and z

4. (a) A student in examination write $s = ut + at^2$ as one of the equations of motion he goes on to check dimensionally its homogeneity and he gets satisfied that he has quoted the write equation. What is the problem with the formula?

(b)The volume of the fluid flowing through a narrow tube per unit time depends on pressure gradient $\frac{p}{l}$, viscosity constant η of the fluid and radius r of the tube. Obtain the expression for time rate of flow Q.

5. (a) the periodic time T of a simple pendulum is assumed in the form of

$$T = k m^x l^y g^z$$

Where k, m, l and g constant, mass of the bob, length of the thread, and acceleration due to gravity in the same order, Find the numerical values of x, y, and z. Give your comment on the expression.

(b)A force experienced by an object moving in a circle depends on mass m of the object, the velocity v at which it moves and radius r of the circle it describes. By using dimension obtain the expression for the force.

1.2 ERRORS

Error is the deviation of measured value from the exact value.

Physics is the subject which deals with measurements of physical quantities such as mass, length, time, temperature, and electricity to mention but a few. The instruments which are used in measuring that quantity are varying depending on nature and magnitude of the quantity being measured. For instance the instrument requires to measure the height of man may not necessarily to use the same instrument for measuring the waist or diameter of his hair strand. The scales on instruments are varying degrees of accuracy. This type of length measured by meter rule is not the same as that measured by micrometer screw gauge. The scale on meter rule is less accurate than the scale on micro meter screw gauge. When measuring time, the scale on stop clock is not accurate than that on digital stopwatch

Accuracy of number

There are different categories of numbers, Our interest will be on two types of numbers, the counting numbers and decimal numbers. These numbers are used more often. The numeral like 1, 2, 3 etc are used denote number of complete objects which naturally do not exist in parts or fractions. The numbers such as 1, 2, 2.5, 3.0 etc are known as decimal numbers because they consists some fractions. All scientific measuring instruments bear scales that cater for decimal number and for this matter all values read from measuring instrument must be recorded in decimal form and the answer after calculations using decimal numbers should be written in standard form $A \times 10^n$

Where A lies between 1−9 ($1 < A < 9$) and n can be positive or negative integers.

Below we have a 5 numbers written in various accuracies:

5 (accurate to nearest unit)

5.0 (accurate to nearest tenth)

5.00 (accurate to nearest hundredth)

5.000 (accurate to nearest thousandth)

Etc.

The more decimal places the more the accurate is the number.

ERROR ANALYSIS

Errors are involved in all measurements. There is a need to know the effects of errors in final results. When one has obtained a result, it is important to have some indication of its accuracy. For instance error of length measured by a meter rule may be given as $x \pm 0.5\ mm$ where x is the measured value and *0.5mm*, is half of the smallest unit in measuring x which is considered as an error in measurement.

There are two main types of errors;

1. Systematic errors
2. Random errors

SYSTEMATIC ERRORS

These are errors due to experimental apparatus e.g an incorrect zero adjustment of the measuring devices (beam balance, voltmeter, galvanometer) Starting or stopping the clock, scale calibration, use of incorrect value of constant in calculation.These types of errors have a tendency of affecting reading in the same direction.

MINIMIZATION OF SYSTEMATIC ERROR

These cannot be corrected by arranging a large number of reading. They must be recognized in advance by means of careful survey. These errors can be determined by suitable treatment of observation or careful setting of apparatus over the zero reading check up before using the instruments.

RANDOM ERRORS

These errors gives a spread in answer of repetition equally, likely to be in either direction. These have equal chance to be –ve or +ve. They are caused due to;

i. Fluctuation in the surrounding e.g temperature and pressure..

ii. Lack of the perfection of the observer due to parallax.

iii. Insensitivity of the instrument/apparatus.

MINIMIZATION OF RANDOM ERROR

- These errors can be minimized by repeating observation of a particular quantity.
- Experiments should be carefully designed, use of highly sensitive instruments and finding mean of the measured value,
- An experiment is accurate when the systematic error is relatively small.

MISTAKE
A mistake is done when the measurement is carried out in wrong way and as a result an error is introduced in value recorded. For example using instrument without checking for zero error or arrange instruments without following instructions properly can cause unnecessary errors which can otherwise be avoided.

PRECISION OF ERROR

This indicate the closeness with which measurements agree with one another quite independent to any systematic error.

BLUNDER
This is a mistake done several times.

How to avoid or minimizing errors

It is possible to avoid or if not to minimize by the first reading the instructions of a given experiment and undertaking them. Apparatus should be checked thoroughly before put them into uses. In most cases diagrams are given to show how apparatus must be arranged and connected, so follow the instructions. The apparatus must be securely connected to avoid accidents in the middle of the experiment. The following are some of precautions to be taken in minimizing the error in each case:

(i) Instrumental errors

In this case of errors that arise from measuring devices themselves, the precautions should be:

Regular maintenance and repair of apparatus

Proper storage of apparatus in special ductless rooms

Careful handling of apparatus when transferring them between places

Regular dusting, cleaning and oiling

(ii) Observation errors

The precautions to be taken to minimizing these kinds of errors during experiment are as follow:

Avoid parallax by reading the value from the scale perpendicular from position

Study the scale of instrument before operating it

Fix the apparatus according to the instructions and securely as possible to avoid unnecessary movement during experiment

Write the values read from the scale within the accuracy of instrument

Record the values actually made but not imaginary or cooked ones

(iii) Adjustment errors

It is good practice to check apparatus including measuring devices before put them into use. Together with checking the observer must:

Adjust the measuring device to remove zero error and where it is not possible record it somewhere for further reference

Fix each item of the apparatus in the right position and more importantly for instruments like meter rules, thermometers etc must kept vertical or upright

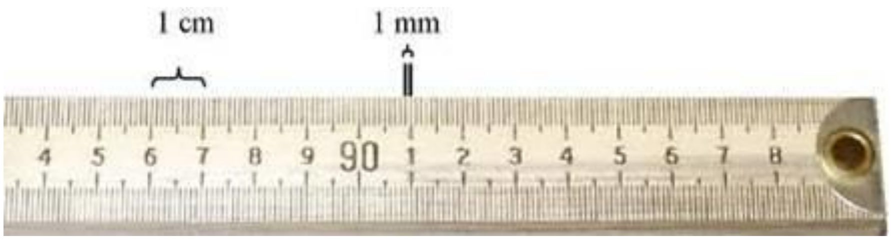
(iv) Random errors

In situation where every trial gives a different value, it is advisable to take as many as measurement as possible and find the average. A good example is determination of the diameter of wire. It good to take the measurement in different positions along the wire and the average value calculated. This is because the wire may not be uniform, so taking only value may not give the best results.

Accuracy of measuring instruments

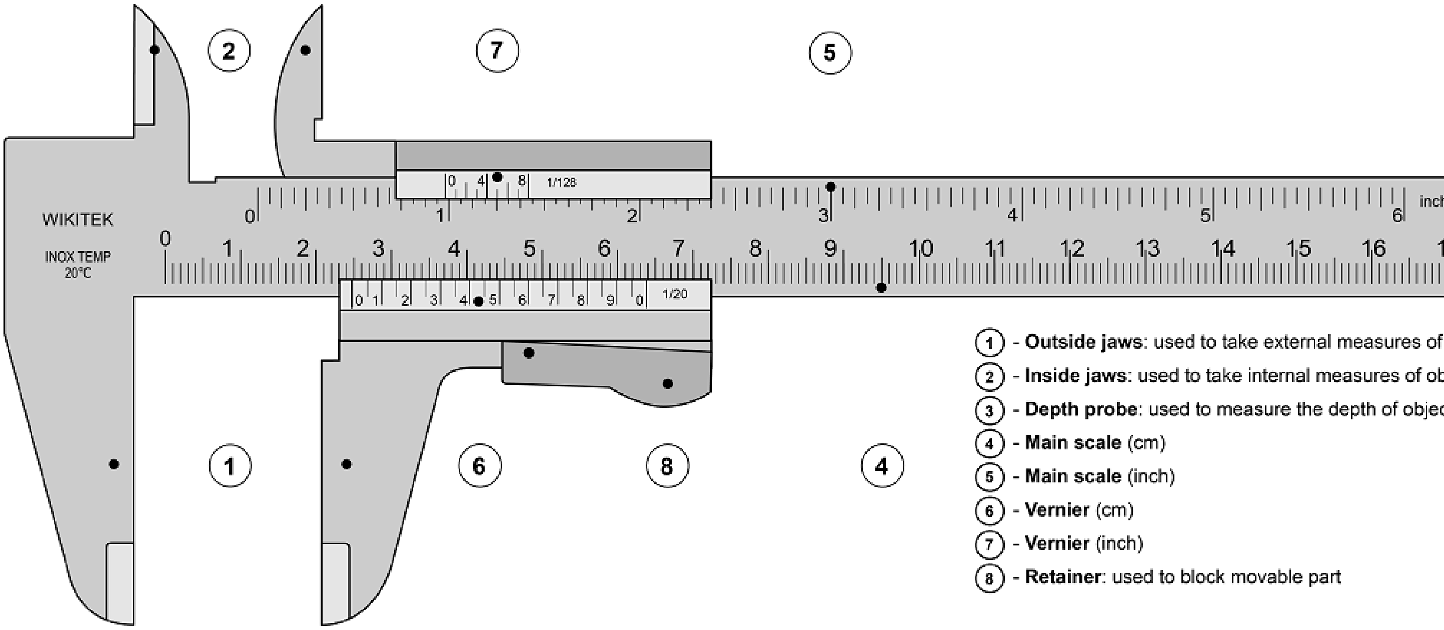
(a) The meter Rule

The accuracy of the measuring instrument depends on the smallest unit it can possibly measure. We shall use the instruments for measuring lengths to explain this concept of accuracy. Take an example of a meter rule in fig 1.1 the total length it can measure is 1m which is subdivided into 1000 partitions each 1mm long. Thus the smallest length a meter rule can possibly measure is a millimeter. This means that the accuracy of a meter rule is 1 part out of 1000 parts that make the whole.



(b) The vernier calipers

The Vernier caliper is the measuring device for determine inner and outer diameter of hollow objects like test tubes, pipes etc. It has got two scale the main scale and vernier scale. The main scale reads up to one decimal place where as vernier scale reads up to the second decimal point of centimeter. Fig 2.2 represents the schematic diagram of vernier calipers.



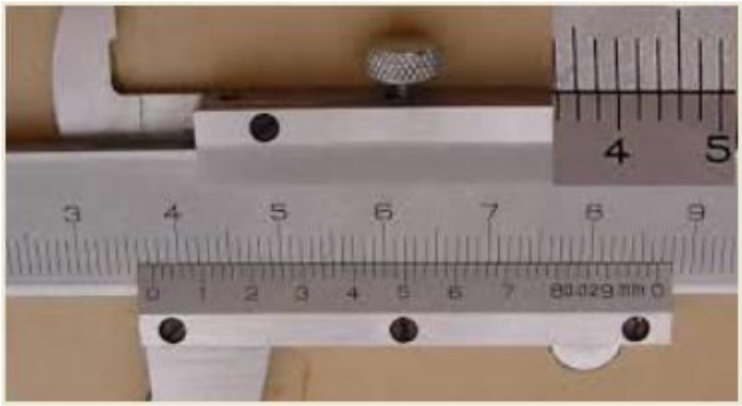
How to operate it;

Before taking the measurements, the gap between fixed outer jaw and movable jaw and movable jaw is closed by pushing a roller. When the gap is closed, zero mark (first line) on the vernier scale must coincide with zero mark (first line) on the main scale not seen in the diagram. To measure inner diameter of the hollow objects, the inner jaws are pushed inside the object and the roller used to move the jaws apart until they touch the inner wall of the object. The screw is tightened to avoid accidental change in distance. The scales are read and values are recorded as;

d_1 = value on the main scale + Value on vernier scale

For the outer diameter, the outer jaws are opened and the object placed in between. After tightened the screw the scales are read once again and value recorded as;

d_0 =Value on main scale + Value on vernier scale



The smallest unit the vernier calipers can possibly measure is 0.01 cm or 0.1 mm. Thus the error that can arise as a result of using the device is therefore ± 0.01 cm. From the fig 1.3, the reading on the vernier scale is read ad recorded as $d = 3.85$ cm, that is the value of main scale is 3.58 cm and on the vernier scale is 0.05 cm

OR

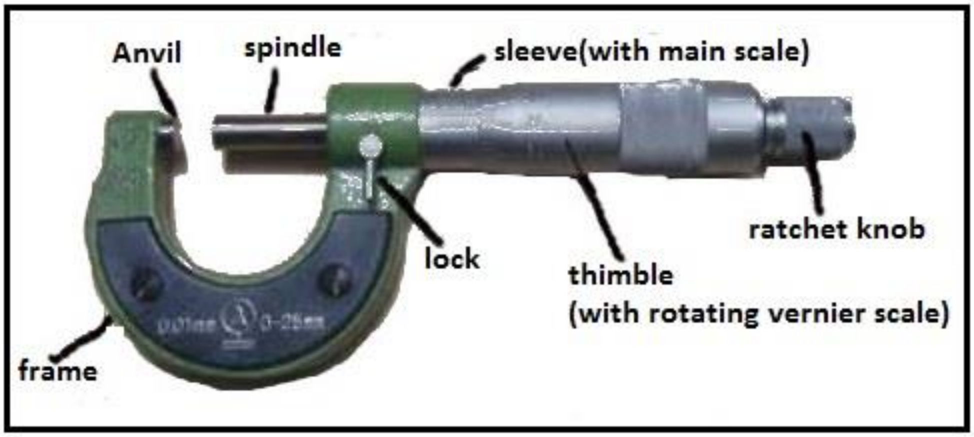
D =3.8 + 0.05

=3.85 cm

To read this value, the units cleared by zero mark of the vernier scale are first counted i.e. 3.8 and the value between 0.8 and 0.9 is obtain by looking for the line on the vernier scale that coincide with the line on the main scale and that is line number 5 on the vernier scale. Because this value is supposed to be the second decimal of a centimeter, it is recorded as 0.05 cm

(c) The micrometer screw gauge

Another very important measuring instrument is a micrometer screw gauge. It measures the length of the magnitude of 1 mm and less. There two scales on the device the sleeve scale and thimble scale as shown in fig 1.4



A micrometer screw gauge is used in measuring diameters of the wires thickness of the metal sheets, diameter of ball bearings and other tiny lengths. Before using it the gap between anvil and spindle has to be closed to check for zero error. An object to be measured is placed between anvil and spindle. By means of ratchet to make sure the object is gently held. Stop screwing when the ratchet makes the crackle sound. The value of the diameter obtained is the sum of the readings from the sleeve scale and thimble scale i.e.

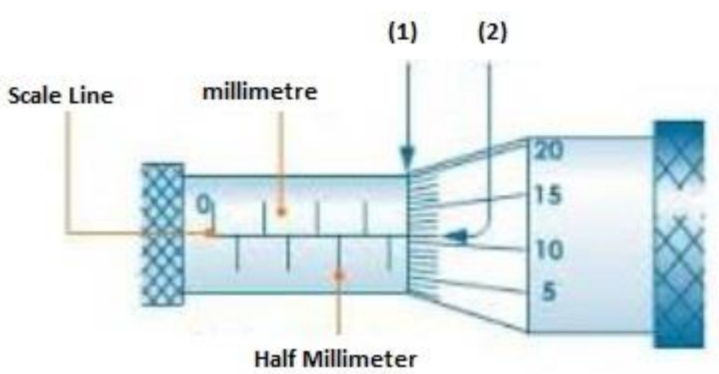
Figure 1.5 shows the procedure for reading and recording the value from two scales. The carries a millimeter scale along horizontal line on the upper side of line each interval represents 0.5 mm. The scale on the thimble has 50 units all round such that when the thimble turn once, it either advances or retreats by 0.5 mm along the sleeve. This means that;

50 divisions = 0.5 mm

I.e 50 × 1 division = 0.5 mm

∴ 1 division = 0.5/50 = 0.01mm

Thus the smallest unit that a micrometer screw gauge can possibly measure is 0.01 mm



From the fig 1.5 the reading can be recorded as follow:

d = $\left(6.00 + \frac{9}{50} \times 0.5\right)$ mm

= (6.00 + 0.09) mm

= 6.09 mm

This means that the diameter of wire

d = reading on sleeve scale + reading on thimble scale

The reading on the sleeve scale is found by counting the interval cleared by edge of the thimble and these are 6, meaning 6.00 mm. The value on the thimble is obtained by looking for the line on the thimble that coincide with the horizontal line on the sleeve which happen to be line number 9. These are 9 unit outs of 50 units round the thimble that make 0.5 mm

i.e. $\frac{9}{50}$ of 0.5 mm.

Absolute error

The absolute error is the magnitude of an error regardless of the sign. If d is the quantity then $\pm\delta d$ is an error positive or negative such that d = $d_0 \pm \delta d$ where d_0 is an actual value. The absolute error in d therefore is written as $|\delta d|$

Relative error

The absolute error alone shows the size of error but does not tell how serious the error is in relation to the actual value. By taking the ratio of error to the actual value we can see how many times an error is as big as the actual value. This is known as relative error is expressed in decimal number.

Relative error = $Absolute\ error/Actual\ error$

If $|\delta x|$ = absolute error in x and x_0 = actual value of x, then the relative error is given as:

$$\varepsilon_o = \frac{|\delta x|}{x_0}$$

Percentage error

The relative error expressed in decimal form multiplied by 100 gives percentage error. It is more convenient to express in percentage rather than as fraction or decimal number. The percentage error is written as

$$|\delta x|/x_0 \times 100$$

Calculation of errors

In experiment the measurement is done to more than one parameter and the values it obtained are substituted in a mathematical expression the gives the relationship between the parameter involved. The parameters concerned may require the same measuring device or different devices such as a meter rule and a micrometer screw gauge or a thermometer and a stopwatch. As we have seen above these instruments have differed accuracies and therefore contributing different errors in the values obtained. Using the values in a formula, the result is likely to contain a compounded error. Therefore, it is important to find the ways of calculating the error in the result. We are going to look; the operations such as addition, subtraction, multiplication and division of errors. Fin; we shall deal with compounded error in expressions involving indices.

(a) Addition of errors

Consider two quantities x and y given as x = $x_0 \pm \delta x$ and y = $y_0 \pm \delta y$ respectively.

Their sum s is s = x + y. We would like to find an error δs in terms of δx and δy. The procedures are as follow:

The range of *x* is $x - \delta x < x < x + \delta x$

The range of y is $y - \delta y < y < v + \delta y$

The maximum possible sum is $S_{max} = (x + \delta x) + (y + \delta y)$

The minimum possible sum is $S_{min} = (x - \delta x) + (y - \delta y)$

The absolute error in S is $|\delta S| = 1/2[S_{max} + S_{min}]$

$= 1/2[(x + \delta x) + (y + \delta y) - (x - \delta y) - (y - \delta y)]$

$= 1/2[2(\delta x + \delta y)]$

$= \delta x + \delta y$

From the results, the absolute error in the sum of two quantities is S the individual error in those quantities. Therefore the sum

$$S = S_0 \pm \delta S$$

Where S0 = ($x_0 + y_0$) the sum of the actual values of x and y.

We can easily find the relative error and the percentage error in S as

$\varepsilon_{0} = |\delta s/s|$ and âƒ' $\delta p/p_0| \times$ 100

(b) Subtraction of error

To subtract is to find the difference. Taking the same quantity x and y, let the difference between them be d = x-y; the error in d is found as follows:

The range of x and y are $x_0 - \delta x < x < x_0 + \delta x$ and $y_0 - \delta y < y < y + \delta y$ respectively.

The maximum possible difference $d_{max} = (x_0 + \delta x) - (y_0 - \delta y)$

The minimum possible difference $d_{min} = (x_0 - \delta x) - (y_0 + \delta y)$

The absolute error in d

$$|\delta d| = 1/2[d_{max} - d_{min}]$$

$$= 1/2[(x_0 + \delta x) - (y_0 \delta y) - (x_0 \delta x) + (y_0 + \delta y)$$

$$= 1/2[2(\delta x + \delta y) = \delta x + \delta y$$

$$\therefore |\delta d| = \delta x + \delta y$$

In far as addition and subtraction of quantities are concerned, error are always added not subtraction.

(c) Multiplication of errors

For quantities x and y consisting of errors δx and δy respectively multiplied together, the product p contains an error δp found as follows:

The product of x and y is $p = xy$

The maximum possible product p_{max} = $(x_0 + \delta x)(y_0 + \delta y)$

$= x_0 y_0 + x_0 \delta y + y_0 \delta x + \delta x \delta y$

The minimum possible product p_{min} = $(x_0 - \delta x)(y - \delta y)$

$= x_0 y_0 - x_0 \delta y - y_0 \delta x + \delta x \delta y$

The absolute error in p therefore is

$|\delta p| = 1/2[p_{max} - p_{min}]$

$= 1/2[2(x_0 \delta y + y_0 \delta x)]$

$= x_o \delta y + y_0 \delta x$

The relative error in p, $\delta p/p_0$âf' $= (x_0 \delta y + y_0 \delta x)/x_0 y_0 = \delta x/x_0 + \delta y/y_0$

(d) Division errors

If y is divided by x, a quotient q is formed, that is q $= y/x$. The results is supposed to be represented as q = $q_0 \pm \delta q$. The error in q is found by the following procedures:

The range of x and y are $x_0 - \delta x < x < x_0 + \delta y$ and $y_0 - \delta y < y < y_0 + \delta y$

The maximum possible quotient, $q_{max} = (y + \delta y)/(x - \delta x)$

The minimum possible quotient, $q_{min} = (y - \delta y)/(x + \delta x)$

The absolute error in q is therefore,

$|\delta q| = 1/2\,[q_{max} - q_{min}]$

$= 1/2[(y + \delta y)/(x - \delta x) - (y - \delta y)/(x + \delta x)]$

Take (x-$\delta x)(x+\delta x)$as LCM multiply we have;

$|\delta x|$ = $1/2[(2x\delta y+2y\delta x)/(x^2-(\delta x)^2$)

As $\delta x \rightarrow 0, (\delta x)^2 \rightarrow 0$

$\therefore$ $|\delta q|$ =$1/2[(2x\delta y+2yx)/x^2$

The relative error can be calculated as

âƒ'$\delta q/q_0$âƒ' = $\delta x/x_0$+$\delta y/y_0$

Example 1: Two quantities x and y have values (10$\pm 0.01)$ and (5$\pm$0.01) respectively. F int the absolute error, relative error and percentage error in their;

(a) Sum S

(b) Difference d

©Product p

(d) Quotient q = y/x

Solution

(a) Given x = $\pm 0.01)$ *and* $y = 5 \pm 0.01$

For the sum, âƒ'δs | = âƒ'$\delta x+\delta y$ |

$\delta x = 0.01$ and $\delta y = 0.01$

$\delta s = 0.01 + 0.01$

=0.02

The relative error in s is âƒ'$\delta s/s_0$âƒ' where s_0 = (x_0+y_0),x_0 = 10 and y_0= 5

$\therefore$ |$\delta s/s_0$âƒ' = 0.02/15 =0.0013

The percentage error is

âƒ’ $\delta s/s_0$ âƒ’ ×100 = 0.0013 ×100

= 0.13%

(b)For the difference âƒ’ δd | = $\delta x + \delta y$

= 0.01 + 0.01

= 0.02

Relative error in d is âƒ’ $\delta d/d_0$ âƒ’ where d_0 = ($x_0 - y_0) = 10 - 5 = 5$

âƒ’ $\delta d/d$ | $=$ 0.02/5 =0.004

Percentage error is

âƒ’ $\delta d/d_0$ âƒ’ × 100 = 0.4%

(c) For the product p = $p_0 \pm \delta p \; where \; p_0 = x_0 y_0$

The absolute error âƒ’ δp | = $x_0 \delta y + y_0 \delta x$

= 10(0.01) +5(0.01)

= 0.10 +0.05

= 0.15

The relative error in p is

âƒ’ $\delta p/p_0$ | $= 0.15/50$ = 0.003

The percentage error in p

| $\delta p/p_0$ âƒ’ × 100 = 0.003 × 100

= 0.3%

(d) For the quotient, q = $q_0 \pm \delta q$ where $q_0 = y_0/x_0$

The absolute error in q is âƒ’δq | = ($x_0\delta y + y_0\delta x)/x^2$

= 10(0.01) +5(0.01)/10^2 =0.015/100 = 0.0015

The relative error in q is

âƒ’$\delta q/q_0$ = 0.0015/0.5 = 0.003

The percentage errors in q is therefore

âƒ’$\delta q/q_0$âƒ’ × 100 =0.003 × 100

=0.3%

Use of natural logarithms in error analysis

The calculations of errors we have seen above are used in simple cases with only two parameters to deal with. There are situations where the mathematical expressions into more than two variables raised to some powers in which case .using the method above. may not be adequate to produce the answer required. Application of logarithms may / quicken the process towards the answer. As an example, consider the following problem:

Example 1.2: A quantity Q is connected to another quantity p, r, and η by the expression

Q = ð□œ«pr^4/8lη . Obtain an expression for

(i) Relative error in Q

(ii) Absolute error in Q

Solution

Given that Q = ð□œ«pr^4/8lη

Take natural logarithms on both sided of the equation

$\ln Q = \ln(\Pi p r^4$/8lη)

$\ln Q = ln\,\Pi/8 + ln\,p + 4ln\,r - ln\,l - ln\,\eta$

Differentiating this we get

$$\delta Q/Q_0 = {}_0\pm \delta p/p_0 \pm 4\delta r_{/}r_0 \pm \delta l_{/}l_0 \pm \delta \eta_{/\eta}$$

(i) The relative error in Q is

$$\delta Q/Q_0 {}_= \pm \delta p/p_0 \pm 4\delta r/r_0 \pm \delta l/l_0 \pm \delta \eta/\eta_0$$

(ii) The relative absolute error in Q is

$$|\delta Q/Q_0| = |\delta p/p_0| + 4|\delta r/r_{0_{\hat{a}f}}’+ |\delta l/l_{0_{\hat{a}f}}’+ |\delta \eta/\eta_{0_{\hat{a}f}}’$$

The absolute error in Q is

$$|\delta Q| = [|\frac{\delta q}{p_0}|] + 4|\frac{\delta r}{r_0}| + |\frac{\delta l}{l_0}| + |\frac{\delta \eta}{\eta_0}|]\ Q_0$$

Exercise

1. State the accuracy of each of numbers below

i/7 ii/3.2 iii/5.00 iv/11.03

2. (a) give the difference between error and mistake.

(b)Mention the types of errors and their sources.

(c) Explain how you would minimizing the magnitude of random error in an experiment.

3. (a) State the smallest unit each of instruments below can possibly measure and mention possible errors

(i) The meter rule

(ii) The vernier caliper

(iii) The micrometer screw gauge

(b) A box P contain smaller packages p_1, p_2 and p_3 weighing (12$\pm$0.2) kg, (20$\pm$0.35) kg and (16$\pm$0.25) kg respectively.

(i) State the range off mass of each package

(ii) Calculate the absolute error in the mass P

(iii) What is relative error in mass P

(v) Determine the percentage error

4. (a) Imagine you are doing an experiment on a simple pendulum to determine the value of acceleration due to gravity g at your location. What are possible errors are like to affect your result and what precaution will you take?

(b) Find the maximum possible error in the measurement of force on the object of mass M moving with velocity V ALONG THE CIRCULAR PATH OF RADIUS r given that M, V, and r are (3.5$\pm$0.1) kg, (20$\pm$1) ms^{-1}, and (12.5± 0.05) m respectively.

5. During the experiment to determine acceleration due to gravity by using simple pendulum, the length l of the pendulum and periodic time T were measured an recorded as (120$\pm$0.1) cm and (2.25$\pm$0.01) s respectively. Given the relationship between l, g and T is; T = 2$\Pi\sqrt{\frac{l}{g}}$,

Calculate;

(i) Absolute error in g

(ii) The percentage error in g

6. (a) What is the meant by relative error?

(b) A quantity Q is expressed in terms of other quantities F, A, v and l by the equation

Q = $\sqrt[4]{F/A/v/l}$

Where F= 5$\pm$0.21, A= 0.05$\pm$0.005, $v = 10 \pm 0.06$ and $l = 3.6 \pm 0.25$

Calculate the percentage error in Q

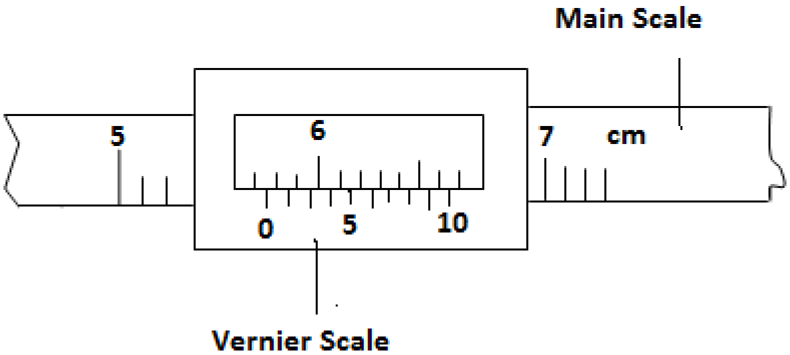
7. (a) Distinguish between random error and systematic error

(b) Four trials in experiment of the diameter of the wire gave the diameter values 0.2 mm, 0.25 mm, 0.23 mm and 0.22 mm.

(i) What is the diameter of wire if each reading contains an error of $\pm$0.001?

(ii) Calculate the absolute relative error in the cross-section of the wire.

(c) Fig 2.6 shows parts of vernier calipers adjusted during the experiment



(i) Read and record the value registered by the instrument

(ii) State the range and the accuracy of the value you have recorded

8. A rectangular sheet of metal measures 47.21 cm × 23.79 cm. If the error in each dimension is $\pm$0.01 cm

(a) State the range of each dimension

(b) Calculate the percentage error in its

(i) Perimeter (ii) Diagonal (iii) Area

9. (a) In an experiment to determine velocity of liquid, the data bellow were collected:

Pressure at inlet = (4000.2) pa

Pressure at outlet = (2000.2) pa

Length of capillary tube L = (100.001) m

Diameter of capillary tube d = (40.1) cm

Discharge Q = (5000.2) liter/sec

Assume Poiseulli's formula apply find relative error in viscosity.

(b) (i) What is the meaning by the physical quantity?

(ii) The maximum velocity of the particle moving with simple harmonic motion can be determined from……

$$V_{max} = \sqrt{\frac{KA}{M}}$$

Where A, M, and K are amplitude, mass and constant, find dimension of K

NEWTON'S LAWS OF MOTION

4.1 Motion

Motion occurs when a body covers distance with time. The quantity of motion is the product of mass of a body and the speed at which it moves. Momentum represents the quantity of motion and it is the time rate of change of momentum that determines which keeps-the body moving. Through ages man has observed motions of various bodies both in space and on smooth and rough planes. Out of these observations some laws that govern motion in general. It was Isaac

Newton who formulated the three laws we now call Newton's, laws of motion. By applying these laws with certain conditions motion problems can be solved.

4.2 Laws of motion

(a) First law of motion

"A body remains in a state of rest or uniform motion in a straight line unless acted upon by the external force".

This law is sometimes referred to as the law of inertia. Inertia means reluctance of a body to be set into motion or to stop if already moving. This inertia depends very much on the mass the body possesses? A body with less mass has small inertia and vice versa. On the other hand mass is the measure of inertia of a given body. The greater the mass of the body the less the acceleration when an external force is applied.

Acceleration $\propto 1/mass$

a $\propto 1/m$

If a force *F* is applied on a body of mass M_1, and then on another body of mass M_2 as shown in figure, the corresponding accelerations a_1 and a_2 are related by

$a_1/a_2 = 1/m_1 / 1/m_2$ OR $a_1/a_2 = m_1/m_2$

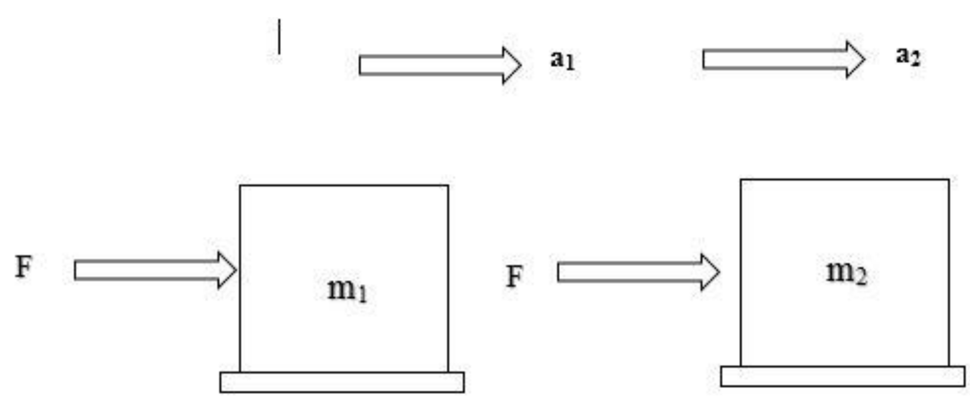


Figure 4.1

Mass is an inherent property of a body. It is independent of its surroundings and the method used in measuring it. Under the first law if there is no external force, a Stationary body is supposed to remain in one position forever; likewise a body in motion continues moving along the same direction indefinitely. However in reality when a body is pushed along a horizontal plane, it just moves for a short time and stops. If the plane is polished and the same body pushed over it, it moves for a little longer time before coming to rest.

(b) Second law of motion.
"The rate of change of momentum of a body is directly proportional to the external force applied and takes place in the direction of the force" We can use the above statement to derive an expression for the force that keeps the body accelerating or decelerating. To do this let us consider the momentum and momentum change of the body under the influence of the force.

Momentum

Momentum is the product of mass of a moving body and the velocity at which it moves
Momentum = mass x velocity

$$p = mv$$

The unit of momentum is a $kgms^{-1}$

Momentum change

For a solid body, what can possibly change during motion is the velocity because the mass remains constant. If initially a body moves at a velocity V and after sometime t the velocity changes to v then its momentum changes from initial value *P,* to the final value

P_f as illustrated below

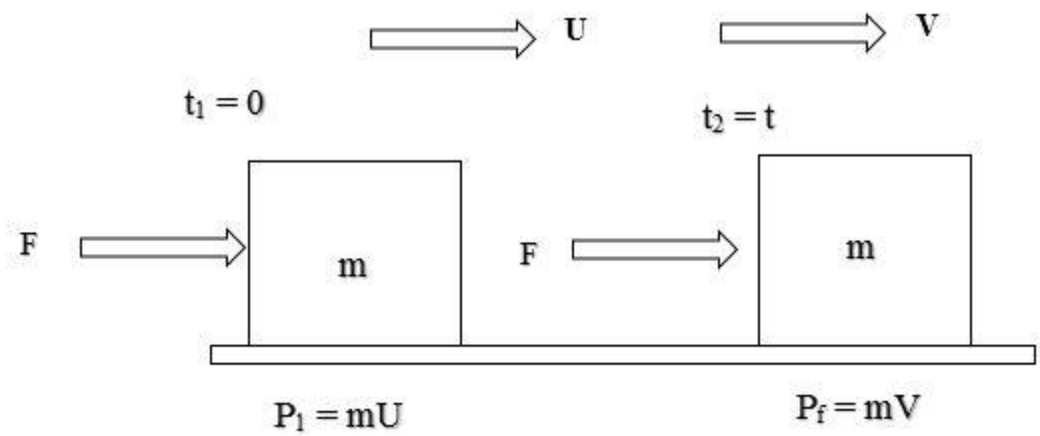


Figure 4.2

The change in momentum = Final momentum - Initial momentum $\Delta p = p_f - p_i$

The time rate of change in momentum = Change in momentum/Time interval = $\Delta p/\Delta t$

It is this rate of change of momentum which is proportional to the applied force F. $^{\Delta P}/_{\Delta t}\, \alpha\, F$

$^{\Delta P}/_{\Delta T} = kF$

Since $^{\Delta P}/_{\Delta t} = \frac{P_f - P_i}{t_2 - t_1} = \frac{mv - mu}{t - 0} = m\left(\frac{v-u}{t}\right)$ and $\frac{v-u}{t} = a$ initially $t_1 = 0$

The unit of force is **Newton** by definition:
A Newton is a force that makes a mass of 1kg to move with an acceleration of 1 ms^{-2}

Which means that, when m = 1kg and $a = 1\ ^{m}/_{s}\ ^{-2}$, the force F = 1N
Substituting for m, a and F we have

$$1kg \times 1ms^{-2} = k \times 1N$$

$$1kg \times 1ms^{-2} = 1N$$

And therefore $1N = k \times 1N$

$$k = 1$$

Thus the expression for the force is given by

$$F = ma \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4.1)$$

(c) **The third law**
For every action there is an equal and opposite reaction
Fig 4.3 gives an illustration on the third law of motion. A body rest on the top reaction of the table exerts the weight W in return the table through the point of contact supports it by exerting a reaction force R equal to that of W only that the two are acting in opposite direction.

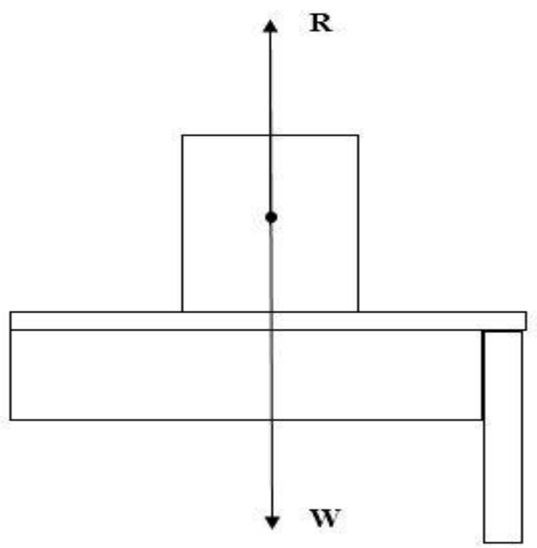


Figure 4.3

4.3 Implications of Newton's laws of motion
The laws we have seen above can be used to explain some of the occurrences that we encounter in daily life. Things like friction, impulse, weightlessness, collisions, motion in fluids and so on. We shall consider few of them in general,In common cases and try to use the appropriate laws to explain the situation.

(a) Friction and Frictional force

Friction is the rubbing between two bodies in contact when moving relative to one another or stationery. If the surfaces in contact are rough there is opposition that takes place giving rise to the friction force. To illustrate this consider a block resting on a plane as in fig 4.4. The molecular moles sticking out of each surface interlock when the surfaces come in contact. The extent of interaction between these protrusions depends on the weight a block exerts on the plane. When the action force *F* is applied to the body with an intention of pulling it, the obstruction by the interlocking protrusions translates into a friction 'f' force / as in fig 4.4.

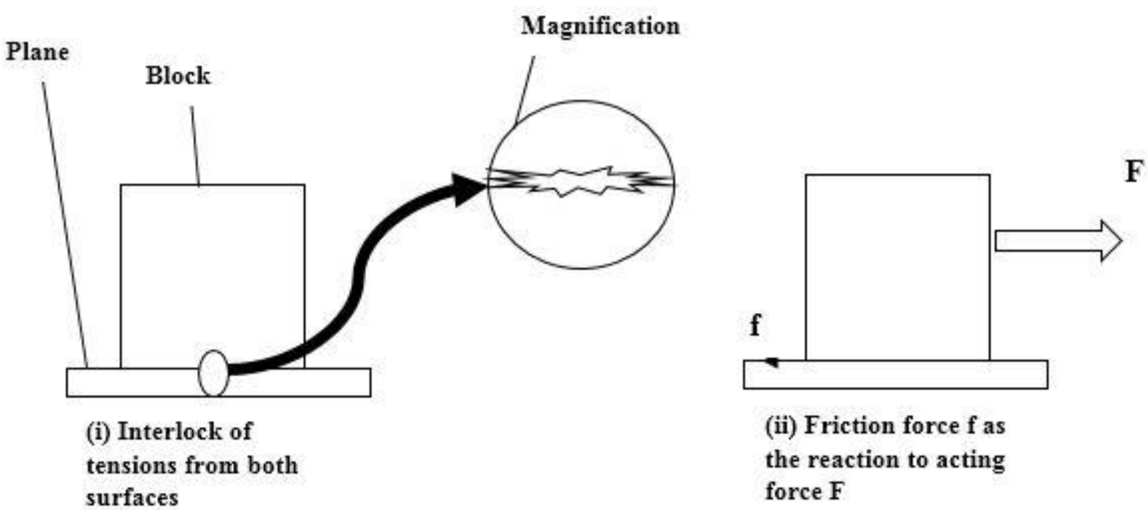


figure 4.4

To maintain the motion of the block, the applied force F must be greater than factional force "ƒ" such that the difference *(F - ƒ)* produces the acceleration *'a'*

$$F - f = ma$$

This means the applied force partly goes into overcoming frictional force and partly accelerates the body. The difference *(F - ƒ)* is sometimes referred to as the effective force that causes acceleration. If the molecular protrusions are crashed and leveled, the surfaces become smoothed and the opposition to motion is drastically reduced to the extent that very little force is required to keep the body moving. In addition if the surface is polished, frictional force is further minimized to the level where it can be regarded as smooth. In solving mechanics problems the terms smooth or frictionless are used to mean that the frictional force is zero. When there is no frictional force the applied force wholly goes into accelerating the body.

(ii) Relationship between friction force and normal reaction

There is relationship between the friction "f " and the normal reaction R. The more is the reaction force , the greater the force of friction in another words friction force is direct proportion to the reaction. The ratio of the friction force to the normal reaction is called the coefficient of friction.

Coefficient of friction (μ) = $friction\ force/normal\ reaction$

$$\mu = f/R$$

From which, $f = \mu$

(iii) Coefficient of static friction μ_s

A body resting on the plane does not begin to move until the external force is applied to overcome friction force. The maximum friction force that is to be overcome by applied force is sometimes referred to as limiting force f_s

$$f_s = \mu_s R$$

One of the methods for determining the value of μ_s is shown in fig 4.5.

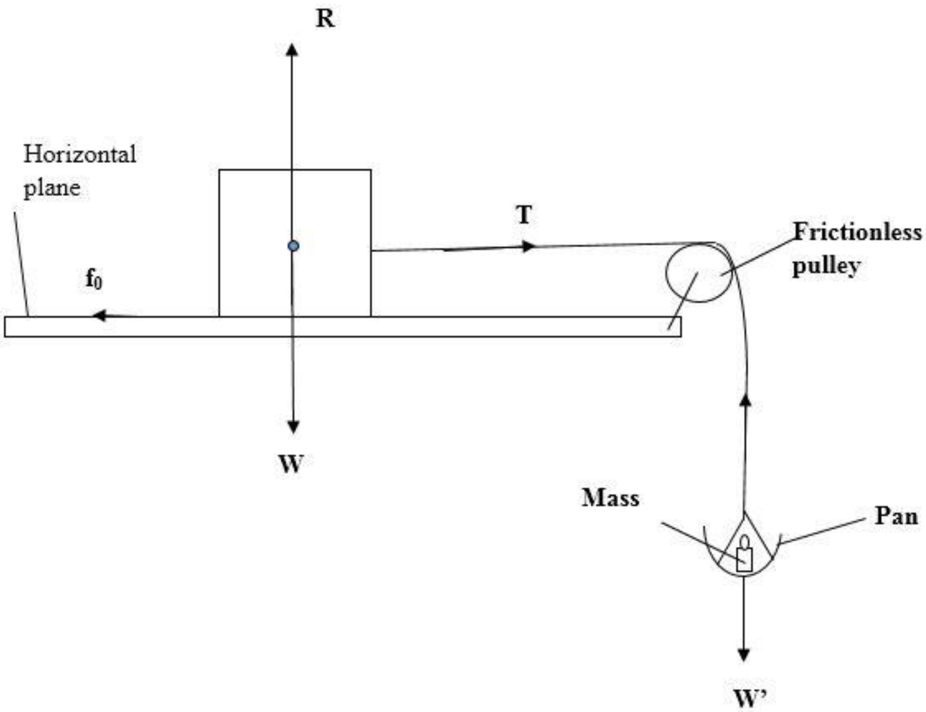


figure 4.5

In this experimental a block of known mass is placed on the horizontal plane whose coefficient of friction is sought. The plane consists of a frictionless pulley at one end. A string with a pan at one end is tied to the block and made to pass over the pulley. Before adding anything on the pan, the only forces on the plane are the weight "W" of the block and the reaction R from the plane. As soon as the mass is added to the pan, the tension "T" created in the string tends to pull the block and at the same time the frictional force takes charge. Initially the weight "*W*" arising from the mass on the pan may not be strong enough to move the block but as more and more masses are placed on the pan the frictional force grow bigger and bigger. The time comes, when the frictional force can no longer hold the block and therefore block just begins to slide. It is at this moment we say that the tension "T" is just equal to the frictional force "f_s". This is the maximum frictional force the plane can offer in preventing the block to slide over it, By recording the total mass on the pan, the magnitude of the weight W' can be found and from the equation

$$f_s = T$$

Where T = W' and W' = m'g the numerical value for μ_s is evaluated.

$$\mu_s = m'g/R$$

Definition

The coefficient of static friction is the ratio of the frictional force when the body is on the average slide.

Once the block has gain momentum, the coefficient of friction is now referred to as the coefficient of kinetic friction μ_k and its value is less than that of μ_s. Table 4.1 shows the value of μ_s and μ_k for the surface of different materials.

Table 4.1: coefficients of friction for some materials

Surface in contact	μ_s	μ_k
Steel on the steel	0.74	0.57
Copper on steel	0.61	0.47
Aluminium on steel	0.53	0.36
Rubber on concrete	1.00	0.80
Wood on wood	0.25-0.50	0.20
Glass on glass	0.94	0.40
Ice on ice	0.1	0.03

(b)Motion on the horizontal plane

Here we shall consider a body sliding along rough plane as a general case. Being rough, the plane offers frictional force to the body on sliding it. If the body is just projected with initial velocity u , its velocity keep on decreasing until it come to the halt due to the frictional force that opposes motion persistently as in fig 4.6

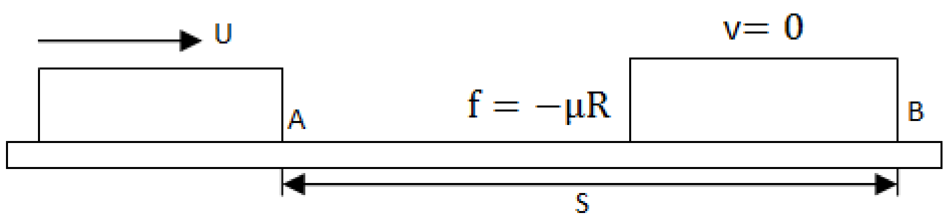


figure 4.6

The question arising from this example would be;
How far does the body go?
What is the deceleration?
What is the frictional force?

How far does the body go?
To find how far the body reaches we can start with the third equation of linear motion $v^2 = u^2 + 2as$ in which the acceleration $a = -\mu R/m$

such that; $s = v^2 - u^2/2a$ = $0^2 - u^2/-\frac{\mu R}{m}$ = $mu^2/\mu R$

Where R = mg and therefore S = mu^2 / μmg or $u^2/\mu g$

(b) Motion on a rough inclined plane

Consider a block projected up the inclined plane with an initial velocity u, as it climbs, there are two forces which opposes the motion, one being frictional force and other is the component of weight parallel to the plane as shown in fig 4.7

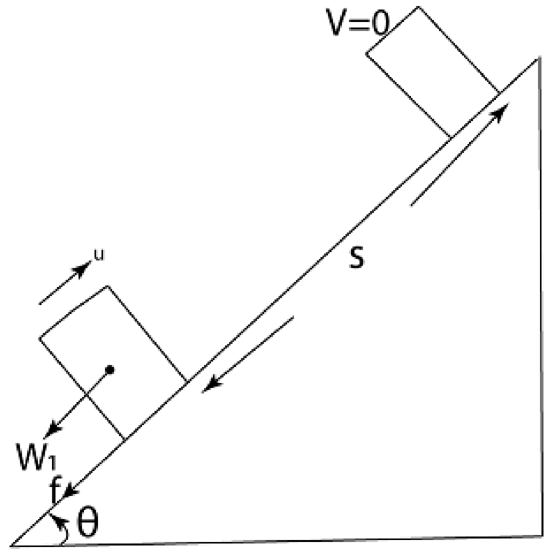


figure 4.7

The two forces opposing the motion simultaneously bring the block to a standstill somewhere along the plane after traveling a distance S. To find the magnitude of the distance covered we can use the same method as in the case of the horizontal plane only that the component of the block has to be considered. The total force opposing the motion is

$$F = W_{\shortparallel} + f$$

Where F = ma, $W_{\shortparallel} = mgsin\theta$ and $f = \mu R = \mu mg$

$$\therefore ma = mgsin\theta + \mu mg$$

a = gsin$\theta + \mu g$

a = $(sin\theta + \mu)g$

From the third equation of linear motion,

Note:

Motion in viscous fluids.

1.

$$v^2 = u^2 + 2(-a)s$$

Where v = 0 (given), a = ($\mu + sin\theta)g\ (calculated)$

$$0^2 = u^2 - 2(\mu + sin\theta)gs$$

From which $s = u^2/2(\mu + sin\theta)g$

If the inclined plane is smooth i.e. $\mu = 0$, there is only one opposing force and that is $W_{\shortparallel}$ and the distance becomes

$$s = \frac{u^2}{2gsin\theta}$$

(C)Motion of connected bodies

(i) On smooth horizontal plane

Fig 4.8(a) shows two bodies A and B of masses m_1 and m_2 respectively connected by the light in extensible string that passes over a frictionless pulley such that the body A rest on the smooth surface and B hang freely.

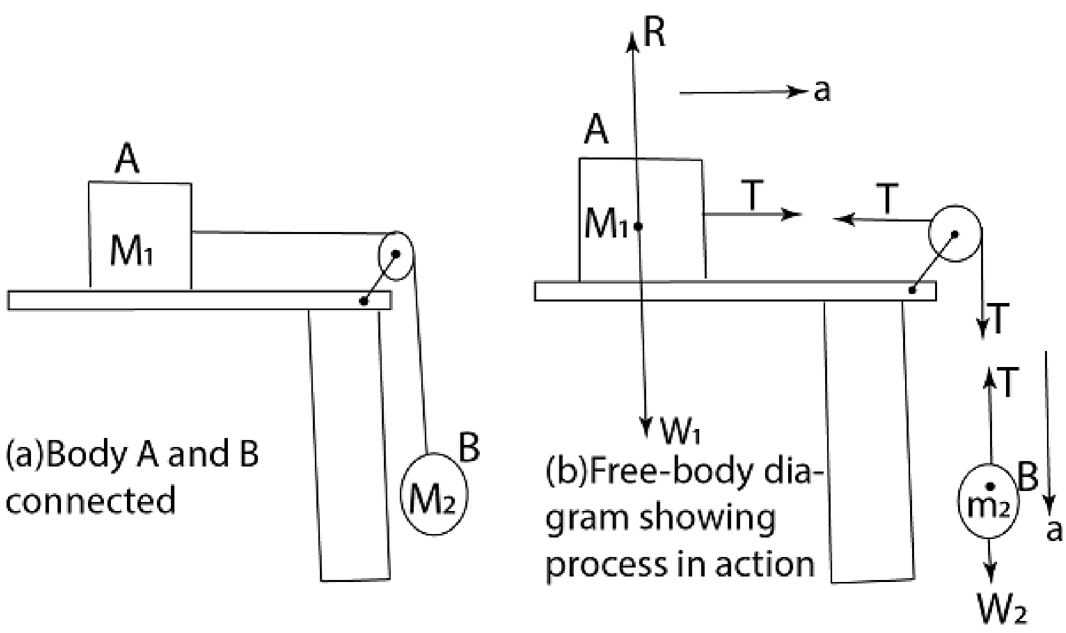


figure 4.8

As soon as the system is let free and given that $m_2 > m_1$, body B falls under its own weight W_2 pulling body A via a string and both move with the common acceleration as shown in fig 4.8(b). The tension in the string is uniform throughout. Using the free-body diagram it is easy to determine both the tension and the acceleration of the system. The process is as follows:

Body B falls vertically downwards because $W_2 > T$

$\therefore W_2 - T = m_2 a$

But $W_2 = m_2 g$

$\therefore m_2 g - T = m_2 a$ ………………(1)

Body A moves horizontally in direction of T without opposing because the surface is smooth

$T = m_1 a$ ……………………… (2)

Solving equations (1) and (2) we have

$m_2 g = (m_1 + m_2)a$

$\therefore a = m_2 g / m_1 + m_2$

Substituting for a in equation (2) we get

$T = [m_1 m_2 / m_1 + m_2]g$

(ii)On rough inclined plane

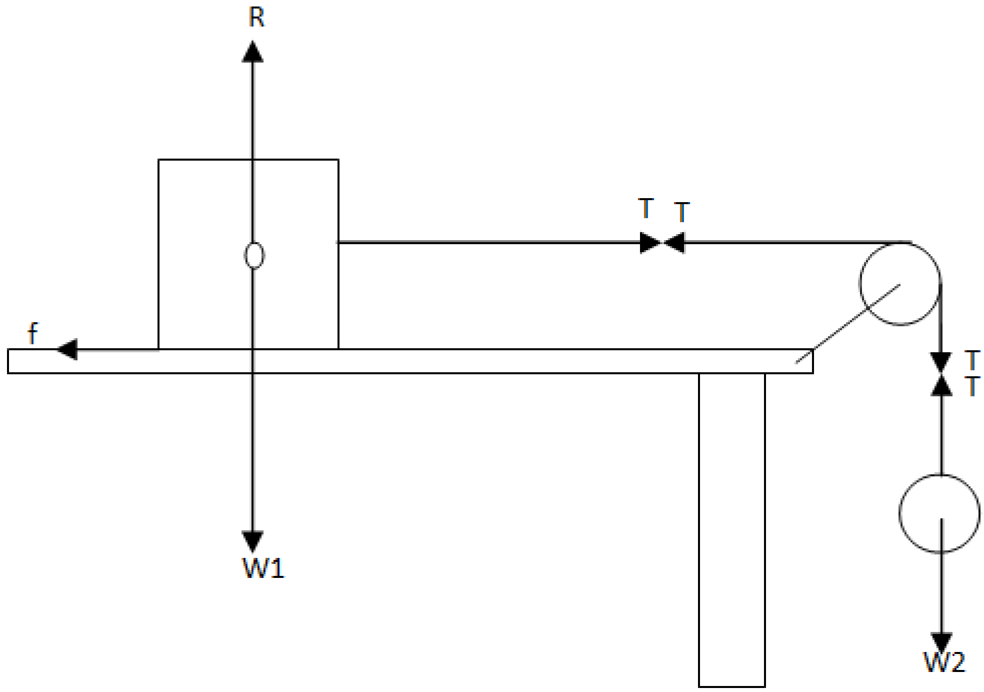


figure 4.9

For a rough plane frictional force play part by opposing the motion of body A as the system is free to move. Equation (i) above remains the same $m_2g - T = m_2a$…………………(3)

Due to the frictional force equation (2) becomes

$T - \mu R = m_1a$

But $R = m_1g$

$\therefore T - \mu m_1g = m_1a$……………………(4)

Add (3) and (4)

$(m_1 - m_2)g = (m_1 + m_2)g$

$\therefore a = [{}^{m_2 - \mu m_1}/_{m_1 + m_2}]g$

Substitute for a in equation (4) and simplify $= [^{1+\mu}/_{m_1} + m_2]m_1m_2$

T

(iii) Connected bodies on inclined plane

Consider a situation where two bodies connected by an extensible string that passes over a frictionless pulley rest on two planes inclined at an angle θ and $\emptyset$ to the horizontal as shown in fig 4.10(a)

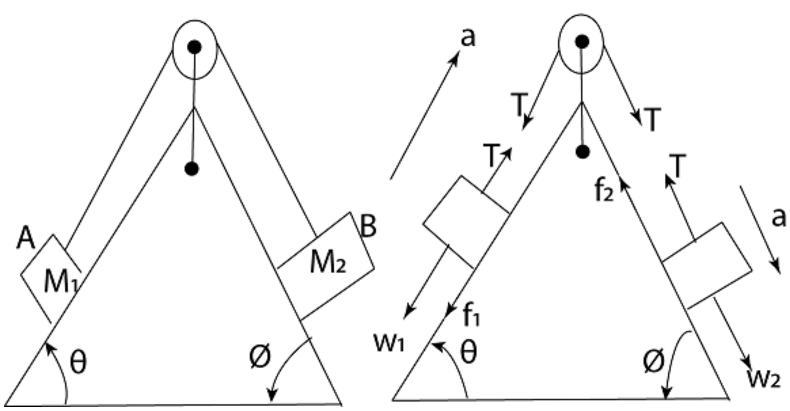


Figure 4.10 (a) Figure 4.10 (b)

When let it free, the bodies may move as in fig 4.10(b) provided $m_2 > m_1$ and the surface of inclined plane are alike i.e their coefficients of friction are equal. The forces involved are indicated their direction and as a results both tension T and acceleration a of the system can be found using similar procedures as in the case of horizontal plane. Since body A climbs the plane it implies that T $> (W_{1_\circ} + f_1)$ i.e

$T - (W_{1_\circ} + f_1) = m_1 a$

But, $W_{1_\circ} = m_1 sin\theta$ and $f_1 = \mu R_1 = \mu m_1 g$

$\therefore T - m_1(sin\theta + \mu)g = m_1 a$………(1)

Likewise since body B moves down the plane it means $W_{2_x} > (T + f_2) i.e$

$$W_{2_x} - (T + f_2) = m_2 \text{ a}$$

Where

$W_{2_x} = m_2 g sin\emptyset$ and $f_2 = \mu R_2 = \mu m_2 g$

$\therefore m_2 g sin\emptyset - \mu m_2 g - T = m_2 a$

Or

$m_2 (sin\emptyset - \mu) g - T = m_2 a$------------(2)

Add (1) and (2) to obtain $m_2 (sin\emptyset - \mu) g - m_1 (sin\theta + \mu) g = (m_1 + m_2) a$

$(m_2 sin\emptyset - m_1 sin\theta) g - \mu (m_1 + m_2) g = (m_1 + m_2) a$

$$a = \left(\frac{M_2 sin\theta - M_1 sin\theta}{M_1 - M_2} - \mu \right) g$$

Substitute for a in (2) and simplify to get

$$T = m_2 \left(m_1 sin\emptyset + m_2 sin\theta /_{m_1} + m_2 \right) g$$

(iv)Connected bodies on pulley

Again consider two unequal bodies of masses $m_1 \; and \; m_2$ connected by a light inextensible string that passes over a frictionless pulley as in fig 4.11

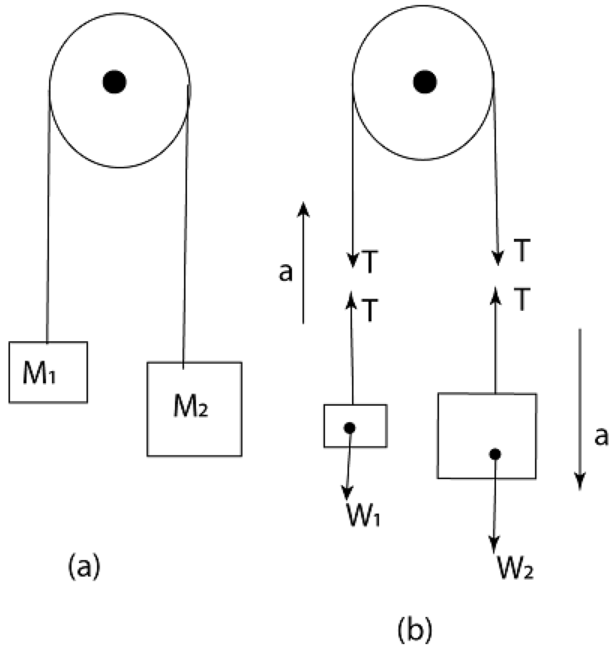


Figure 4.11 (a) **Figure 4.11 (b)**

In figure 4.1 1(a) the masses are mounted on a pulley with $m_2 > m_1$. When released, the weight of the larger mass overcomes that of smaller mass and the motion takes place as in figi4.1 l(b). Using the observation and reasoning we can determine the acceleration"*a*" of the system 'and the tension "*T*" in the string. For the mass moving downwards $W_2 > T$ and therefore;

For the mass moving vertically upwards $T > W_1$ and hence

$$W_2 - T = m_2 a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$T - W_1 = m_1 a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$W_2 - W_1 = (m_1 + m_2)a$$

Adding (1) and (2) we have

$$W_2 - W_1 = (m_1 + m_2)a$$

$$a = \frac{w_2 - w_1}{m_1 + m_2}$$

Since $W_1 = m_1 g$ and $W_2 = m_2 g$

$$a = \left({m_2 - m_1}/{m_1 + m_2}\right)g$$

From (1) $T = W_2 - m_2 a$

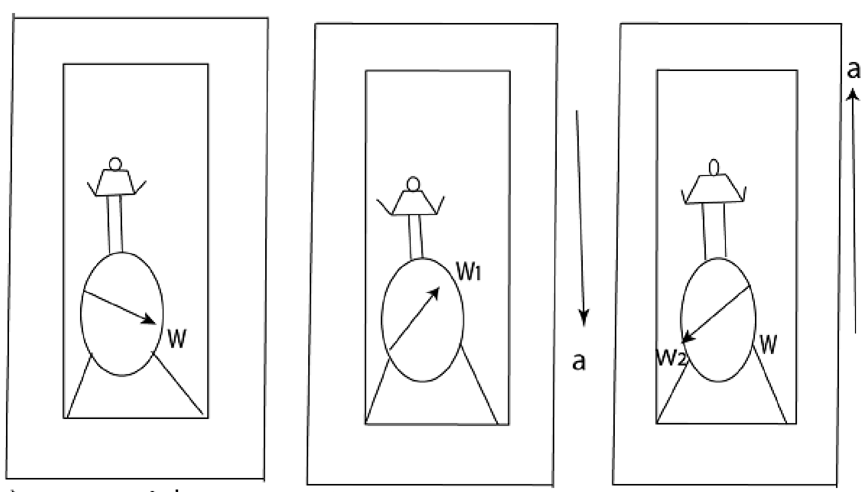
$$= m_2 g - m_2\left[{m_2 - m_1}/{m_1 + m_2}\right]g$$

$$\therefore T = 2\left({m_1 m_2}/{m_1 + m_2}\right)g$$

(v)Motion in a lift

If you have been in a lift, you may have felt a difference in your weight as a lift ascends or descends with constant acceleration. When ascending one feel heavier than normal by pressing hard on the lift floor and when descending, one becomes apparently lighter than normal by losing weight.

To get the idea of what happens consider a body of mass *m* on a pan of a weighing machine on the floor in a lift. Before the motion along the vertical plane begins, the weight of the body is taken to be normal, say *W* as in fig 4.12.



(a)Normal weight a body when the light is not accelerates

(b)The body appers to weight less os it accelerates downwards

(c)The body appers

Figure 4.12(a) (b) (c)

When a lift accelerates uniformly downwards, the scale of a weighing machine shows that there is a drop in weight by indicating a lower value W_1 as in fig 4.12(b) and when it accelerates upwards the machine indicates a higher value in weight *Was* in fig 4.12(c). To explain these observations, let us consider the body on the scale pan in each case separately.

In the body exerts the normal weight W on the pan and in return the pan exerts force of figure 4.12 (a) reaction R as shown in fig 4.13. Since the lift is at rest, $a = 0$ meaning that

$$W - R = 0$$

Figure 4

But $W = mg$ and therefore

$$R = mg$$

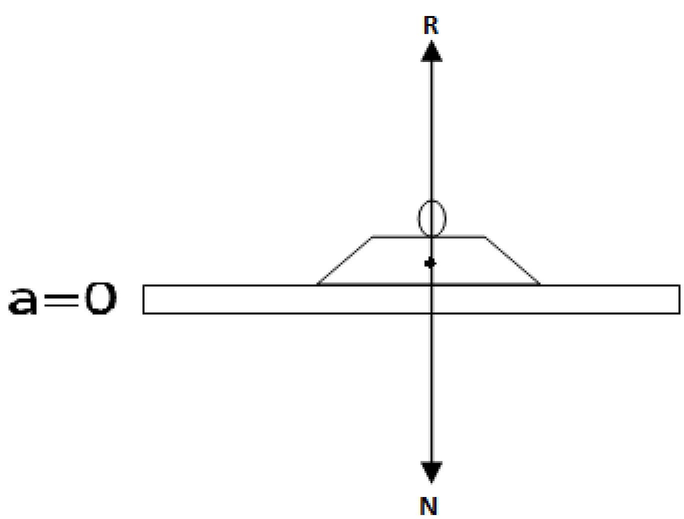
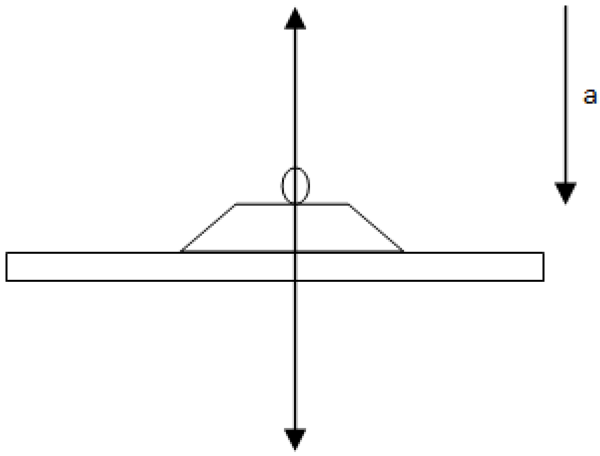


figure 4.13

In the lift accelerates uniformly vertically downwards, this causes the apparent loss in weight meaning that the weight W_1 indicated on the scale equals to the reaction R_1 such that $R_1 < R$ as in fig 4.14

Figure 4.14.



The apparent loss in weight as the lift goes down is

$$W - R_1 = ma$$

$$mg - R_1 = ma$$

$$R_1 = mg - ma$$

$$R_1 = m(g - a)$$

The reason for the apparent loss *in* weight in a body is that the surface on which the body is resting runs away or falls faster in such a way that the contact between; the pan and the body becomes lighter and hence less reaction from the surface. In fact if the lift falls with acceleration equal to that of gravity, the reaction on the body from the surface would be zero i.e. when a = *g, .R, = m(g - g)* = 0 and the pointer on the scale would indicate zero weight. Under such condition the body would appear weightless.

In figure 4.12(c), the lift accelerates vertically upwards causing the scale to indicate a larger weight W_2. The apparent increase in weight as the lift ascends arises from the fact that the pan on which the body sits tends to rise faster the body itself and therefore pressing harder underneath it and hence the reaction R_2 as in fig 4.15. The effect of the pan pressing hard on the body is the apparent gain in weight W_2 registered on the scale. The apparent gain in weight is given as

$$R_2 - W = ma$$

Or

$$R_2 - mg = ma$$

$$R_2 = m(g + a)$$

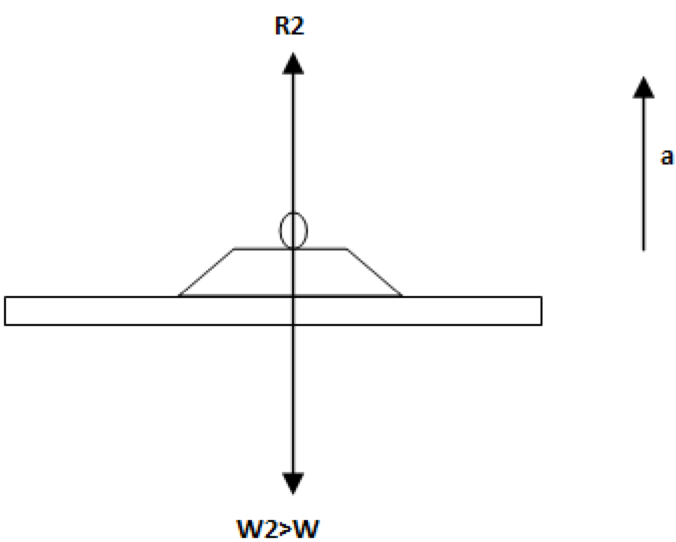


Figure. 4 .15

If the lift ascends with acceleration equals to that of the gravity, the reaction R_2 would double. That is if $a = g$, then R_1 =m $(g + g) = 2mg.$

4.4 Collision

A collision is the process in which two or more bodies suddenly smash into each other. An impact at the point of collision causes an impulse on each of the colliding bodies' results into change in momentum. There are mainly two types of collision.

a) Elastic collision.
b) Inelastic collision.

(a)Elastic collision

The collision whereby the colliding bodies take very short time to separate is known as elastic collision. In this kind of collision, both the momentum and kinetic energy are conserved. Fig 4.18 illustrates the collision of two spherical bodies with masses $m_1$and m_2 initially moving with velocities $u_1$and u_2 respectively where$u_1 > u_2$. *At* the point of collision, the rear body exerts a force F_1on the front body and at the same time the front body exerts an equal but opposite force $-F_2$ on the rear body. After collision the rear body slows down to, velocity $v_1$whereas the front body picks up the motion attaining velocity v_2 .

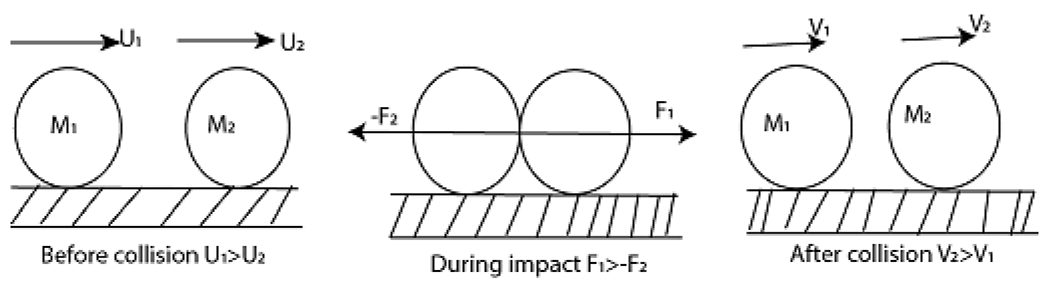


Figure 4 .17

Impulse,

The impulse of a force is the product of force applied and time interval remain in action, that is

$$Impulse = Force\ applied\ \times time\ interval$$

$$I = F\Delta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.25)$$

The unit of impulse is Newton-second (Ns). From the Newton's second law of motion

$$F = {mv - mu}/_t = {\Delta p}/_{\Delta t}$$

we have also seen that

And therefore$F\Delta t = \Delta p$. This means the impulse is equal to the change in momentum

$$I = mv - mu \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . (4.26)$$

This means that the impulse is a kilogram-meter per second ($kgms^{-1}$)

Principle of conservation of linear momentum:

"In system of colliding particles, the total momentum before collisions is equal to the total momentum after collision so long as there is no interference to the system"

From Newton's third law of motion, action and reaction are equal but opposite.

For example at the point of impact in fig 4.18

$$F_1 = -F_2$$

Since the action and reaction are taken at an equal interval of time Δt to remain in action each body experiences the same impulse that is

$$F_1\Delta t = -F_2\Delta t$$

Where F_1 causes change in momentum on m_2 and F_2 causes change in momentum on m_1 such that $F_1\Delta t = (m_2v_2 - m_2u_2)$ and $F_2\Delta t = -(m_1v_1 - m_1u_1)$

$$\therefore m_2v_2 - m_2u_2 = -(m_1v_1 - m_1u_1)$$

Or

$$m_2v_2 - m_2u_2 = -m_1v_1 + m_1u_1$$

Collecting initial terms together and final terms together we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . (4.27)$$

Equation (4.27) summarizes principle of conservation of momentum.

Conservation of kinetic energy:

"Work is done when the force moves a body through a distance, $W = Fs.$ In motion the work done is translated into a change kinetic energy as it can be shown from the second law of motion and third equation of linear motion".

$F = ma(second\ law\ of\ motion)$ and

$Fs = mas(work)$

But from

$$v^2 = u^2 + 2as(third\ equation\ of\ linear\ motion)$$

$$as = 1/2\,(v^2 - u^2)$$

$$W = mas = m\left(v^2 - {u^2}/{2}\right) = \tfrac{1}{2}m(v^2 - u^2)$$

In the case of collision we talk in terms virtual distance and therefore virtual work done the forces of action and reaction.
The virtual work done on m_2 by F_1 is given by $W_1 = \frac{1}{2}m_2(v_2^2 - u_2^2)$

Likewise the virtual done on;
m_1 by F_2 is $W_2 = -\frac{1}{2}m_1(v_1^2 - u_1^2)$ and hence

$$W_2 = W_1$$

$$-\tfrac{1}{2}m_1(v_1^2 - u_1^2) = \tfrac{1}{2}m_2(v_2^2 - u_2^2)$$

$$-\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_1u_1^2 = \tfrac{1}{2}m_2v_2^2 - \tfrac{1}{2}m_2v_2^2$$

Collecting the initial quantities together on one side and the final quantities together on the other side we get

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 - \tfrac{1}{2}m_2v_2^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.29)$$

Equation (4.29) is the summary of the conservation of kinetic energy in a system of colliding particle provided the collision is perfectly elastic.

(b)Inelastic collision

There are certain instances whereby the colliding bodies delay in separating after collision has taken place and at times they remain stuck together. Delaying to separate or sticking together after collision is due to in elasticity and hence elastic collision. In this case it only the momentum which is conserved but not kinetic energy. The deformation that takes place while the bodies are exerting onto each other in the process of colliding, results into transformation of energy from mechanical into heat and sound the two forms of energy which are recoverable. Once the energy has changed into heat we say that it has degenerated, it is lost to the surroundings. When the two bodies stick together after impact they can only move with a common velocity and if they do not move after collision then the momentum is .said to -have been destroyed.

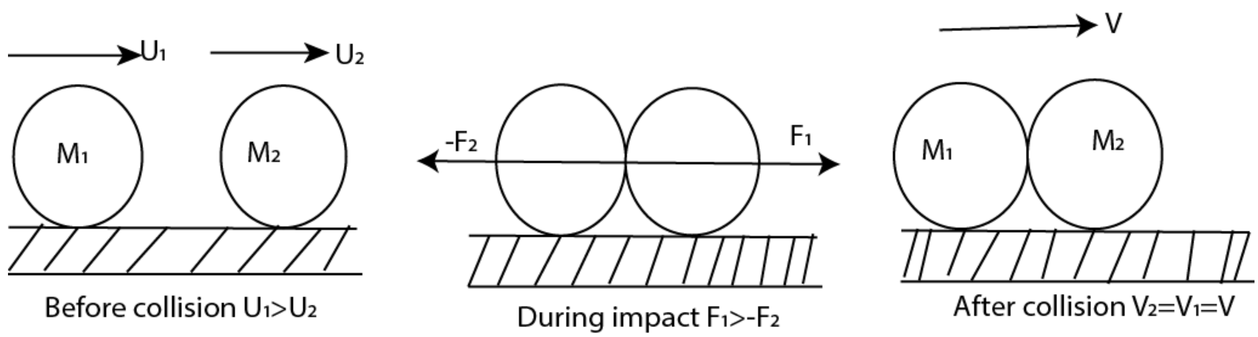


Figure 4.18

Coefficient of restitution

One of the measures of elasticity of the body is the ratio of the different in velocity after and before the collision. Before colliding, the space between the particles decreases as the rear body overtakes that in front but after collision the space between them widens as the front particle run away from the rear one. The difference in velocity before collision is called velocity of approach and that after collision is called velocity of separation. The ratio of velocity of separation to velocity of approach is known as coefficient of restitution.

Let $e =$ coefficient of restitution, ($v_2 - v_1) =$ velocity of separation, $(u_1 - u_2) =$ velocity of approach.

$$e = v_2 - v_1/u_1 - u_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.30)$$

For perfectly elastic collision, $e = 1$ in this case perfectly inelastic collision $e = 0$. But collision result into explosion, $e > 1$ otherwise in normal circumstances, $0 \leq e < 1$ the coefficient of restitution cannot be 1 due to the fact that it does not matter how hard the colliding bodies are they always undergo deformation at the moment of impact and hence take longer to recover to their original shape while separating. We only assume $e = 1$ to make calculation simpler.

4.5 Oblique collision

In the previous discussion on collision we dealt with direct impingement of one body onto the other along the line joining their common center. However there are situations in which bodies collide at an angle. This is known as oblique collision. Fig 4.19 illustrates the oblique collision of two bodies of mass m_1 and m_2 initially moving at velocities u_1 and u_2 respectively in the *x*-direction. After collision the body in front moves along the direction making an-angle $\emptyset$ with the initial direction whereas the rear body goes in the direction making an angle θ with initial direction. Given the initial conditions of the colliding bodies, the final velocities and directions after collision can be found

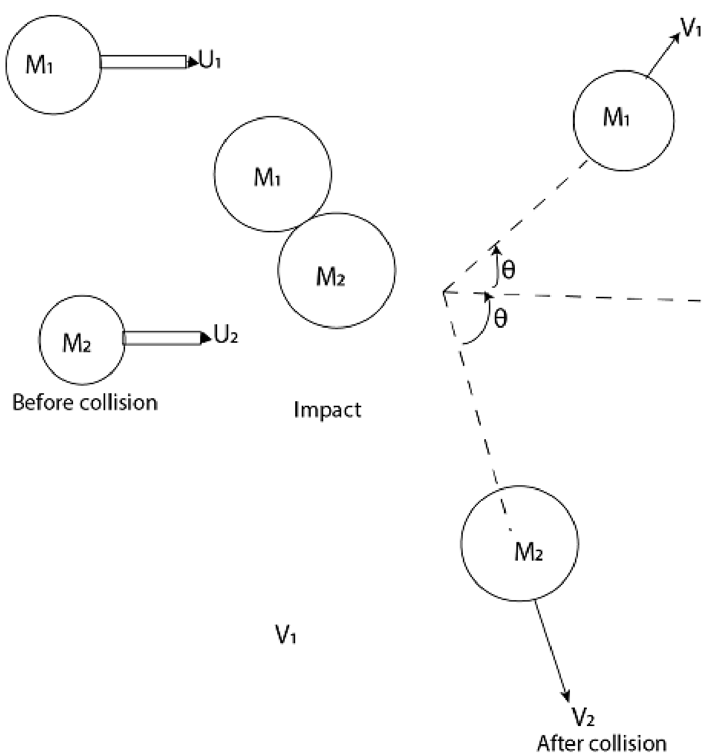


Figure 4.19

Applying the principle of conservation of linear momentum in equation (4.27) we can come up with more,equations for solving problems on oblique considering the motion in x - and y – directions

(a)Motion along x-direction

$$m_1u_{1_x} + m_2u_{2_x} = m_1v_{1_x} + m_2v_2$$

Where

$u_{1x} = u_1$, $u_{2x} = u_2$, $v_{1x} = v_1 cos\theta, v_{2x} = v_2 cos\emptyset$

$$\therefore m_1u_1 + m_2u_2 = m_1v_1cos\theta + m_2v_2cos\emptyset \ldots (4.31)$$

(b) Motion along y-direction

$$m_1u_{1_y} + m_2u_{2_y} = m_1v_{1_y} + m_2v_{2_y}$$

If initially the bodies are not moving in y-direction then $u_{1_y} = 0$ and $u_{2_y} = 0$

$$\therefore 0 + 0 = m_1v_1sin\theta + m_2(-v_1sin\emptyset)$$

Which is

$$m_1v_1sin\theta - m_2v_2sin\emptyset = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.32)$$

Applying the definition of coefficient of restitution in equation (4.30), we have

$$e = v_{2_x} - v_{1_x}/u_{1_x} - u_{2_x}$$

Or

$$e = v_2cos\emptyset - v_1cos\theta/u_1 - u_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.33)$$

4.6 The ballistic balance

Ballistic balances are used in determining velocities of bullets as well as light comparison of masses. To do this a wooden block of mass M is suspended from light wires so that it hangs vertically. A bullet of mass m is fired horizontally towards a stationary block. If the bullet is embedded inside the block, the two swings together as a single mass this is inelastic collision. The block will swing until the wires make an angle θ with the vertical as in figure 4.20

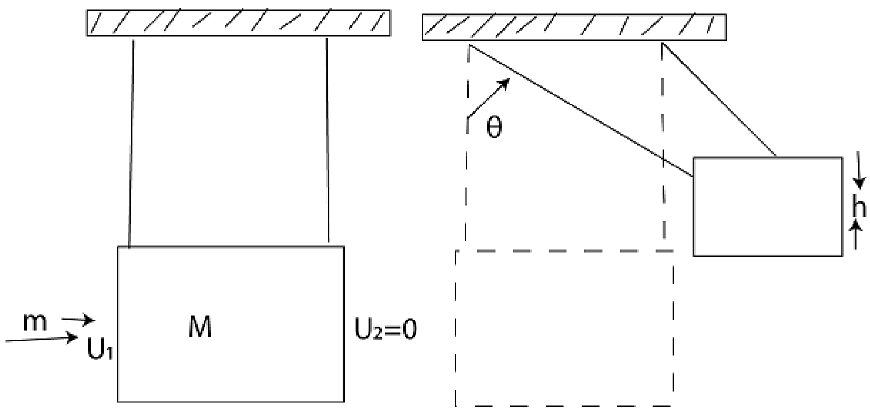


Figure 4 .20 shows the ballistic pendulum.

Since the collision is inelastic, only the momentum is conserved. If m and u_1 are the mass and initial velocity of the bullet and *M*, u_2 the mass and initial velocity of the block, then by principle of conservation of linear momentum.

$$M_1 + {}_1 + M_1 U_1 = (M + m) V$$

From which

$$\frac{v}{u_1} = \frac{m}{M+m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4.34)$$

After impact the kinetic energy of the system at the beginning of the swing is transformed into gravitational potential energy at the end of the swing and therefore

$$\frac{1}{2}(M + m)v^2 = (M + m)gh$$

$$\frac{1}{2}v^2 = gh$$

$$\therefore v = \sqrt{2gh} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4.36)$$

Substituting for v in equation (4.34) we get

$$\sqrt{2gh}/u_1 = m/M + m$$

Thus the initial velocity of the bullet is found to be

$$U_1 = \frac{(M + m)\sqrt{2gh}}{M}$$

In fig 4.21, suppose the length of the wire is l before the block swings. After swinging, the center of gravity of the block rises by a distance h reducing the vertical distance to $(l-h)$. By forming the triangle of displacements the values of h and θ can easily be found as shown in fig 4.21

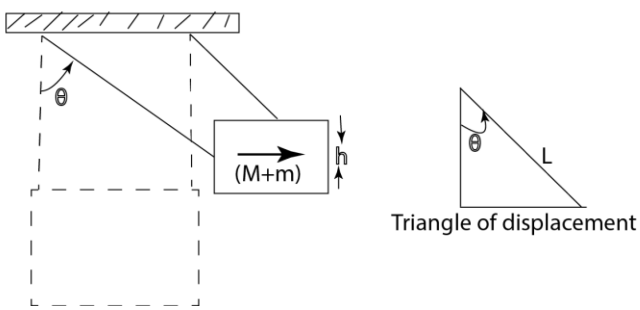


Figure 4.21

$$cos\theta = l - h/l$$

The height h is

$$h = l - lcos\theta \text{..}(4.37)$$

Or

$h = l(1 - cos\theta)$

The angle the wire makes with vertical is

$\therefore \theta = \cos\,(l - h/l)$ ………………………………………(4.38)

4.7 Reaction from a jet engine

The operation of a jet engine depends on the third law of motion where the escaping mass of hot gases exerts force on jet enabling it to move forward. Air is first sucked in through the front side then compressed, the oxygen contained in this air intake is used in burning the fuel producing gases which when expelled at a very high speed through the rear action forces are created and hence forward thrust . Fig 4.22 illustrate the principle of a jet in which the mass of air Ma is taken in at the rate of $dM_a/_{dt}\,kgs^{-1}$ with relative velocity $v_a ms^{-1}$ and passes through the engine at the rate $\mathrm{dM_a}/_{\mathrm{dt}}\mathrm{kgs^{-1}}$ with relative velocity of $v\ ms^{-1}$.The mass of gases M_g produced at the rate of $dM_g/_{dt}\,kgs^{-1}$ by combustion is ejected at a relative velocity $v\ ms^{-1}$.The total rate of change of momentum of the system is therefore given as

$$\frac{dp}{dt} = \left(dM_g/_{dt} + dM_a/_{dt}\right) - dM_a/_{dt}$$ …………(4.39)

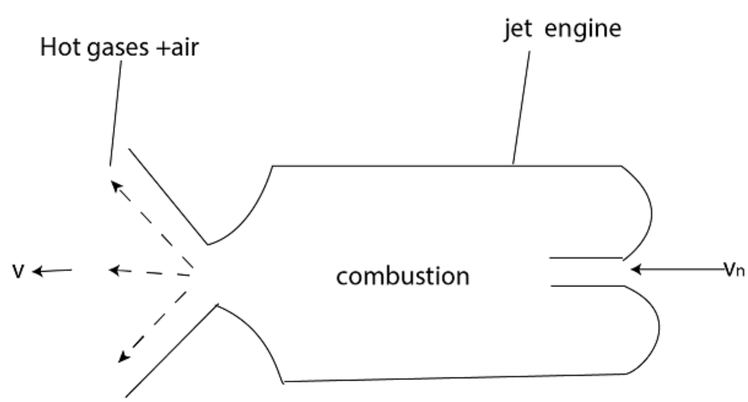
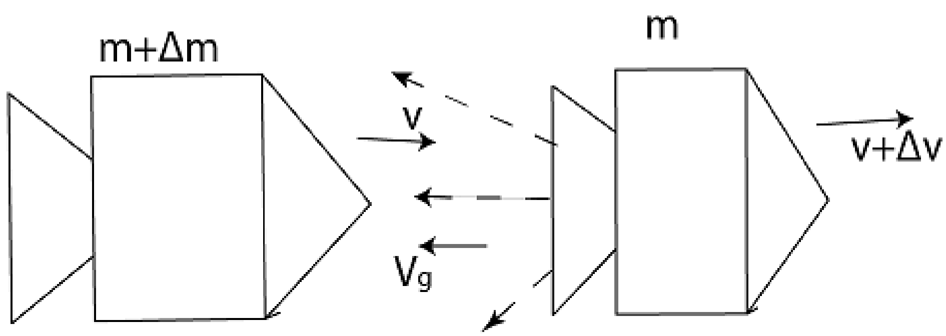


Figure : 4 .22 Jet engine

From Newton's second law of motion, the rate of change of momentum is equal to force. Therefore equation (4.39) represents the forward thrust on the jet aircraft. Some jet aircraft have two identical engines and others have four. The total thrust is the product of number of engines and thrust of one engine.

4.8 Reaction from a rocket

Unlike the jet engine, the rocket carries all of its propellant materials including oxygen with it. Imagine a rocket that is so far away from gravitational influence of the earth, then all of the exhaust hot gases will be available for the propelling and accelerating the rocket. Fig 4.24 is an illustration of a rocket of mass *m* carrying the fuel of mass Δm such that the total mass at time t is ($m + \Delta m$) moving horizontally far away from the earth surface. As the fuel burns and gases formed expelled from the rocket at a velocity of . V_g after sometime Δt, the mass of the rocket becomes *m* but its velocity increases to ($v + \Delta v$) as in fig.4.23(b) whilst the velocity of the ejected gases decreases from v to ($v - \Delta v_g$)



(a)The rocket propellant at time t (b)v+rocket at time t +Δt

Figure. 4 .23

From the principle of conservation of linear momentum

$$(m + \Delta m)v = \Delta m(v - v_g) + m(v + \Delta v)$$

$mv + v\Delta m = v\Delta m - v_g \Delta m + mv + m\Delta v$

$m\Delta v - v_g \Delta m = 0$

Hence

$\Delta v = {}^{v_g}/_{m}\, {}^{\Delta m}/_{\Delta t}$

$\therefore m\, {}^{\Delta v}/_{\Delta t} = v_g\, {}^{\Delta m}/_{\Delta t}$

Since the time rate of change in momentum is equal to thrust or force (F) on the rocket by the escaping mass of the gases the above relation can be written as $F = v_g\, {}^{\Delta m}/_{\Delta t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.40)$

If the large thrust is to be obtained, the rocket designer has to make the velocity v_g at which the hot gases are ejected and the rate at which the fuel is burnt ${}^{\Delta m}/_{\Delta t}$ high as possible.

4.9 Rocket moving vertically upwards

Let us consider the rocket fired vertically upwards from the surface of the earth as shown in fig 4.24

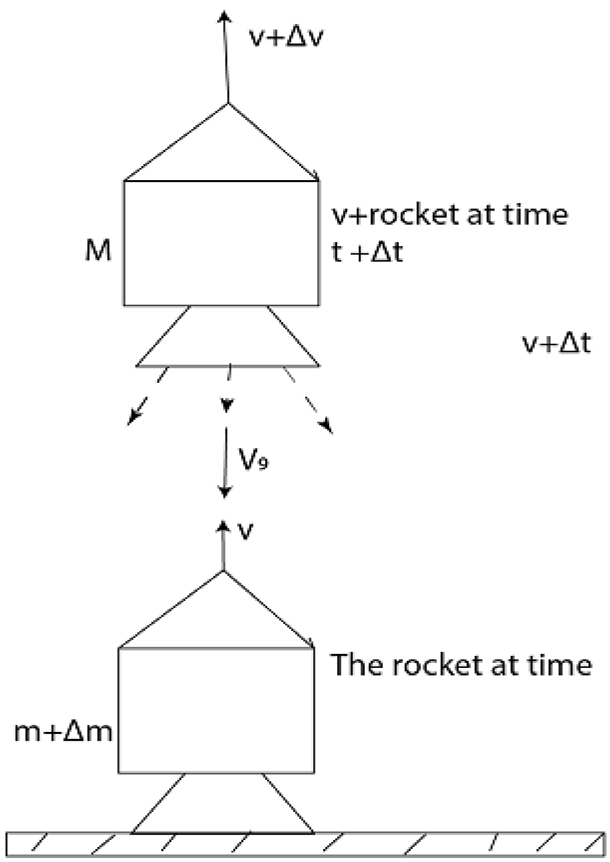


Fig 4.25; The rocket in a vertical motion from earth surface

Figure. 4. 24

The thrust developed during combustion must be greater than the weight of the rocket if at all it is to accelerate vertically upwards $i.e\ F > mg$ which means that $m\,{}^{\Delta v}/_{\Delta t} = v_g\,{}^{\Delta m}/_{\Delta t} - mg \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.41)$

4.10 Reaction from the hose pipe

If a hose pipe connected to the running tap on the smooth horizontal surface, the free end that issues water seems to move backwards as the water flow out. This is yet another example of action and reaction forces. Again if a jet of water from a horizontal hose pipe is directed at a vertical wall, it exerts an equal but opposite force on the water.

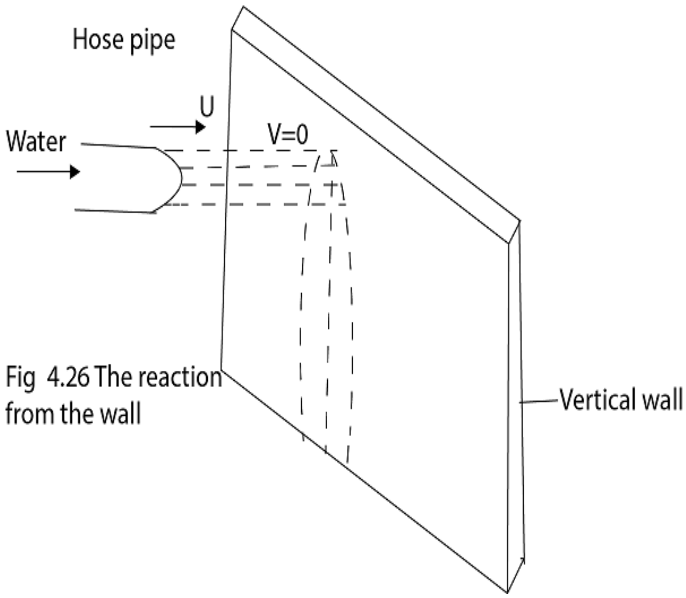


Fig 4.26 The reaction from the wall

Figure. 4.25

Let u be the initial velocity of water when leaving the pipe. On striking the wall its final velocity $v = 0$ assuming that the water does not rebound. If is the density of water, *A* is the cross-sectional area of the pipe, then the mass of water hitting the wall per second is given by

$$m = V\rho$$

Where $V = volume\ per\ second\ expressed\ as$

$$V = Au$$

$$m = Au\rho$$

The time rate of change in momentum of water is therefore

$$\Delta P/_{\Delta t} = m\Delta v = Au\rho\Delta v$$

Where $\Delta v = (v - u)\ \Delta P/_{\Delta t} = F$ and $v = 0$

$$\therefore F = Au\rho(-u)$$

i.e.

$$F = -A\rho u^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.42)$$

The negative sign means the force is the reaction of the wall on water. Thus the force exerted by the water on the wall is $F = A\rho u^2$

4.11 Reaction on a gun

Consider a gun mass m_g with bullet mass m_b in it initially at rest. Before firing the gun, their total momentum is zero as in fig. 4.27(a). At the point of firing there are equal opposite internal forces as in fig 4.27(b). As the bullet leaves the gun the total momentum of the system is still zero as in fig 4.27(c).

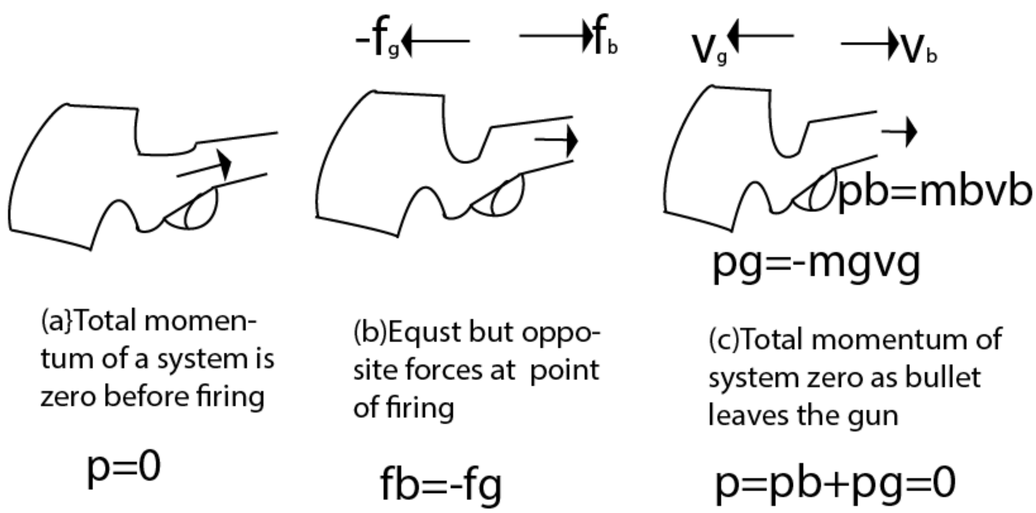


Figure. 4. 26

Initially the velocity of the gun and that of the bullet are zero

$i.e\ u_g = u_b = 0$

Therefore the initial momentum

$$P_1 = m_g u_g + m_b u_b$$

$$= m_g(0) + m_b(0)$$

$$= 0$$

When the bullet leaves the gun with final velocity v_b, the gun recoil with velocity of -v_g and the final total momentum

$$P_2 = m_b v_b + m_g v_g$$

From the principle of conservation of linear momentum

$$P_1 = P_2$$

Thus

$$m_b v_b + m_g v_g = 0$$

$$\therefore m_g v_g = -m_b v_b$$

4.12 Equilibrant forces

A body is said to be in static equilibrium if it does not move under the action of external forces. For example in fig 4.3, a block is in equilibrium since it neither moves up nor down under forces R and W. These two forces are action and reaction which cancel each other out such that the net force on the body is zero. The net external force is an algebraic sum of all the forces acting on the body that is

$$\sum F = 0$$

In this case

$$\sum F = W + R$$

$$W + R = 0$$

$$\therefore R = -W$$

The forces that keep the body in equilibrium are called equilibrant forces. These are the forces whose resultant is zero. Fig 4.28(a) shows a body in equilibrium under the action of three forces, hanging vertically. The weight W of the body establishes the tensions T_1 and T_2 in the sections AB and BC of the string which make angles θ and φ respectively with the horizontal point B along the string experiences three forces as shown In fig 4.28(b). If this point is taken as an origin and two perpendicular axes drawn, the tensions T_1 and $T_{2,}$ appear to make angles θ and φ with the *x* - axis. The body remains in equilibrium when no motion occurs either horizontally or vertically and for that reason the net force along the horizontal direction is zero

$$\sum F_x = 0$$

Likewise the net force on the body along the vertical direction is zero

$$\sum F_y = 0$$

To obtain the net forces we have to find the x- and y-components of tensions T_1 and T_2 as shown in fig 4.28 (C). For the horizontal components of two tension gives

$$\sum F_x = -T_{1_x} + T_{2_x}$$

Where

$$T_{1_x} = T_1 cos\theta \; and \; T_{2_x} = T_2 cos\varphi$$

$$\sum F_x = -T_1 cos\theta + T_2 cos\varphi$$

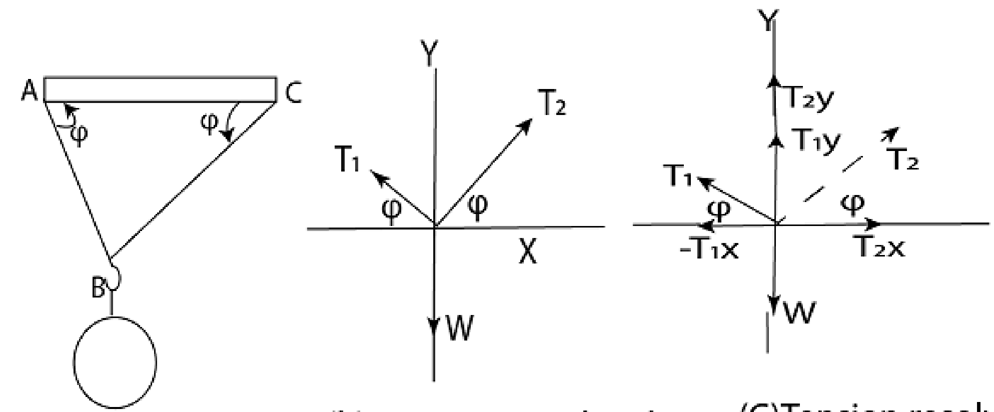
$$\therefore -T_1 cos\theta + T_2 cos\varphi = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (4.43)$$

For the y-direction the net force

$$\sum F_y = T_{1_y} + T_{2_y} + (-W)$$

Where $T_{1_y} = T_1 sin\theta \; and \; T_{2_y} = T_2 sin\varphi$

$$\therefore T_1 sin\theta + T_2 sin\varphi + (-W) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.44)$$



(a)A body hanging from the string

(b)Tension T1 and T2 developed in srting by a weight W

(C)Tension resolved along X -and y -axis

Figure. 4. 27

Solving for T_1 and T_2 in terms of W from equations (4.43) and (4.44 we get

$$T1 = {}^{W\cos\varphi}/_{\sin\,(\theta+\ \varphi)} \text{ and } T_2 = {}^{W\cos\theta}/_{\sin(\theta+\ \varphi)}$$

Exercise 4.0

4.1

(a) State Newton's laws of motion

(b) Give three examples in which Newton's third law applies

(c) With the aid of labeled diagram explain what causes frictional force.

4.2 (a) A body of mass *m* rests on a rough inclined plane with angle of inclination θ

(i) Explain why the body does not slide down the plane,

(ii)Draw the diagram indicating all the forces acting on the body and give each force its name, direction and magnitude.

(b) If the mass of the body in (a) is 5kg and the angle of inclination is 20° then find

(i) The force that keeps the body in contact with the plane (ii) The force that prevents the body to slide down the plane.

4.3(a) Differentiate between

(i) A rough surface and a smooth surface

(ii) Static friction and kinetic friction

(iii) Coefficient of static friction and coefficient of dynamic friction .

(b) A body of mass 20kg is pulled by a horizontal force P. If it accelerates at *1.5* ms^{-1} *and* the coefficient of friction of the plane is 0.25, what is the magnitude of P? (Take $g = 9.8ms^{-2}$)

4.4 (a) From the second law of motion show the expression for the force.

$F = ma$, where m and a are the mass and acceleration respectively

(b)Using the third law of motion, show that for the two colliding bodies of masses m_1 and m_2 moving along the common line at velocities u_1 and u_2 , before collision and at velocities v_1 and v_2 respectively just after collision, the total momentum of the system is conserved and represented by relation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$. Assume elastic collision.

4.5 (a) Fig 4.28 shows two bodies of masses *M* and *m* connected by the light string, if the body of mass *m* rests on a rough plane whose coefficient *of friction is μ* on releasing the system free, obtain an expression for

(i) The acceleration of the system.

(ii) The tension in the string.

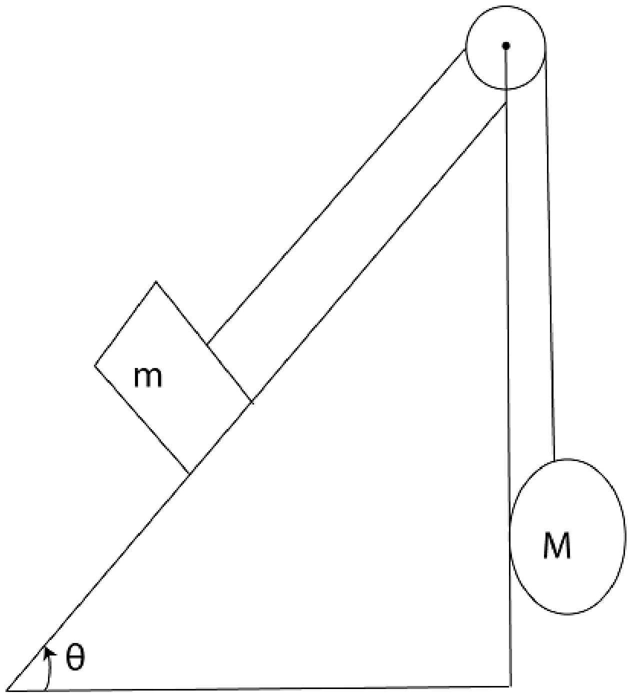


Figure. 4 .28

4.6 (a) (i) What is the difference between the coefficient of restitution and the coefficient of friction?

(ii) Explain in details the implications of the following about the coefficient of restitution *e*

- when $e > 1$
- when $e = 0$
- when $e < 1$
- when e = l

(b) Two bodies A and B of masses 3kg and 2.5kg are moving towards each other along a common line with initial velocities 4ms^{-1} and 2.5m^{-1} respectively, after sometime they eventually collide elastically.

Determine their final velocities.

4.7 (a) A body is hanging in equilibrium as shown in fig 4.29, find the tensions

T_1, T_2 *and* T_3 .Take $g = 9.8ms^{-2}$

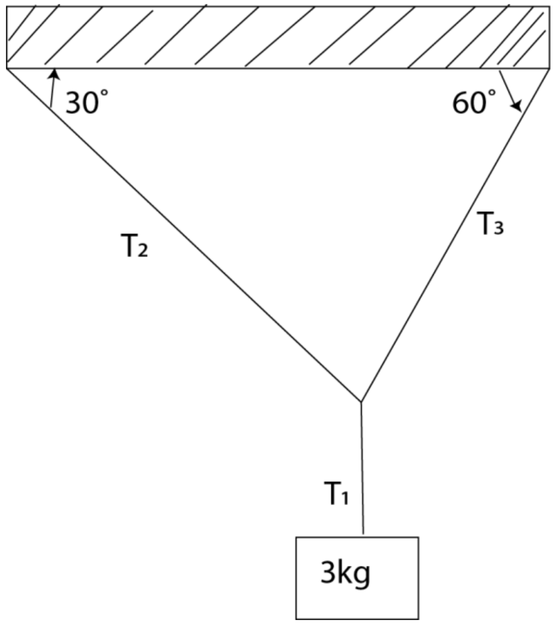


Figure. 4. 29

(b) A body of mass 2kg sits on a horizontal plane, and then the plane is accelerated vertically upwards at $4ms^{-2}$. Determine the magnitude of the reaction on the body by the plane.

4.8 The engine of a jet aircraft flying at $400 ms^{-1}$ takes in $1000m^3$ of air per second at an operating height where the density of air is $0.5 kgm^{-3}$, The air is used to burn the fuel at the rate of $50 kgs^{-1}$ and the exhaust gases (Including incoming air) are ejected at $700 ms^{-1}$ relative to the aircraft . Determine the thrust.

PROJECTILE MOTION

3.1 PROJECTILES

A projectile is any object that when given an initial velocity it moves freely in space under the influence of gravity. The path it follows is known as trajectory which is in the form of a parabola $y = ax-bx^2$ *as* shown in fig 3.1

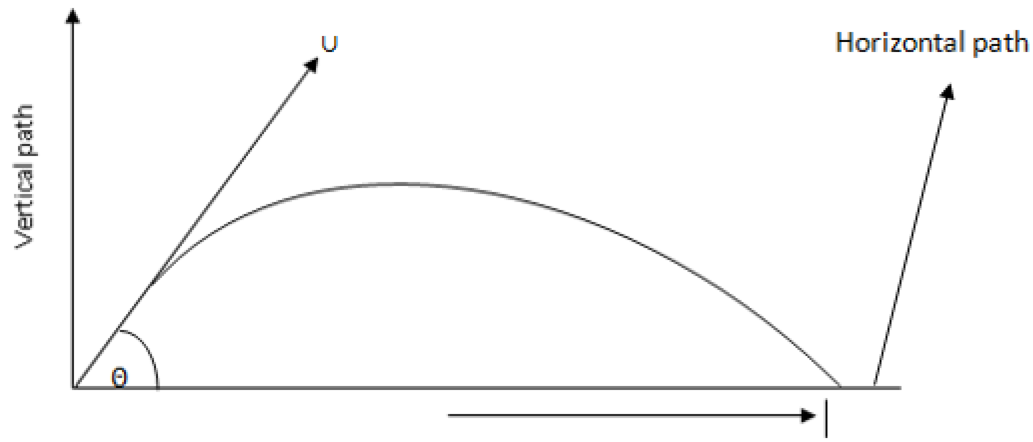


Fig 3.1: Trajectory of a projectile

Projectile motion is common in warfare, sports, hunting fire fighting and irrigation. To describe the motion of a projectile we shall make use of the equation of linear motion. This is because a projectile moves in two directions simultaneously, that is, it moves along the x-direction and along the y-direction.

3.2 Initial velocities at the point of projection

Suppose a projectile is projected with an initial velocity *u* along the direction making an angle to the horizontal plane as in fig 3.1. This velocity is divided into two components u_x along the x-direction and u_y along the y-direction. Fig 3.1 shows how these components can be obtained by first forming the triangle of velocities and then using the trigonometric ratios of the right-angled triangle.

From the right angled -triangle in fig 3.2 we can establish the expressions of the initial velocities as follows:

For x - direction

$$\frac{u_x}{u} = \cos\theta$$

$$\therefore u_x = u\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

For y-direction

$$u_y/_u = \sin\theta$$

$\therefore u_y = u\sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.2)$

After a time t, the velocities of the projectile v_x *and* v_y can be obtained from the first equation of linear motion $v = u + at$

Along the x-direction

$v_x = u_x + a_x t$

Where

$a_x = 0$ *and* $u_x = u\cos\theta$

$v_x = u\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.3)$

This means that at any time t when the projectile is in flight the horizontal component of the initial velocity remains constant.

Along the y-direction

$v_y = u_y + a_y t$

Where

$u_y = u\sin\theta$ *and* $a_y = -g$

$v_y = u\sin\theta - gt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.4)$

At the same time the horizontal and vertical displacements x *and* y can be obtained from the second equation linear motion $i.e\ \ s = ut + {}^1/_2\, at^2$

For horizontal displacement

$x = u_x t + {}^1/_2\, a_x t^2$

Since $u_x = u\cos\theta \quad and \quad a_x = 0$

$$x = u\cos\theta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

For vertical displacement

$$y = u_y t + \frac{1}{2} a_y t$$

Given that $u_y = u\sin\theta \ \ and \ \ a_y = -g$ (Shows that the acceleration is against the gravity)

$$y = u\ sin\ \theta t - \frac{1}{2} g t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

3.3 Equation of a trajectory

The mathematical relationship between horizontal and vertical displacements that defines the path followed by the projectile.
This can be obtained by combining-equations (3.5) and (3.6) as follows:

From equation, (3.5) t = $^x/_{u\cos\theta}$. Substitute for t in (3.6)

$$y = u\sin\theta(^x/_{u\cos\theta}) - {}^1/_2\, g(x/u\cos\theta)^2$$

On simplification we have

$$y = x\tan\theta - gx^2/2u^2\ cos^2\ \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

From the above equations it is now possible to get other expressions for the maximum height, time of flight, range and maximum range.

3.4 Maximum height

The maximum height is the vertical distance above the horizontal plane a projectile can possible attain for given initial velocity and angle of projection. This is the position where a projectile ceases to move upwards in This case $v_y = 0$ as shown in the figure 3.3.

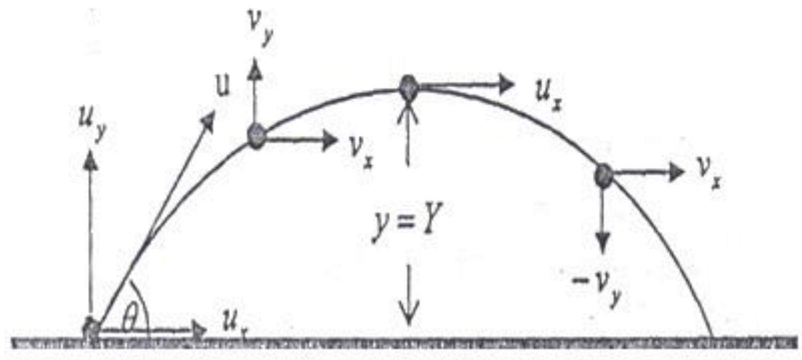


Figure **3.3:**
Maximum height and range of a projectile

From equation (3.4) when $v_y = 0,\ u\sin\theta - gt = 0$

$\therefore t = {u\sin\theta}/{g}$

This is the time taken by the projectile to attain the maximum height y. Substituting for t in equation (3.6) we get

$$Y = u\sin\theta({u\sin\theta}/{g}) - {1}/{2}\, g(u\sin\theta/g)^2$$

When simplified this expression become

$$Y = \frac{1}{2}\frac{u^2 sin^2\theta}{g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.8)$$

3.5 Time of flight

The time taken by the projectile to move from the point of projection to the target' horizontally is known as time of flight. In other words it is the time a projectile remains in air. When the projectile lands on the target, the vertical displacement becomes zero which means *y* - 0. Using this condition in equation (3.6) we can solve for t as follows.

$$u\sin\theta t - 1/2\, gt^2 = 0$$

$$(u\sin\theta - 1/2\, gt)t = 0$$

Either $t = 0\ \ or\ u\sin\theta - 1/2\, gt = 0$

$$t = 2(u\sin\theta/g)$$

Thus the time of flight is twice the time taken by projectile to reach the maximum height.

3.6 The Range (R)

The range of a projectile is the horizontal displacement between the point of projection and the target. Since the horizontal component of the initial velocity of the projectile remains constant, the range is obtained from equation (3.5)

$$X = u\cos\theta t$$

When

$$t = 2u\cos\theta/g\,, x = R$$

$$\therefore R = u\cos\theta(2u\sin\theta/g) = 2u^2\cos\theta\sin\theta/g$$ =U 2

$$R = 2u^2\cos\sin\theta/g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.9)$$

R = u^2 $\sin^2\theta/g$ ... ... **(3.9).**

3.7 The Maximum Range

When a projectile is projected at different angles with the horizontal, the ranges covered differ. However there is a single angle for which the range is greatest of all. We can get this angle from the range expression in equation (3.9).

Write the range expression as

$$R = 2u^2\cos\theta\sin\theta/g$$

Rearrange the expression

$$R = U^2\,(2\cos\theta\sin\theta)/g$$

From the trigonometric identities

$$2\cos\theta\sin\theta = \sin 2\theta$$

∴ R = $U^2 \sin^2\theta)/g$

For maximum range $\sin 2\theta = 1,\ i.e\ 2\theta = 90^0\quad or\quad \theta = 45^0$

$$\therefore R_{max} = u^2/g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.10)$$

Example 3.1: A projectile is launched from a point on a level ground with a velocity of $50\ ms^{-1}$ in the direction making an angle of 60° to the horizontal.

(a) Sketch the trajectory of the projectile

(b) What are the initial velocities of the projectile along the x- and y-directions?

(c) Find the velocities of the projectile along the horizontal and vertical planes 3seconds after launch

(d) Determine the greatest height attained by the projectile

(e) What is the time of flight?

(f) What is the range?

Solution

The main assumptions to consider in a projectile motion are such as the effect of air resistance is reflected the effect of earth curvature and it is rotation are neglected.

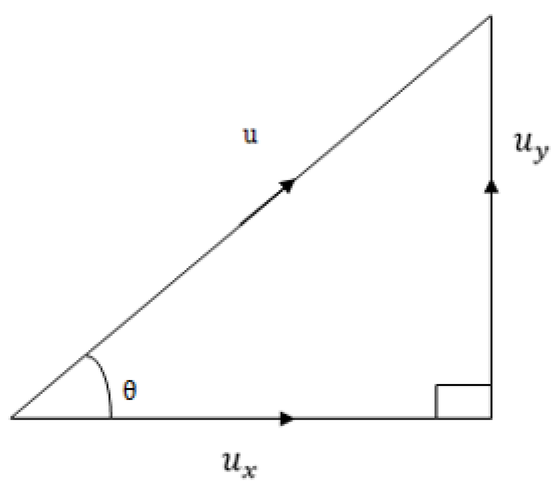
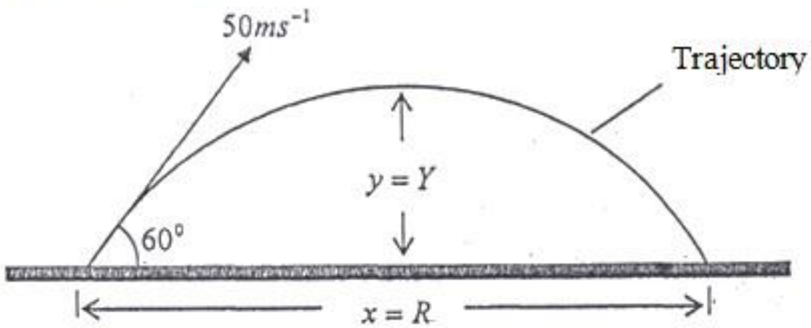


Figure 3.2

(a) The sketch of the trajectory



(b) The initial velocities u_x and u_y

(i) $u_x = u\cos\theta$

$U_x = 50\left(\frac{1}{2}\right) = 25\, {}^{m}/_{s}$

$u_x = 50(1/2)25\ ms^{-1}$

(ii) $u_y = u\sin\theta$

$u = 50\ ms^{-1}, \theta = 60^0\ \ and\ \sin 60^0 = \sqrt{3}/2$

$u_y = 50\left(\sqrt{3}/2\right) = 25\sqrt{3}\ \ or\ \ 43.3\ ms^{-1}$

(c) The velocities $v_x\ \ and\ \ u_y$

$(i)\ \ v_x = u_x = 25\ ms^{-1}\ \ (Horizontal\ velocity\ remains\ constant)$

$(ii)\ \ v_y = u\sin\theta - gt$

$u = 50\ ms^{-1}, \theta = 60^0, \sin 60^0 = \sqrt{3}/2\ \ and\ t = 3s$

$v_y = 50\ \sin 60^0 - 9.8(3)$

$= 43.3 - 29.4$

$= 13.9\ ms^{-1}$

(d) The greatest height Y

$$Y = \frac{\frac{1}{2}(U_2 \sin^2\theta)}{g}$$

$$= 50^2\left(\sqrt{3/2}\right)^2/2(9.8) = 2500 \times 3/4 \times 2 \times 9.8 = 36750\text{ M}$$

(e) The time of flight

$$t = 2u\sin\theta/g$$

$$= 2(50)\sin 60^0/9.8 = 100 \times 0.866/9.8 = 8.84s$$

(f) The range R

$$R = 2u^2\cos\theta\sin\theta/g$$

$$= 2(50)^2\cos 60^0\sin 60^0/9.8$$

$$= 2(2500)(0.5)(0.866)/9.8$$

$$= 220.9\, m$$

3.8 Projectile fired from raised ground

Let us consider the projectile fired from the top of a cliff. Here the projectile may be launched horizontally or at an angle with the horizontal. The equations used to describe the motion in this case are the same as what have been derived so far but with some modifications.

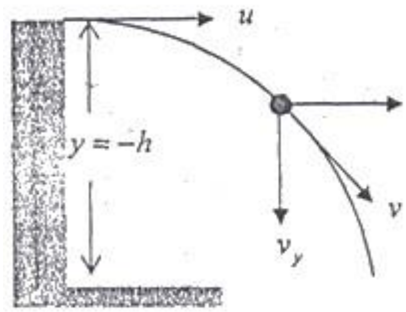


Figure 3.4 projectile fired horizontally

Projectile fired horizontally from the top of a cliff

Consider a projectile launched horizontally with velocity u from the top of a cliff as shown in fig 3.4. The gravity causes the projectile to follow the trajectory as it advances the nature of which is identical to that in fig 3.1. The following are the factors to observe:

The horizontal velocity remains constant throughout the motion

The initial velocity along the horizontal direction u = u_x and that along the vertical direction Uy = 0

The vertical distance projectile fall through is always assigned negative sign.

The time the projectile takes to move along the trajectory equal to the time it takes to fall through the vertical distances h

The resultant velocity v is obtained by Pythagoras's theorem $V^2 = V^2{}_x + V_Y{}^2$

Example 3.2: A bullet is fired

horizontally from the gun with muzzle velocity of 200 ms1from the top of a cliff 120m high.

Find.

(a) The velocities of the bullet after it has fallen $\frac{3}{4}$ of the vertical distance

(b) The time it takes to land on a level ground

(c) The distance from the foot of a cliff to where it lands

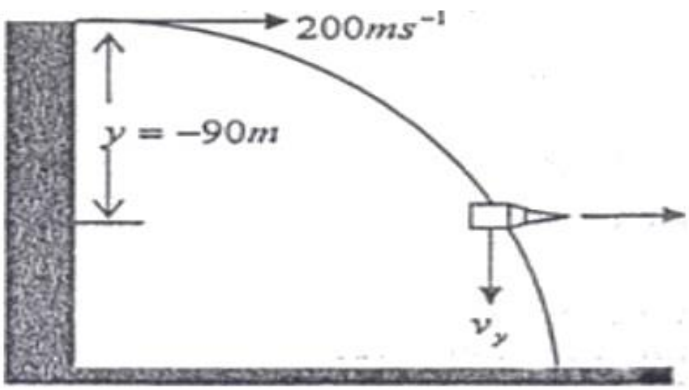


Figure. 3.4 A bullet fired from top of the cliff.

(a) To find v_x and v_y

Let *t = time taken*

From $y = u\sin\theta t - {}^1/_2\, gt^2$

$since\ \theta = 0^0, \sin\theta = 0. y = -90\ m\ \ and\ g = 9.8\ ms^{-1}$

$-90 = 0 - {}^1/_2(9.8)t^2$

$4.9t^2 = 90\ \ or\ \ t^2 = {}^{90}/_{4.9}$

$t = \sqrt{{}^{90}/_{4.9}} = \sqrt{{}^{900}/_{49}} = {}^{30}/_7\ \ or\ 4.2857s$

$(i)\ v_x = 200\ ms^{-1}\ \ (constant)$

$(ii)\ v_y = u \sin\theta - gt$

$V\gamma = 0 - 9.8\ (4.2857) = -41.99986\ ms^{-1}$

$\approx -42\ ms^{-1}$

(b) By the moment it lands on the ground, the vertical distance covered is $y = 120\ m$

From

$y = u \sin\theta t - {}^{1}/_{2}\, gt^2$

$-120 = 0 - {}^{1}/_{2}\,(9.8)t^2$

$120 = 4.9t^2$

$t^2 = {}^{120}/_{4.9}$

$\therefore t = \pm\sqrt{{}^{120}/_{4.9}} = \pm\sqrt{24.5}\ s$ i.e $t = -24.5s$ or t

$= \sqrt{24.5}\ s\quad t = 4.948\ s$

-It takes the bullet *4.948 seconds* to land on its target

(c) Since the horizontal velocity is constant through out the motion

$x = v_x t$

= 200 (4,948)

= 989 . 6 m.

Example 3.3: A shell is launched from the top of a hill 90m above the plane ground with a velocity of 150 ms^{-1} in the direction making an angle 30° to the horizontal. Find

(a) The velocities along the x- and y-directions when it is half-way between the top and ground

(b) The horizontal distance covered by the moment it lands on the ground

The velocity and direction as the shell hits the ground. (Use $g = 9.8m/s^2$

Solution

(a) Let *t = time for the shell to descend 45m*

From

$y = u\sin\theta t - {}^{1}/_{2}\, gt^2$

y = - 45, u = 150 ms^{-1} θ = 30°, sin 30, sin 30° = 0.5 and g = 9.8 ms^{-1}

$-45 = 150(0.5)t - {}^{1}/_{2}(9.8)t^2$

$-45 = 75t - 4.9t^2$

This is a quadratic equation in t that can be solved by the formula

$t = {}^{-b \pm \sqrt{b^2 - 4ac}}/_{2a}$

$a = 4.9, \quad b = -75 \;\; and \;\; c = -45$

$\mathbf{t} = -\mathbf{7.5} \pm \;\; \sqrt{(-75)^2 - 4(4.9)(-45)}/_{2(4.9)} = {}^{75 \pm \sqrt{5625 + 882}}/_{9.8}$

$= 75 \pm \sqrt{{}^{6507}/_{9.8}} \;\; = 75 \pm {}^{80.67}/_{9.8}$

$\therefore t = 15.9\, s \;\; or - 0.6\, s$

(i) $v_x = u\cos\theta$

$= 150\cos 30^0 = 150(0.866)$

$= 129.9\, ms^{-1}$

(ii) $v_y = u\sin\theta - gt$

$$= 150(0.5) - 9.8(15.9)$$

$$= 75 - 155.82 = -80.82\ ms^{-1}$$

(b) When it lands on the plane the distance covered is now -90 m and therefore from

$$y = u\sin\theta t - {}^{1}/_{2}\,gt^2$$

$$y = -90\ m, \theta = 30^0\ \ and\ \ g = 9.8\ ms^{-1}$$

$$-90 = 75t - 4.9t^2$$

$$4.9t^2 - 75t - 90 = 0$$

$$t = \frac{-(-75) \pm \sqrt{(-75)^2 - 4(4.9)(-90)}}{2(4.9)} = \frac{75 \pm \sqrt{5625 + 1764}}{9.8}$$

$$= \frac{75 \pm \sqrt{7389}}{9.8} = \frac{75 \pm 86}{9.8}$$

$$t = \frac{75 + 86}{9.8} = \frac{161}{9.8} = 16.4s\ \ or\ \ t = \frac{75 - 86}{9.8} = -\frac{11}{9.8} = -1.1s$$

The shell takes 16.4s to the land on the target and by then the horizontal distance is

$$x = v_x t$$

$$= 129.9(16.4) = 2130.36\ m$$

(c) Let $v = the\ velocity$

$$v_x = the\ horizontal\ velocity$$

$$v_y = the\ vertical\ velocity$$

By forming the triangle of vector we can use Pythagoras theorem

$$v^2 = v_x^2 + v_y^2$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$v_x = 129.9\, ms^{-1}, \qquad v_y = -80.82\, ms^{-1}$

$$\therefore v = \pm\sqrt{(129.9)^2 + (-80.82)^2} = \pm\sqrt{16874.01 + 6531.87} = \pm\sqrt{23405.8824}$$

$$= \pm 153\, ms^{-1}$$

The velocity of the shell is 153 ms^{-1}

The direction of the shell at this moment is given by

$$\varphi = tan^{-1}\left(\frac{|-v_y|}{|v_x|}\right) = tan^{-1}\left(\frac{80.82}{129.9}\right) = tan^{-1}(0.6222) = 32^0$$

Exercise 3.0

3.1. A projectile is launched from a point on a horizontal ground with an initial velocity of 100ms"[1] along the direction making an angle of 60° with the horizontal.

a) (a)Sketch a labeled diagram showing how the projectile moves.

(b) What are the initial velocities at the point of projection

(c) Determine the velocities of the projectile 4 seconds after projection.

what is the highest point does the projectile reach while in motion?

d) (d) Calculate the range of the projectile.

3.2 (a) Show that the equation of the trajectory of a projectile is,

$$y = x\tan\theta - \frac{gx^2}{2u}\sec^2\theta$$

Where $\theta =$ angle of projection, *u*= initial velocity, *x* =horizontal distance, *y* = vertical distance and *g* = acceleration due to gravity.

(b) State the condition for a projectile to attain the maximum range and show that the maximum range is $R_{max} = u^2/g$

3.3 Find the two possible angles of projection for a projectile to just clear the wall 10m high if the point of projection is 40m from the wall and the initial velocity of a projectile is $50ms^{-1}$.

3.4 A man standing on a cliff 50m high sees a dog running away 20m from the footnote cliff. He throws a stone horizontally with velocity of $30ms^{-1}$, if the stone hits the dog, find

(a) The distance of the dog from the cliff

(b) The speed of the dog by the time it is hit by the stone.(Take $g = 9.8ms^{-2}$)

3.5 A projectile is fired with an initial velocity of u at an angle θ to the horizontal as in figure 3.3. When it reaches its peak, it has, (x, y) coordinates given by $1/2\,R, h$ and when it strikes the ground, its coordinates are $(R, 0)$, where R is the horizontal range,

(a) Show that it reaches the maximum height, Y, given by $Y = u^2\ sin^2\ \theta/2g$

(b) Show that its horizontal range is given by $R = u^2 \sin 2\theta/g$

3.6 In fig 3.6, a projectile is fired at a falling target. The target begins falling at the same time as projectile leaves the gun. Assuming that the gun is initially aimed at the target, show that the target will be hit.

3.7 An object slides from rest along a frictionless roof 8m long, inclined 37°to the horizontal. While sliding, the object accelerates at $5m/s^2$ towards the edge of the roof which is 6m above the 'ground. Find (a) the velocity components when it reaches the edge of the roof (b) the total time it remains in motion (c) the distance from the wall of the house to where the object hits the ground.

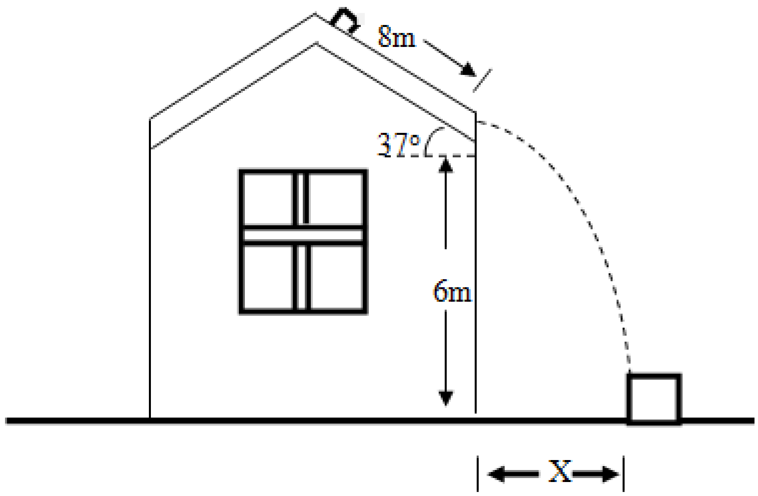


Figure 3.6

3.8. A particle is projected at point on a level ground in such a way that its horizontal and vertical components of the initial velocity are 30m/s and 30*m/s* respectively. From this information ;find.

(a) The highest point above the ground reached, by the particle

(b) The horizontal distance covered after landing on the ground

(c) The magnitude and direction of the initial velocity.

3.9 A warplane flying horizontally at l000kmh^{-1} releases a bomb at a height of 1000m.| The bomb hits the intended target, what was the distance of the plane from the target when the bomb was released?

3.14 A basketball player 1.8m tall throws a ball at a velocity of 10m/s in the direction 40º with the horizontal. The ball passes through the basket fixed at a height of 3m above the ground level. Find the horizontal distance of the basket from the point where the ball was thrown.

UNIFORM CIRCULAR MOTION

CIRCULAR MOTION

Is the motion of the body around the circular track

There are two types of circular motion

(i) Uniform circular motion

(ii) Non-uniform circular motion

(i)UNIFORM CIRCULAR MOTION

This refers to the motion of a particle in circular path moves with a uniform speed.The word "uniform" refers to the constant speed.It means that in uniform circular motion,the object covers equal distances along the circumference in equal intervals of time i.e. speed is constant.Although the magnitude of velocity (speed) remains constant, the direction of the velocity is changing continuously.Therefore, the object is undergoing acceleration. This is called centripetal acceleration and is directed radically towards the center of the circle as shown shown in the figure 1;

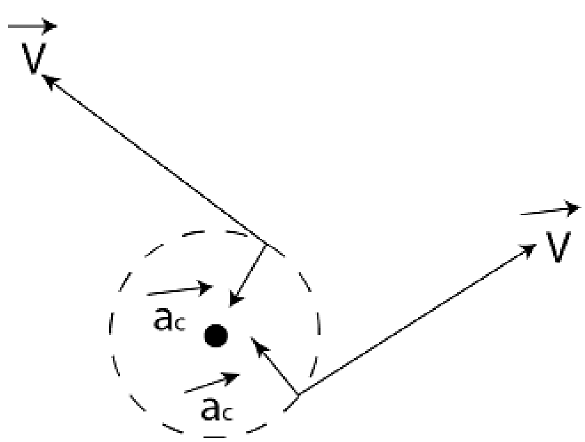


Figure .1

(ii)NON UNIFORM CIRCULAR MOTION

This refers to the motion of the particle in circular path moves with a non uniform speed.

The speed of the particle in circular motion is different at different points along the circular path.

RELATIONSHIP BETWEEN LINEAR DISPLACEMENT AND ANGULAR DISPLACEMENT

Consider the particles of mass m moving around the circular track with angular speed (ω) as illustrated on the figure 2

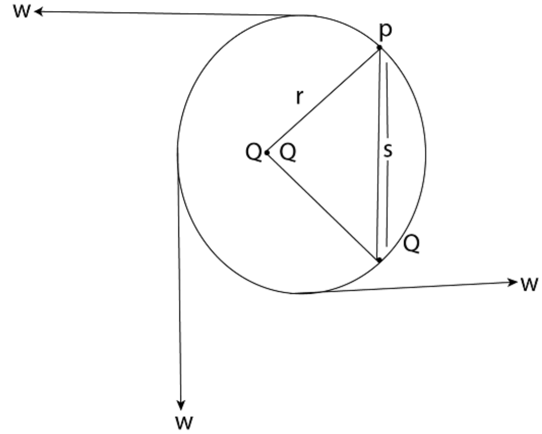


Figure.2

From the figure above

O = Origin of the circle (circular track)

Q = Angular Displacement

S = Linear Displacement

R = Radius of the circular track

ω = Angular velocity

Consider the particles moving from Q to P from the figure above

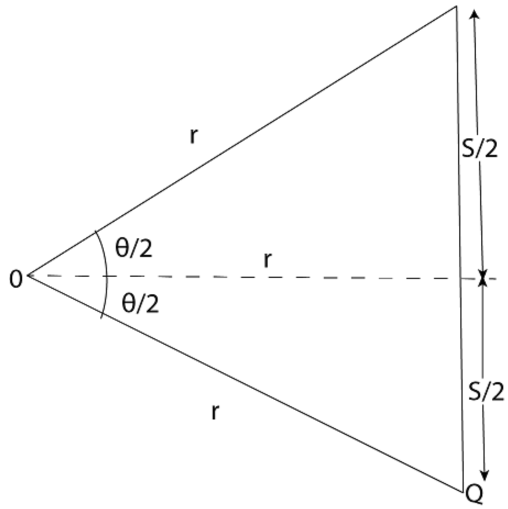


Figure.3

From the figure above

$$sin[\theta/2] = {S/2}/r$$

$$sin[\theta/2] = S/2r$$

If $\theta/2$ is very small

$$sin[\theta/2] = \theta/2$$

Therefore

$$\theta/2 = S/2r$$

$$2S = 2r\theta$$

$$S = r\theta$$

Note

The angle θ in radian is given by

$\theta = {}^{S}/_{r}$

$\theta = {}^{Length\ of\ the\ arc}/_{Radius}$

When $\theta = 360,$ the length of the arc which is the circumference of the circle is $2\pi r$

TERMS RELATING TO CIRCULAR MOTION

Angular Displacement (θ)

Is the angle in radian traced at the center of the circle by the body moving around the circular track.

Or

Is the angle turned by an object moving along a circular path in a given time.

Consider an object moving along a circular path with center O as shown in figure 4.

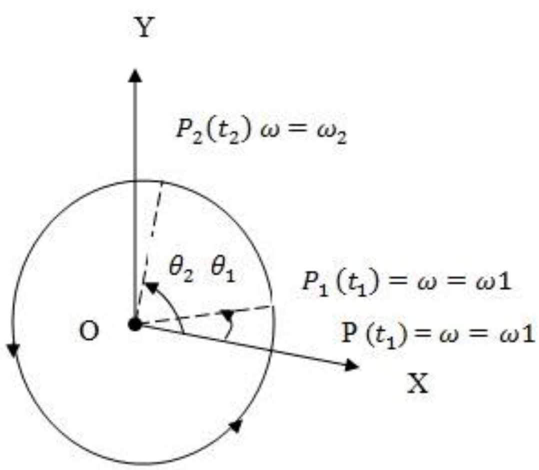


Figure .4

Let us consider O as the origin of our coordinate system.

Suppose initially at t = O, the object is at point P.

At time t_1, the object is at point P_1 and its angular position is θ_1

At time t_2, the object is at point P_2 and its angular position is θ_2

During the time interval $\Delta t = t_1 - t_2$ the angular displacement is $\Delta\theta = \theta_1 - \theta_2$

Angular Velocity (ω)

Is the angular displacement traced per unit time.

Or

Is the rate of change of angular displacement of a body moving along a circular path

If the angular displacement of an object is $\Delta\theta = \theta_2 - \theta_1$ during the time interval $\Delta t = t_2 - t_1$ then the angular velocity of the object is

$$\omega = \Delta\theta / \Delta t$$

$$\omega = \theta_2 - \theta_1 / t_2 - t_1$$

SI unit of angular displacement is rad/s

The angular velocity of an object moving along the circular path at any instant of time is called instantaneous angular velocity

It is denoted by ω. It is given by the limit of $\Delta\theta / \Delta t$ as Δt approaches zero

$$\omega = \Delta\theta / \Delta t$$

lim (Δt→0)

$$\Delta\theta / \Delta t = d\theta / dt$$

$$\omega = d\theta / dt$$

Angular Acceleration α.

Is the rate of change of angular velocity

$\propto = {}^{\omega}/_{t}$

If change in the angular velocity of an object is $\Delta\omega = \omega_2 - \omega_1$ during time interval $\Delta t = t_2 - t_1$, the average angular velocity is given by

$\propto = {}^{\Delta\omega}/_{\Delta t}$

$\propto = {}^{\omega_2 - \omega_1}/_{t_2 - t_1}$

The SI unit of angular acceleration is rad/s^2

The angular acceleration of an object moving along a circular path at any instant of time is called instantaneous angular acceleration.

It is denoted by α. It is given by the limit of ${}^{\Delta\omega}/_{\Delta t}$ as Δt approaches zero.

$\propto = {}^{\Delta\omega}/_{\Delta t}$

lim $\Delta t \rightarrow 0$

$\propto = {}^{d\omega}/_{dt}$

Time Period (T)

Is the time taken by the body moving along a circular path to complete one revolution.

It is denoted by T and its unit is second

For example, if an object completes 120 revolutions in 30 seconds, its time period is given by

$$T = \frac{30}{120} = 0.25_s$$

It means that the object will complete one cycle in 0.25_s

Frequency (f)

Is the number of revolution completed by the object moving along a circular track in one second.

It is denoted by f and its SI unit is s^{-1} or hertz (Hz)

Thus in the above case, the object completes 120 revolutions in 30 seconds. Therefore, the frequency of the object is

$$f = \frac{120}{30} = 4 \text{ Hertz}$$

It means the object will complete 4 revolutions in one second.

Relation between T and f

Suppose an object executing circular motion has frequency f.

$$f = \frac{1}{T}$$

It means that the object completes f revolutions in 1 second.

- Therefore, the time taken to complete one revolution is 1/f.

Relation between ω, f and T

When an object executing circular motion completes one revolution, angular displacement θ and time taken is T.

From

$$\omega = \theta/T$$

$\theta = 2\pi$

$\omega = {2\pi}/{T}$

$\omega = 2\pi f$

Relation between V and ω

As an object moves along the circumference of the circle, it has linear velocity V which is always Tangent to the circular path at every instant as shown in figure 5.

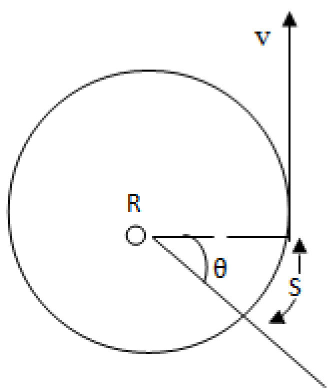


Figure . 5

The relation between the linear velocity V and angular velocity w of the object can be found as under

$v = {ds}/{dt} = {d(s)}/{dt}$

But

$s = r\theta$

$v = {d(r\theta)}/{dt} = r\,{d\theta}/{dt}$

$v = r\,{d\theta}/{dt}$

But

$\omega = {}^{d\theta}/_{dt}$

$v = r\omega$

This is an important relationship between the circular motion of an object and the linear motion that results from rotation.

Relation between linear acceleration and angular acceleration.

The relation between linear acceleration θ and angular acceleration α can be found as follows.

$a = {}^{dv}/_{dt} = {}^{d(v)}/_{dt}$

$a = {}^{d(v)}/_{dt}$

$v = r\omega$

$a = {}^{d(r\omega)}/_{dt} = {}^{rd\omega}/_{dt}$

$a = {}^{rd\omega}/_{dt}$

$\propto = {}^{d\omega}/_{dt}$

$a = r \propto$

EQUATIONS OF UNIFORM MOTION AS APPLIED TO CIRCULAR MOTION.

Consider the particle of mass m moving around the circular track with uniform angular acceleration.

1) To Derive $\omega = \omega_0 + \propto T$

From

$\propto = {d\omega}/{dt}$

$d\omega = \propto dt$

At $t = 0, \omega = \omega_0 \; and \; at \; t = t, \omega = \omega$

$\int_{\omega_0}^{\omega} d\omega = \int_0^t \propto dt$

$[\omega]_{\omega_0}^{\omega} = \propto [t]_0^t$

$\omega - \omega_0 = \propto t$

$\omega = \omega_0 + \propto t$

2) To derive $\theta = \omega_0 t + {1}/{2} \propto t^2$

$\omega = {d\theta}/{dt}$

From

$d\theta = \omega dt$

At $t = 0, \theta = 0 \; and \; at \; t = t, \theta = \theta$

$\int_0^{\theta} d\theta = \int_0^t \omega dt = \int_0^t (\omega_0 + \propto t) dt$

$\int_0^{\theta} d\theta = \int_o^t \omega_0 dt + \int_0^t \propto t dt$

$[\theta]_0^{\theta} = \omega_0 [t]_0^t + \propto \left[{t^2}/{2}\right]_0^t$

$\theta = \omega_0 t + {1}/{2} \propto t^2$

3) To derive $w^2 = w_o^2 + 2\alpha Q$

From

$\omega = {d\theta}/{dt}$

$\propto = {d\omega}/{dt}$

$\propto = {d\omega}/{dt} . {d\theta}/{dt}$

$\propto = {d\theta}/{dt} . {d\omega}/{dt}$

$\propto = {\omega d\omega}/{d\theta}$

$\omega d\omega = \propto d\theta$

When $\theta = 0, \omega = \omega_0$ $and\ when\ \theta = \theta, \omega = \omega$

Centripetal Acceleration (a_c)

Is the acceleration possessed by the body which is moving around the circular track and always directed towards the center.

It is also called radial acceleration because it always acts radially towards the center of the circle.

This acceleration must be in the same direction as it passes *e* towards the center of the circle.

For a body which is moving with constant angular velocity ω along a circular path of radius r, the magnitude of the centripetal acceleration to be given by

$$a_C = \omega^2 r$$

If the linear speed of the particle is V, then the centripetal acceleration is given by

$$a_C = {v^2}/{r}$$

It's SI unit is m/s^2

For non uniform circular motion, acceleration has two components, centripetal component and tangential component (at).
The magnitude of the resultant acceleration is

$$a = \sqrt{a_C^2 + a_t^2}$$

$$a_t = r \propto$$

To show that the centripetal

Acceleration = $\frac{V^2}{r}$

Consider a particle moving with constant speed V along an arc NOP as in figure 6.

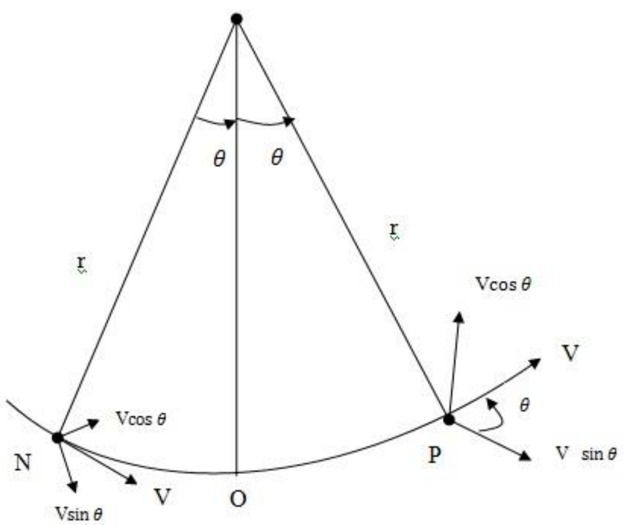


Figure. 6

The x – component of velocity of the particle has the same value at P as at N and therefore its x-component of acceleration a_x is zero

$$a_x = O$$

As the particle moves from N to P its y-component of velocity changes by 2Vsinθ.

If this takes place in a time interval t its y – component of acceleration, ay is given by

$a_y = \frac{v_y}{t}$

$a_y = {}^{2V\sin\theta}/_{t} - - - - - - - - - - - - - - - -(i)$

The speed of the particle along the arc is V, and therefore

$t = {}^{Arc\ length\ NOP}/_{V}$

$t = {}^{2\theta r}/_{V} - - - - - - - - - - - - - - - -(ii)$

Sub equation (ii) into equation (i)

$a_y = {}^{2V\sin\theta}/_{t}$

$a_y = {}^{2V\sin\theta}/_{1} \times {}^{V}/_{2\theta r}$

$a_y = {}^{V^2}/_{r} . {}^{\sin\theta}/_{\theta}$

If N and P are now taken to be coincident at O, then θ approaches to zero, and ${}^{\sin\theta}/_{\theta}$ has its limiting value of 1 in this case.

$a_y = {}^{V^2}/_{r}$

Thus at O, a_x = O and a_y = ${}^{V^2}/_{r}$ and therefore the acceleration is directed along Oz i.e. towards the center of the circle.

Alternately

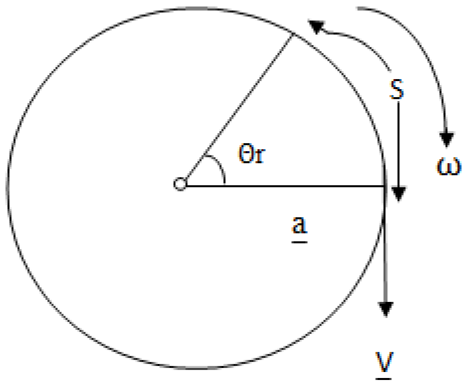


Figure 7

From the figure 7 above, the distance of displacement S arc is given by

S = r^{θ}

Differentiate by product rule

$S = r\theta$

$d\theta = d(r\theta)$

$dS = rd\theta + \theta dr$

But per body moving in circular path of common radius (Since radius is constant)

$dr = O$

Then

$ds = rd\theta$

Make $d\theta$ the subject

$d\theta = {}^{ds}/_{r} - - - - - - - - - - - - - - -(i)$

From

$v = {}^{s}/_{t}$

$s = vt$

Differentiate by product rule

$ds = d(vt)$

$ds = tdv + vdt$

At, $t = 0, when\ the\ body\ starting\ the\ motion$

$dt = 0$

$ds = tdv -(ii)$

Substitute equation (ii) into equation (i)

$d\theta = {}^{ds}/_{r}$

$d\theta = {}^{tdv}/_{r}$

Divide by t on the $R.H.S$

$d\theta = {}^{t/t\ dv}/_{r/t}$

$d\theta = {}^{dv}/_{r/t}$

Since

$V = {}^{r}/_{t}$

$d\theta = {}^{dv}/_{v} - - - - - - - - - - - - - - - (iii)$

Also from

$ds = vdt + tdv$

When the body is moving around the circular track, velocity remains constant

$dv = 0$

$ds = vdt - - - - - - - - - - - - (iv)$

Substitute equation (iv) into equation (i)

$d\theta = {dS}/{r}$

$d\theta + {vdt}/{r} - - - - - - - - - - - - - - (v)$

Equate the equation (iii) and equation (v)

${dv}/{v} = {vdt}/{r}$

$dv = {v^2}/{r}\, dt$

${dv}/{dt} = {v^2}/{r}$

$a = {v^2}/{r}$

Centripetal Force (F_c)

Is the force possessed by the body moving around circular path and always directs the body towards the centre

OR

Is the force acting on a body moving along a circular path with uniform speed and is directed towards the centre of the circle.

$F_c = ma_c$

$F_c = {mv^2}/{r}$

From

$V = \omega r$

$v^2 = \omega^2 r^2$

$F_c = {m\omega^2 r^2}/{r}$

$F_c = m\,\omega^2 r$

If

$\omega^2 = {4\pi^2}/{T^2}$

$F_c = {4\pi^2 mr}/{T^2}$

Also

$F_c \propto \frac{1}{T^2}$, since $f = \frac{1}{T}$ then

$F_c = 4\pi m f^2 r$

More over

$F_c = {mv^2}/{r}$

$F_c = mv.{v}/{r}$

$V = r\omega$

$linear\ momentum\ p = mv$

$\omega = {V}/{r}$

$F_c = p\omega$

From

$F_c = m\,\omega^2 r$

$F_c = r\omega . m\omega$

But

$V = r\omega$

$F_c = m.v\omega$

No work is done by the centripetal Force

Work done $=$ F.S Cos θ

$= F.S\cos 90^0$

Work done = O

Some Common Examples of Centripetal Force

(i) In the case of planets orbiting around the sun the centripetal force is provided by the gravitational force of attraction between the planets and the sun.

(ii) In the case of an electron moving around the nucleus of the atom, the centripetal force is provided by the electrostatic force of attraction between the electron and proton.

(iii) When a particle tied to a string and whirled in a horizontal circle then the tension in the string provides the centripetal force.

(iv) When charged particle describing a circular path in a magnetic field, then magnetic force exerted on a charged particle that set up the centripetal force .

(v) When a vehicle moves in a circular path on a level road the force of lateral friction between the wheels and the road provides the centripetal force.

Application of Centripetal force in Every Day Life

(a) In separating honey from bees wax

(b) In separating cream from milk

(c) In separating sugar crystals from molasses

(d) In spin drier machines, water particles fly off tangentially through holes in the wall of the machine.

Centrifugal Force (Fictitious Force)

Is the force which does not really act on a body but appears due to the acceleration of the frame

In order to move a body in a circular path, a centripetal force F_c is required. This force acts along the radius towards the centre of the circle.

The reaction of this centripetal force is the centrifugal force.

Both these forces are equal in magnitude but opposite in direction and act on different bodies.

Force example, consider the case of a stone tied at one end and rotated in a circle as shown below.

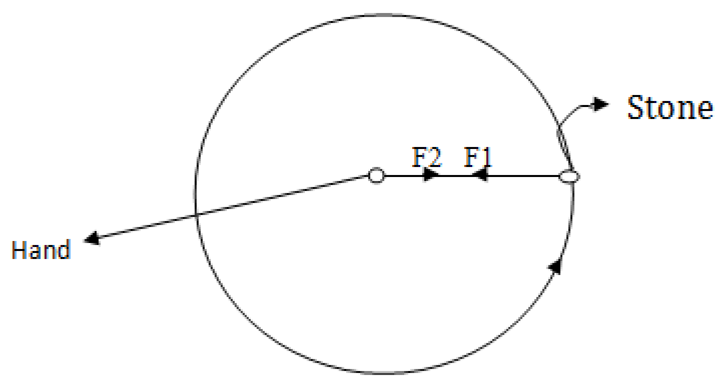


Figure 8

Note that the centripetal force F_1 is applied on the stone by the hand and acts towards the center.

The centrifugal force F_2 acts on hand and pulls it away from the center of the circle.

Centrifugal means center - Fleeing or away from the center.

Therefore, the outward force in circular motion is called centrifugal force.

The magnitude of centrifugal force is the same as that of the centripetal force and its direction is opposite to that of the centripetal force.

$$|F_2| = {mv^2}/{r}$$

$$|F_1| = {mv^2}/{r}$$

APPLICATION OF CIRCULAR MOTION

Circular motion is applied in the following phenomena.

1. Conical pendulum
2. Motion in a vertical circle
3. Looping the loop
4. Banking of roads
5. Cyclist
6. Motorist
7. Centrifuge
8. Rotor
9. Whirling bucket of water around the vertical circle.

The centripetal and centrifugal forces balance each other and therefore; the net force on a particle in circular motion is zero.

1. Conical Pendulum

Is a simple pendulum consisting of a small heavy bob attached to a light inextensible string, the bob is made to r evolve in a horizontal circle with uniform circular motion about a vertical axis, such that the string traces out a cone.

The length of the pendulum ℓ is the distance between the point of suspension and the center of gravity of the bob.

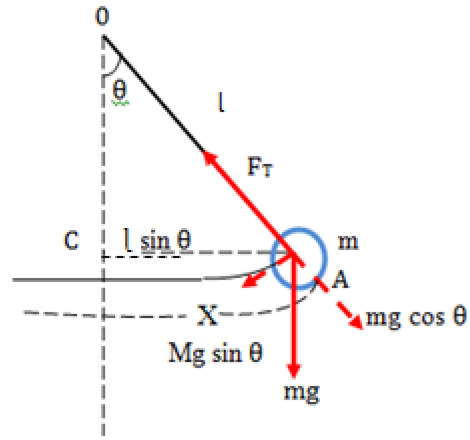


Figure 9

OA = ℓ, length of the pendulum

Let m be the mass of the bob. The forces acting on the bob are weight of the bob mg acting vertically downwards and the tension in the string T directed along the length of the string OA

Let the bob move in horizontal circle r with a uniform speed V.

The centripetal force acting on the bob is $Rc = \frac{mv^2}{r}$

The tension in the string is resolved into two components.

(i) The horizontal component TSinθ provide the necessary centripetal force

(ii) The vertical component TCosθ balances the weight mg

$$T\sin\theta = {mv^2}/{r} ------------- (i)$$

$$T\cos\theta = mg ------------ (ii)$$

Take equation (i) divide by (ii)

$$T\sin\theta / T\cos\theta = {mv^2}/{r} \times {1}/{mg}$$

$$\tan\theta = {v^2}/{rg} ------------- (iii)$$

From the right angled triangle OCA

$$\tan\theta = {}^{AC}/_{OC}$$

$$\tan\theta = {}^{r}/_{h}$$

Substituting the values of $\tan\theta$ in the equation (iii)

$${}^{r}/_{h} = {}^{v^2}/_{rg} - - - - - - - - - - - - - (iv)$$

But

$$v^2 = r^2\omega^2$$

$${}^{r}/_{h} = {}^{r^2\omega^2}/_{rg}$$

$$\omega^2 = {}^{g}/_{h}$$

$$\omega = \sqrt{{}^{g}/_{h}}$$

Let t be the period of revolution of the pendulum.

$$\omega = {}^{2\pi}/_{t}$$

$${}^{2\pi}/_{t} = \sqrt{{}^{g}/_{h}}$$

$$t = 2\pi\sqrt{{}^{h}/_{g}} - - - - - - - - - - - - - (v)$$

Squaring and add equation (i) and (ii)

$$T^2 sin^2\,\theta + T^2 cos^2\,\theta = \left({}^{mv^2}/_{r}\right)^2 + (mg)^2$$

$$T^2(sin^2\,\theta + cos^2\,\theta) = {}^{m^2v^4}/_{r^2} + m^2g^2$$

$$T^2 = \left({}^{mv^2}/_{r}\right)^2 + (mg)^2$$

From equation (iv)

$$v^2 = {r^2 g}/{h}$$

$$T^2 = \left({mv^2 g}/{rh}\right)^2 + (mg)^2$$

$$T = mg\sqrt{1 + \left({r}/{h}\right)^2}$$

The time period of a conical pendulum is the same as that of a simple pendulum of length h where h is the axial height of the cone.

$$\cos\theta = {h}/{l}$$

$$h = l\cos\theta$$

From

$$T = 2\pi\sqrt{{h}/{g}}$$

$$T = 2\pi\sqrt{{l\cos\theta}/{g}}$$

If θ is very small $\cos\theta$ is nearly equal to 1. So h = ℓ

This means the time period is almost independent of θ if θ is very small.

The principle of conical pendulum is used in the construction of the centrifugal governor used for regulating automatically the speed of engines.

Alternativelly

From equation (iii)

$$\tan\theta = {v^2}/{rg}$$

From the figure 9

$$\sin\theta = {r}/{l}$$

From equation (i)

$$T\sin\theta = {mv^2}/{r}$$

$$Tr/l = {mv^2}/{r}$$

$$v^2 = r^2\omega^2$$

$$Tr^2 = mr^2\omega^2 l$$

$$T = m\omega^2 l$$

WORKED EXAMPLES

1. A car is moving with a speed of $30m5^{-1}$ on a circular track of radius 500m. Its speed is increasing at the rate of $2m/s^2$. Determine the value of its acceleration.

Solution

Since the speed of the car moving on a circular track is increasing, the car has centripetal a_c as well as tangential acceleration a_T. The two accelerations are at right angles to each other

$$a_c = {V^2}/{r}$$

$$a_c = {30^2}/{500}$$

$$a_c = 1.8\, m/s^2$$

Resultant linear acceleration a is given by

$$a = \sqrt{a_c^2 + a_T^2}$$

$$a = \sqrt{1.8^2 + 2^2}$$

$$a = \sqrt{7.24}$$

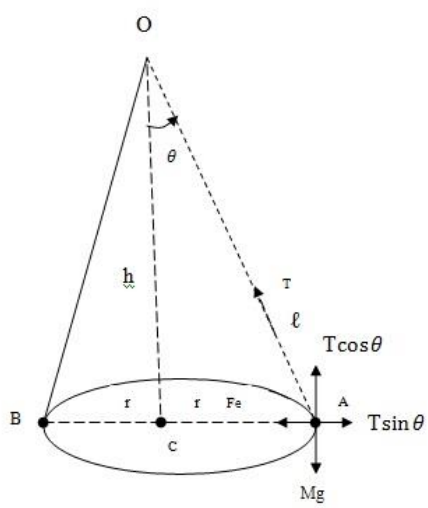
$$a = 2.7\ m/s^2$$

2. A small mass of 1kg is attached to the lower end of a string 1m long whose upper end is fixed. The mass is made to rotate in a horizontal circle of radius 0.6m. If the circular speed of the mass is constant.

Find (i) Tension in the string

(ii) The period of the motion.

Solution



m = 1kð□‘’

$$l = 1m$$

$$r = 0.6m$$

i) Tension in the string

$$T = m\omega^2 l$$

$$= 1 \times 1 \times \omega^2$$

$\sin\theta = {}^{r}/_{l} = {}^{3}/_{5}$

$\cos\theta = {}^{4}/_{5} = 0.8$

$T\sin\theta = {}^{mV^2}/_{r}$

$T\cos\theta = mg$

$T = {}^{mg}/_{\cos\theta} = {}^{9.8}/_{0.8} = 12.25\ N$

(ii) The period of the motion ;

$T\sin\theta = {}^{mv^2}/_{r}$

${}^{3T}/_{5} = {}^{5V^2}/_{3}$

$T = {}^{25v^2}/_{9}$

$v^2 = {}^{9T}/_{25} = {}^{9\times 12.25}/_{25}$

$v^2 = 4.41$

$v = 2.1\frac{m}{s}$

${}^{2\pi r}/_{T} = 2.1$

$T = {}^{2\pi r}/_{2.1}$

$T = 1.8s$

3. A pendulum bob of mass m is held out in the horizontal position and then released. If the length of the string is ℓ.

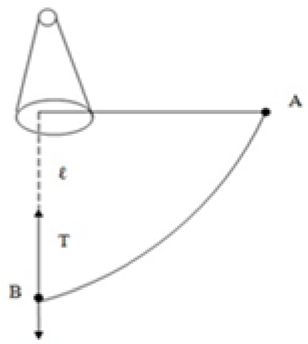
Calculate

(i) The bob velocity

(ii) The force on the string when the bob reaches the lowest position

Solution

Decrease in P.E. of the bob = increase in its K.E. in falling through a height ℓ



i) $mgl = {}^{1}/_{2}mv^2$

$v = \sqrt{2gl}$

At the lowest position B, the forces acting on the bob are re-tension in the string T acting vertically upward and weight of the bob acting vertically down ward. The resultant force gives the centripetal force.

$$\frac{mv^2}{l} = T - mg$$

$$T = mg + \frac{mv^2}{l}$$

But $V = \sqrt{2g\ell}$

$$v^2 = 2gl$$

$$T = mg + \frac{2mgl}{l}$$

$$T = 3mg$$

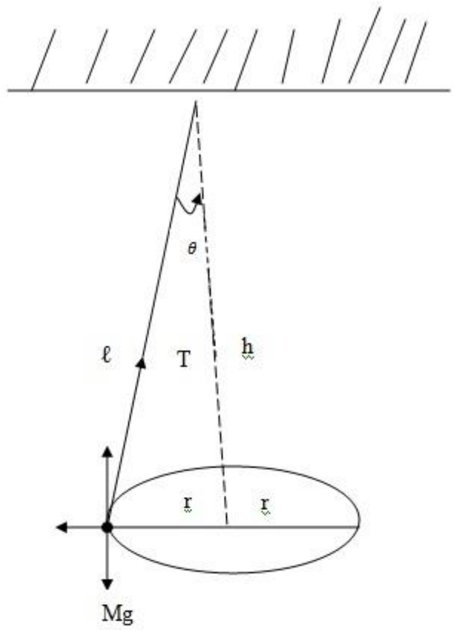
4. A ball of mass 200g is attached to the end of a cord of length 1m and is whirled in a horizontal circle with $\theta = 10^0$

Find

(i) The speed of the ball

(ii) The time period of revolution of the ball

Solution



$\theta = 10^0$

m = 200 × 10^3 kg

$l = 1m$

$T \sin\theta = \frac{mv^2}{r}$ ________ _________ _______ _____ (i)

$T \cos\theta = mg$ ________ _________ _______ (ii)

Take equation (i) ÷ equation (ii)

$$\tan\theta = \frac{V^2}{rg}$$

$$V = \sqrt{rg \tan\theta}$$

But $\frac{r}{\ell} = Sin\theta$

$$r = \ell Sin\theta$$

$$V = \sqrt{\ell Sin\theta \, x \, g \tan\theta}$$

$$V = \sqrt{1xSin10x9.8x\tan 10}$$

(i) \ $V = 0.547m/s$

(ii) $$t = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\ell Cos\theta}{g}}$$

$$t = 2x3.14\sqrt{\frac{1x\cos 10}{9.8}}$$

T = 1.9s

5. A stone is moved in a horizontal circle of radius 1.5m by means of a string at a height of 2m above the ground. The string breaks and the particle fly off horizontally, striking the ground 10m away. Find the centripetal acceleration during circular motion.

Solution

$S = ut + \frac{1}{2}gt^2$

u = O

$$2 = Ut + \frac{1}{2}x9.8t^2$$

t = 0.64 seconds

$$10m = vt$$

$$V = \frac{10m}{t}$$

$$V = \frac{10}{0.64}$$

V = 15.63 m/s.

a_L $\frac{V^2}{r}$ $= \frac{244.14}{1.5}$

$a_ι = 162.76\ m/s^2$

6. A sphere of mass 200g is attached to an inextensible string of length 130cm whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 50cm.

Calculate

(i) Time period of revolution

(ii) Tension in the string

Solution

(i)The period of revolution.

$$T = 2\pi\sqrt{\frac{ι\cos\theta}{g}}$$

$$\cos\theta = \frac{120}{130} = 0.92$$

$$T = 2 \times 3.14\sqrt{\frac{130x\,10^{-2} \times \cos\theta}{9.8}}$$

$$T = 2 \times 3.14\sqrt{\frac{130x\,10^{-2} \times 0.92}{9.8}}$$

T = 2.2 second

(ii) Tension in the string T.

$$T = \frac{Mg}{\cos\theta} = \frac{200x10^{-3}x9.8}{0.92} = 2.12N$$

7. A string of length 1m fixed at one end and carries a mass of 100g at the other end. The string makes $2/\pi$ revolutions per second around a vertical axis passing through its fixed end.

Calculate

(a) The angle of inclination of the string with the vertical

(b) The tension in the string

(c) Linear velocity of the mass

Solution

$\ell = 1m, r = 50x10^{-2}m, m = 100x10^{3}$ kg.

(a) $TCos\theta = mg$

$$Cos\theta = \frac{mg}{T} = \frac{100x10^{-3}x9.8}{1.6}$$

$$Cos\theta = 0.6125$$

$$\theta = Cos^{-1}(0.6125)$$

$$\theta = 52.23^{0}$$

(b)

$$TSin\theta = \frac{MV^{2}}{r}$$

$$Sin\theta = \frac{r}{\ell}$$

$$v^2 = r^2\omega^2$$

$$T = m\omega^2 l$$

$$T = Mx\frac{4\pi^2}{t^2}x\ell$$

$$t^2 = \frac{\pi^2}{4}\ T = \frac{1}{f} = \frac{\pi}{2}$$

$$T = M\ x4\pi^2\ x\ \frac{4}{\pi^2}\ x\ell$$

$$T = 100x10^1x16$$

\ T = 1.6N

1. Motion in a horizontal circle

Consider a body of mass m tied to a mass less inextensible string and whirled in a horizontal circle of radius *r*

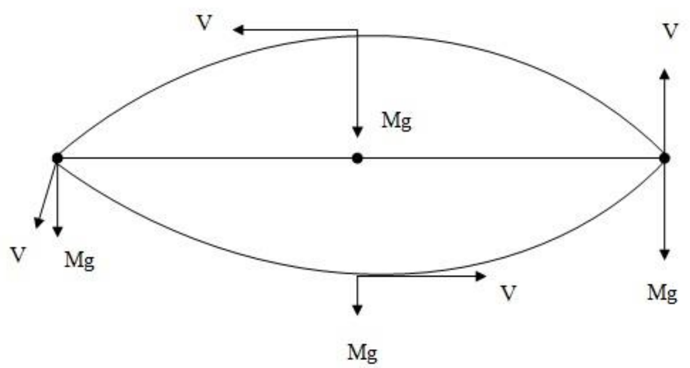


Figure 10

To keep the body in a circular path a force must act on the body directed towards the center of the circle called the continental force.

The direction of velocity of the body is tangential to the circular path and is perpendicular to the weight mg of the body

Since F and mg are perpendicular, mg will not disturb the motion of the body. So it will continue to move in circular path.

2. Motion of a body in a vertical circle

Motion in a vertical circle is a non uniform circular motion described by a particle along a vertical circular path about a fixed point as the center of the path.

The body, whirled in vertical circular path is not uniform since the speed of the particle is different at different point on the circular path.

Consider a body of mass m tied at one end of the string and whirled in a vertical circle of radius *r* as shown below.

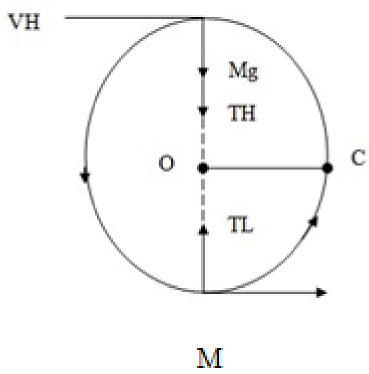


Figure 11

As the body moves from the lowest point L of the vertical circle to the highest point H, the speed of the body goes on decreasing because gravity opposes motion.

On the other hand, as the body moves from the highest point *H* to the lowest point *L*, the speed of the body increases because gravity helps the motion.

Therefore, the speed of the body is minimum at the highest point *H* while the speed of the body is maximum at the lowest point *L*.

Note that; the linear velocity is always along the tangent to the circular path at every instant.

(i) Tension in the string at points *L* and *H* as the body moves in a vertical circle, the tension in the string varies.

It will be maximum when the body is at the lowest point *L* and minimum when the body is at the highest point *H*.

Suppose VL and VH are the velocities of the body. TL and TH are the tensions in the string at the lowest point *L* and at the highest point H respectively.

At the lowest point *L* of the motion, the weight mg of the body acts in opposite direction to the tension TL in the string.

$$T_L - mg = \frac{mv^2_L}{r}$$

$$T_L = \frac{mv^2_L}{r} + mg$$

At the highest point *H* of the motion, both the tension TH and the weight *Mg* of the body act downwards towards the centre *O* of the circle.

$$T_H + \frac{MV_4^2}{r}$$

$$T_H = \frac{mv^2_L}{r} - mg$$

Tension in the string is maximum when the body is at lowest point L. It is here that the string is most likely to break.

However, tension in the string is minimum when the body is at the highest point *H*.

(ii) Minimum velocity at highest point H to complete circle.

It is clear that the velocity at the highest point H will be minimum when TH is minimum.

$$T_{H^-} = O$$

$$O = \frac{MV_4^2}{r} - Mg$$

$$O = \frac{MV_4^2}{r} - Mg$$

$$V_H^2 = rg$$

$$V_H = \sqrt{rg}$$

Thus in order that the body loops the vertical circle, the minimum velocity of the body at the highest point *H* should be $\sqrt{rg}$, the body will fail to loop the circle if velocity at point H is less than $\sqrt{rg}$.

3. Minimum velocity at the lowest point L to complete circle.

In order that the body loops the vertical circle, it must possess certain minimum velocity at the lowest point *L*.

Thus minimum velocity should be such that when the body reaches the highest point *H*, its velocity should be $\sqrt{rg}$.

According to principle of conservation of energy

K.E. of body = (P.E. + K.E.) of body

At point *L* at point *H*

$$1/_2 m v_L^2 = m g h_H + 1/_2 m v_H^2$$

$$h_H = 2r$$

$$V_L^2 = 2mg(2r) + V_H^2$$

$$V_L^2 = 4gr + rg$$

$$V_L^2 = 5rg$$

$$V_L = \sqrt{5rg}$$

Thus, in order that the body loops the vertical circle, the minimum velocity of the body at the lowest point L should be $\sqrt{5rg}$

If the velocity at point L is less than $\sqrt{5rg}$, the body will fail to loop the vertical circle.

4. Minimum velocity when the string is horizontal

In order that the body loops the vertical circle, it must have minimum velocity V_c when the string becomes horizontal at point *C*.

$$\frac{\text{Total energy at C}}{\text{at C}} = \frac{\text{Total energy at L}}{\text{at L}}$$

$$\frac{1}{2}mv_c^2 + mgh_c = \frac{1}{2}mv_L^2$$

$$h_c = r$$

$$V_C^2 + 2gr = V_L^2$$

$$V_C^2 = V_L^2 - 2gr$$

$$V_C^2 = 5rg - 2rg$$

$$V_c^2 = 3rg$$

$$V_c = \sqrt{3rg}$$

Thus, in order that the body loops the vertical circle, the minimum velocity of the body at point L should be $\sqrt{3rg}$.

Difference in tension at the lowest and highest point

TL is the tension at the lowest point and *TH* that at the highest point *H*

$$T_L = {mV_L^2}/{r} + mg$$

$$T_H = {mV_H^2}/{r} - mg$$

$$T_L - T_H = \left[{mV_L^2}/{r} + mg\right] - \left[{mV_H^2}/{r} - mg\right]$$

$$T_L - T_H = {mV_L^2}/{r} - {mV_H^2}/{r} + 2mg$$

$$T_L - T_H = m\left[{V_L^2}/{r} - {V_H^2}/{r} + 2g\right]$$

But

$$V_L^2 = 5rg$$
$$V_H^2 = rg$$

$$T_L - T_H = m\left[5rg/_r - rg/_r + 2g\right]$$
$$T_L - T_H = m[5g - g + 2g]$$
$$T_L - T_H = 6mg$$

Therefore, the tension in the string at the lowest point L is greater than the tension at the highest point H by six times the weight of the body.

Note

For the body to loop the vertical circle, its velocity at points *L, C* and *H* should satisfy the following conditions.

$V_L \geq \sqrt{5rg}$ $\quad$ $V_C \geq \sqrt{3rg}$

$V_H \geq \sqrt{rg}$

Motion of a body in a vertical circle at any point *P* On the circle.

Consider a body of mass M tied at one end of a string and whirled in a vertical circle of radius *r* as shown below

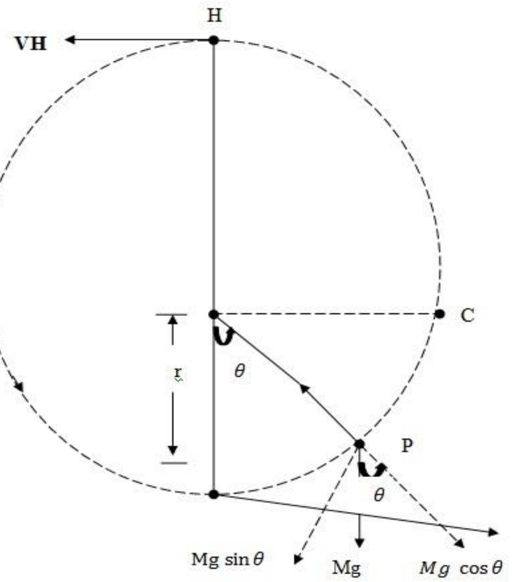


Figure 12

Let at any time, the body be at point P at an angular position θ.

Suppose at point P, T is the tension in the string and V is the linear velocity of the body

The various forces acting on the body are

(i) Weight Mg of the body acting vertically downwards.

(ii) Tension T in the string acting along PO

The weight Mg can be resolved into two perpendicular components

(i) $MgCos\theta$ opposite to *T*

(ii) $Mg\sin\theta$ along tangent to the circle at *P*

The net force on the body at P provides necessary centripetal force required by the body.

$$T - MgCos\theta = \frac{MV^2}{r}$$

This is the general equation describing the motion of the body in a vertical circle.

$\underline{At\ the\ lowest\ point\ L}$

At the lowest point L $\quad T = T_L, V = V_L$ and $\theta = 0^0$

$$T_L - MgCos\theta\, O^0 = \frac{MV_L^2}{r}$$

$$CosO^o = 1$$

$$T_L - Mg = \frac{MV_L^2}{r}$$

$\underline{At\ the\ highestpoi\ nt\ H}$

At the highest point H $\quad T = T_H, V = V_H$ and $\theta = 180^0$

$$T_H - MgCos\,\theta 18O^0 = \frac{MV_H^2}{r} \qquad Cos18O^o = 1$$

$$CosO^o = 1$$

$$T_H - Mg = \frac{MV_H^2}{r}$$

When string is horizontal at point *C*

The string is horizontal at point C where velocity is Vc and $\theta = 90^0$

$$T_c = MgCos90 = \frac{MV_c^2}{r}$$

$$Cos\ 90 = 0$$

$$T_c = \frac{MV_C^2}{r}$$

Looping the Loop

This is an example of circular motion. It involves the motion of the body o the inclined plane and complete a vertical circular track (path) without falling down at the top of the loop.

The body can be described by the figure below,

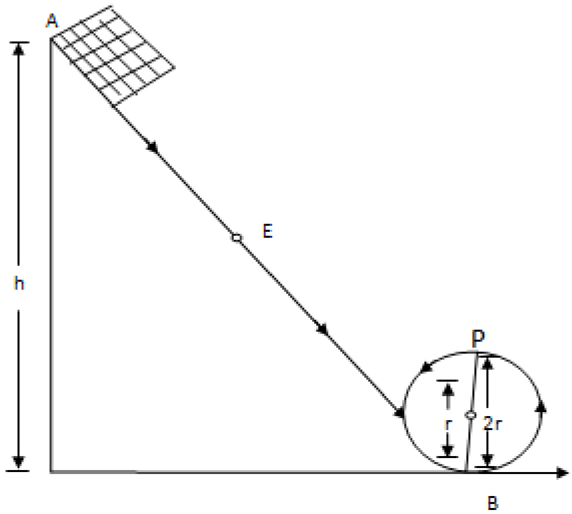


Figure 13

$$\frac{\text{Energy changes as the body}}{\text{moves from } A \text{ to } C}$$

(Ignore frictional effects)

At point A

The body at point *A* has all its energy due to the height *h* i.e. potential energy.

Since it is at rest, there is no kinetic energy.

Hence at point *A* all its energy is potential energy.

At point E

At any instant between point A and B, the body will posses both kinetic energy and potential energy.

$$E_T = K.E + P.E$$

At point B

As it moves along the inclined plane from *A* to *B*, part of its potential energy is converted to kinetic energy.

The body will have all its energy as on kinetic energy at point *B*, where it also possesses maximum speed.

The total energy at *B* is *K.E* with zero potential energy

At point P

As it starts to describe the circular path from *B* to *P*, some of its kinetic energy is converted again to potential energy.

At point P, the body will have part of its energy as kinetic energy and p art of it as potential energy.

It will have all its energy as kinetic energy as it comes again to point *B* after looping the loop

By applying the principle of conservation of energy

Total energy = Total energy

at A at P

$$P.E_A = K.E_P + P.E_P$$

$$Mgh_A = \frac{1}{2}MV_P^2 + Mg_P$$

But

$$h_A = h,$$

$$Hp = 2r$$

$$\backslash mgh = \frac{1}{2}MV_P^2 + 2Mgr$$

$$gh = \frac{V_P^2}{2} + 2gr \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (i)$$

For the body to pass at the highest point P on the vertical circle, tension should be minimum

$$T = \frac{MV_P^2}{r} - Mg$$

When $T = O$

$$\frac{MV_P^2}{r} = Mg$$

$$V_P^2 = rg \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (ii)$$

Sub equation (ii) into equation (i)

$$gh = \frac{V_P^2}{2} + 2gr$$

$$gh = \frac{rg}{2} + 2gr$$

$$h = \frac{r}{2} + 2r$$

$$h = \frac{5r}{2}$$

* If a bucket containing water is rotated in a vertical circle such that its velocity at the lowest point of motion is equal to or greater than $\sqrt{5rg}$, the water will not fall out even if the bucket is at highest point with its open top pointing downwards. It is because the weight Mg of water is less than the required centripetal force MV_H^2 / r of the water in the bucket.

* Similarly, when an aeroplane loops a vertical circle such that its velocity at the lowest point is equal or greater than $\sqrt{5rg}$, the pilot does not fall down even when he is not tied down to his seat in the cockpit.

WORKED EXAMPLES

1. A string 0.5m long is used to whirl a 1Kg stone in a vertical circle at uniform speed of 5m/s. Determine the tension in the string when the stone is

(i) at the top of the circle

(ii) At the bottom of the circle

Solution

$M = 1Kg$ $\qquad$ $r = 5m/s$

-Centripetal force required to keep the stone moving at 5/m/s is

$$F_c = \frac{MV^2}{r}$$

$$F_c = \frac{1x5^2}{0.5} = 50N$$

(i) $T_H = F_c = Mg$

$$T_H = 50N - (1x9.8)$$

$$\therefore T_H = 40.2N$$

(ii) $$T_L = F_c = Mg$$

$$50N - (1x9.8)$$

$$\therefore T_L = 59.8N$$

2. A bucket containing water is tied to one end of a rope 1.25m long and rotated about the other end in a vertical circle. What should be the minimum velocity at the highest and lowest points so that water in the bucket may not spill?

Solution

In order that water in the bucket may not spill, the minimum velocity at the highest point should be

$$V_H = \sqrt{rg}$$

$$V_H = \sqrt{1.25x9.8}$$

$$\therefore V_H = 3.5m/s$$

Minimum velocity at a lowest point

$$V_L = \sqrt{5rg}$$

$$V_L = \sqrt{51.25x9.8}$$

-

3. The road way bridge over a canal is in the form of an arc of a circle of radius 20m. What is the maximum speed with which a car can cross the bridge without leaving contact with the ground at the highest point.

Solution

At highest point

$$T_H + Mg = \frac{MV^2}{r}$$

For maximum velocity Vmax, *TH = O*

$$\frac{MV_{max}^2}{r} = Mg$$

$$V_{max} = \sqrt{rg} \qquad \sqrt{20x9.8}$$

$$\therefore V_{max} \text{ is } 14m/s$$

4. A stone of mass 0.5kg tied to a rope of length 0.5m revolves along a circular path in a vertical plane. The tension I the rope at the bottom point of the circle is 45N. To what height the stone will rise if the rope breaks the moment the velocity is directed upwards? (g = 10m/s2)

Solution

$$T_L = \frac{MV_L^2}{} + Mg$$

M = 0.5Kg
TL = 4SN
r = 0.5m

$$\frac{MV_L^2}{r} = T_L = Mg$$

$$V_L^2 = \frac{(T_L - Mg)r}{M}$$

$$V_L^2 = \frac{(4S - (0.5x10))x0.5}{0.5}$$

$$V_L^2 = 40$$

When the rope breaks, the stone rises to a height h until its entire K.E is converted to P.E.

$$\frac{1}{2}MV^2 = Mgh$$

$$h = \frac{V^2}{2g} = \frac{40}{2x20}$$

$$\therefore \text{h} = 2\text{M}$$

Motion of a car on a level road

When a car moves on a flat horizontal circular road the force of friction between the Tyre and the road provides the necessary centripetal force.

Consider a car of total mass *M* moving with a constant speed *V* on a flat horizontal road.

Let the car move in a curved path of the radius r.

When the car moves around a curved path the inner and the outer wheels experience differential normal reactions, say R1on the inner wheels and *R2* o the outer wheel.

$$R_1 + R_2 = Mg \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (i)$$

The frictional force acting on the wheels are F1 and F2

$F_1 = \mu R_1$ and $F_2 = \mu R_2$

Total frictional force provides the necessary centripetal force.

$$F_1 + F_2 = \frac{MV^2}{r}$$

$$r >> 2d$$

Where

2d is the distance between the inner and the outer wheels.

Thus the maximum velocity with which a car can turn safely

$$V_{max} \leq \sqrt{\mu r g}$$

Let the height of the center of gravity of the car above the horizontal road is h.

The frictional forces and the normal reactions produce torque o the car. The car will not overturn if the net torque about the center of gravity *G* is zero.

$$(F_1 + F_2)h + R_1 d - R_2 d = 0$$

$$\frac{MV^2 h}{r} + R_1 d - R_2 d = 0$$

From eqn (i)

$$R_1 + R_2 = Mg$$

$$R_2 = Mg - R_1$$

$$\frac{MV^2 h}{r} + R_1 d = (Mg - R_1)d = 0$$

$$\frac{MV^2 h}{r} + R_1 d + R_1 d - Mgd = 0$$

$$2R_1 d = Mgd - \frac{MV^2 h}{r}$$

$$R_1 = \frac{Mgd}{2d} - \frac{MV^2h}{2rd}$$

$$R_1 = \frac{Mg}{2} - \frac{MV^2h}{2rd}$$

$$R_1 = \frac{M}{2}\left[g - \frac{V^2h}{rd}\right]$$

Also

From

$$R_1 + R_2 = Mg$$

$$R_1 = Mg - R_2$$

$$\frac{MV^2h}{r} + R_1d - R_2d = 0$$

$$\frac{MV^2h}{r} + R_2d + (Mg - R_2)d = 0$$

$$\frac{MV^2h}{r} + R_2d - R_2d + Mgd = 0$$

$$2R_2d = \frac{MV^2h}{r} + Mgd$$

$$R_2 = \frac{MV^2h}{2rd} + \frac{Mgd}{2d}$$

$$R_2 = \frac{Mg}{2} + \frac{MV^2h}{2rd}$$

$$R_1 = \frac{M}{2}\left[g - \frac{V^2h}{rd}\right]$$

Since $R_2 > R_1$, then the reactions on the outer wheels are greater.

With increase in speed V, the reactions on inner wheels decrease and that o the outer wheels increase.

For a certain speed it is possible that R1 reduces to zero, and then the car may topple:

The condition per toppling

$R_{1 = 0}$

From $$R_1 = \frac{M}{2}\left[g - \frac{V^2 h}{rd}\right]$$

$$O = \frac{Mg}{2} - \frac{MV^2 h}{2rd}$$

$$\frac{MV^2 h}{2rd} = \frac{Mg}{2}$$

$$\frac{V^2 h}{rd} = g$$

$$V^2 = \frac{gdr}{h}$$

$$V = \sqrt{\frac{gdr}{h}}$$

The safe speed with which the vehicle can move around a curved path without toppling is

$$V < \sqrt{\frac{gdr}{h}}$$

MOTION OF A VEHICLE ON A LEVEL CIRCULAR ROAD

When a vehicle goes around a curve, a centripetal force must act on the vehicle to keep it on the circular path. On level circular road this force is provided by the static friction between the tires and the road. If the friction is not great enough the vehicle fails to make a curve and the tires slide sideways. We say the vehicle **Skids.**

Consider a car if mass M going around a curve of radius r with constant speed V on a level circular road as shown in figure below.

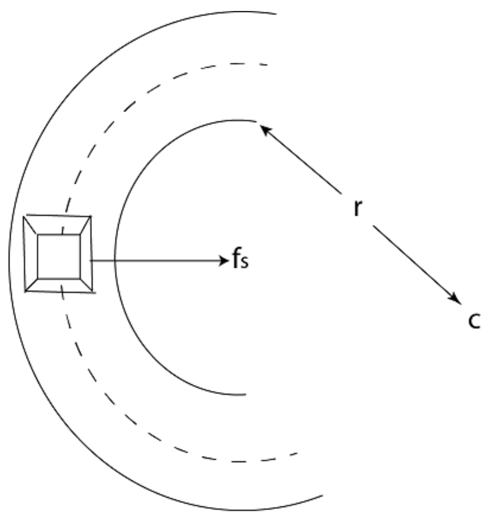


Figure 14 (a)

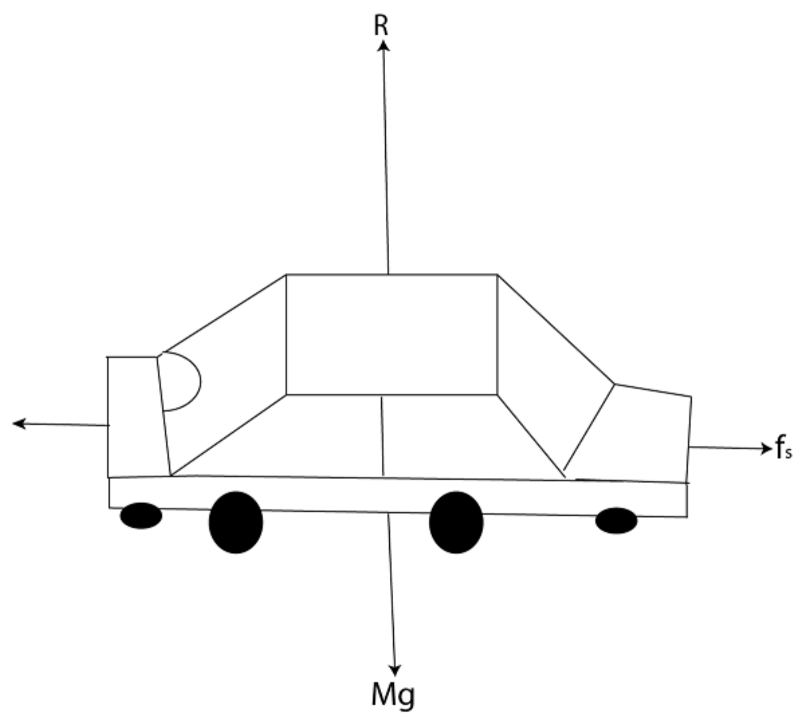
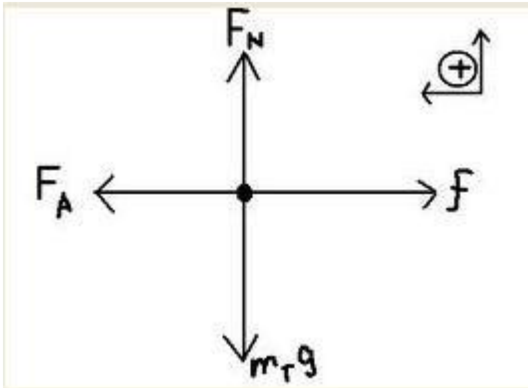


Figure 14 (b)

The car's free body diagram is shown below



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The static friction rather than kinetic friction provides the centripetal force because car's tires do not slide on the road.

Note that Fs acts inward, pointing towards the center C. The reason is simple as a car negotiates the curve, The tires tend to go away from the center of the curve. Consequently, the force of friction acts towards the center of the curve

Since there no vertical motion of the car

R = mg

The centripetal force required by the car to go around the curve is provided by the static friction Fs between the tyres and the road.

$F^S = mV^2/r$ -----------------------------(i)

As the speed v of the car increases, Fs required also increases. But Fs cannot exceed the maximum value

$$F_{S_{(max)}} = \mu_S R$$

The maximum speed Vm corresponding to $F_{S_{(max)}}$ is

$$F_{S_{(max)}} = MV\,m^2 / r$$

$\mu_S R$ = MV m ^{2} / r

$\mu_S Mg$ = MV m ^{2} / r

$$V_m^2 = \mu_S rg$$

$$V_m = \sqrt{\mu_S rg}$$

If the car exceed this speed sliding will begin and the car will no longer travel in a circle.

Vm is independent of the mass of the car but depends upon the coefficient of static fraction and the radius of the curve.

BANKING OF ROADS

Is the process of raising the outer edge of a curved road above the level of the inner edge.

When an automobile goes around a curve the road must exert an inward centripetal force on automobile in order to move in the circular path his force is provided by the friction between tyres and the road.

Generally, this friction is inadequate so that the automobile has the tendency to **skid** $i.e$ to leave the circular path.
In order to avoid this, road curves are banked $i.e$ outer edge of the road is raised by some angle θ above the level of the inner edge.

To avoid **wear** and **tear** of the tire, inclination is given to the road by raising the outer side of the road a little above the inner side.

Banking Angle θ

Is the angle which the surface of the road makes with the horizontal

Safety speed limit

Is a specified speed of vehicle with which curved roads are correctly banked so as to avoid tendency of skidding and therefore no frictional force is required.

Types of banking

(i) Perfect banking

(ii) Imperfect banking

PERFECT BANKING

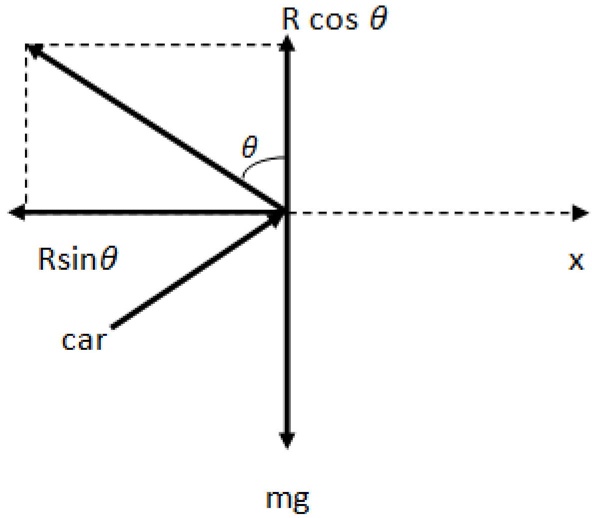
Is the type of banking of roads in which a vehicle moves with safety speed limit round a banked circular road.

In this type of banking, no frictional force is needed as the horizontal components of the normal reaction provide all the necessary centripetal force required.

For each banking angle require, there is one particular speed per which the horizontal component of normal reaction R provides the necessary centripetal force for turning the car without skidding even if there is no friction between the tires and the road.

Figure below shows the curved road AB of radius r banked at an angle θ

Suppose a car goes around this curve with a speed V



Free body diagram of the car

$$R\cos\theta = mg \text{ ------------------ } (i)$$

$$R\sin\theta = {mV^2}/{r} \text{ ------------------ } (ii)$$

Dividing equation (ii) by equation (i)

$$\frac{\text{RSinQ}}{\text{RCosQ}} = \frac{\text{mV}^2}{\text{r}} \cdot \frac{1}{\text{mg}}$$

Tan θ = V^2 / r

Imperfect banking

Is the type of banking of roads in which the vehicle moves with speed which is dependent from safety speed limit round a banked circular road.

In this imperfect banking frictional force is invited along the banked road to contribute part of the necessary centripetal force required as the horizontal component of the normal reaction does not provide sufficient centripetal force on vehicle without skidding.

$Rcos\,\theta = F_S sin\,\theta + mg$

$Rcos\,\theta - F_S \sin\theta = mg$

But, $F_S = \mu_S R$

$Rcos\,\theta - \mu_S Rsin\,\theta = mg$

$R(\cos\theta - \mu_S \sin\theta) = mg - - - - - - - - - - - - - (i)$

$Rsin\,\theta + F_S \cos\theta = {mV^2}/{r}$

$Rsin\,\theta + \mu_S Rcos\,\theta = {mV^2}/{r}$

$R(sin\theta + \mu_S \cos\theta) = {mV^2}/{r} - - - - - - - - - - - - - - - (ii)$

Putting the value of R from equation (i) into equation (ii)

From, R (cos θ - μs Sin θ) = mg

R = $^{m}{}_{2}/ \cos\theta - \mu_s \mathrm{Sin}\theta)$___ __ ___ ___ _ _$(iii)$

Substitute equation (iii) into equation (ii)

$R(\sin\theta + \mu_S \cos\theta) = {mV^2}/{r}$

${(\sin\theta + \mu_S \cos\theta)}/{(\cos\theta - \mu_S \sin\theta)}\,mg = {mV^2}/{r}$

$$V_m^2 = \left[\sin\theta + \mu_S\cos\theta \Big/ \cos\theta - \mu_S\sin\theta\right] rg$$

Divide by $\cos\theta$ on each term in bracket above

$$V_m^2 = \left[\tan\theta + \mu_S \Big/ 1 - \mu_S\tan\theta\right] rg$$

$$V^2 = \sqrt{(\tan\theta + \mu_S) \Big/ 1 - \mu_S\tan\theta \; rg}$$

-

If friction is neglected $\mu_S = 0$

Vm = $\frac{tan\ \theta + 0}{1 - 0}\ rg$

$$V_m = \sqrt{rgtan\,\theta}$$

WORKED EXAMPLE

1. A car traveling at a speed of 500Km/hr tilts at an angle of 30â—¦ as it makes a turn. What is the radius of the curve?

Solution

V = 500 km/hr = 138.9 m/s

θ = 30â—¦

$tan\theta$ = V^2/rg

$$r = {V^2}/{g\tan\theta}$$

$$r = {138.9^2}/{9.8 \times \tan 30}$$

r = 3409. 3m

2.The distance between the inner and outer wheels of a car is 1.75m and its centre of gravity is 0.5m above the ground. What is the maximum speed with which the car can negotiate a curve of radius of 40m without overturning?

Solution

Distance between the inner and outer wheels of car 2d = 1.75m

Height of C.G from the ground h, = 0.5m

Radius of the circular road r = 40m

$$V_{max} = \sqrt{{gdr}/{h}}$$

$$= \sqrt{{9.8 \times 0.875 \times 40}/{0.5}}$$

V_{max} =26.2 m/s

3. A bend in a level road has a radius of 100m. What is the maximum speed which a car turning this bend may have without skidding if the coefficient between the road and tires is 0.3. If the centripetal force is provided by banking. What is the angle of banking

-

r = 100m

μ_s= 0.3

V_{max}= ?

$V_{max} = \sqrt{\mu_s r g}$

$= \sqrt{0.3 \text{ x } 100 \text{ x } 9.8}$

V_{max} = 17.15 m/s

Angle of banking $\tan\theta = V^2/rg$

$tan\,\theta = 17.15^2/100 \times 9.8$

$\tan\theta = 0.3$

$\theta = tan^{-1}(0.3)$

θ = 16.70â—¦

4. A circular disk is rotating at 90 $r.p.m$ and a coin of mass 20g is placed at a distance of 10cm from its centre. What is the centrifugal force acting on the coin?

Solution

M = 20g = 20 $\times$ 10^{-3} Kg

R = 10 cm = 10 $\times$ 10^{-2} m

f = 90 $r.p.m$ = 90/60 $r.p.s$

ω = 2πf

ω = 2π $\times$ 90/60

Centrifugal force (F)

$F = Mr\omega^2$

$F = 0.02 \times 0.1 \times (3\pi)^2$

F = 0.177 N

5. A car of mass 1500kg is moving along a circular level road of radius 150with a maximum speed of 15m/s without skidding. Find

(i) Force of friction

(ii) Coefficient of friction between the tires and road.

Solution

M = 1500kg v = 15m/s r = 150m

(i) $F_s = MV^2/r = (1500 \times 15^2)/150$

$F_s = 2250$ N

(ii) Coefficient of friction force.(μs)

$F_s = \mu_s R$

$F_s = \mu_s Mg$

$\mu_s = F_s/Mg = 2250/(1500 \times 9.8)$

$\mu_s = 0.15$

6. A car whose wheels are 1.5m apart laterally and whose center of gravity is 1.5m above the ground rounds a curve of radius 250m. Find the maximum speed,which the car can travel without toppling.

Solution

2d = 1.5m

d = 0.75

h = 1.5m

r = 250m

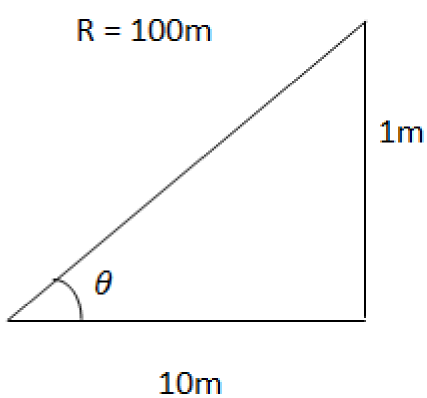
$V_{max} = \sqrt{gdr/h}$

$= \sqrt{9.8 \times 0.75 \times 250/1.5}$

V_{max} = 35m/s

7. A curve on a highway forms an arc of radius 100m. If the road is 10m wide and its outer edge is 1m higher than the inner edge, for what speed the road is banked?

Solution



$\tan\theta = V^2/rg$

$V^2 = rg\tan\theta$

$$V^2 = 100 \times 9.8 \times {}^{1}/_{10}$$

$$\sqrt{V^2} = \sqrt{98}$$

$$\therefore V = 9.89 m/s$$

8. A curve in a road has 60m radius. The angle of bank of the road is **47â—¦**. The maximum speed a car can have without skidding if the coefficient of static friction between tires and road is 0.8

Solution

r = 60m   μs = 0.8

θ = 47â—¦

$$V_{max} = \sqrt{\frac{\tan\theta + \mu s}{1 - \mu s \tan\theta} rg}$$

$$V_{max} = \sqrt{\frac{(tan\ 47 + 0.8) \times 60x9.8}{1 - 0.8 \times \tan 47}}$$

V_{max} = 317 km/hr or 88m/s

9 9. A small toy car moves round a circular track of radius 4m. The car makes one revolution in 10s. Calculate:

(a) The speed of the car

(b) The centripetal acceleration

(c) The centripetal force exerted by the truck if the mass of the car is 200g

(d) The safe speed with which the car can move around without toppling, if the distance between the wheels is 4cm and the height of the center of gravity of the car from the horizontal is 2cm.

Solution

r = 4m m=200 g= 200 $\times$ 10^{-3}

T = 10s 2d = 4cm = 4 $\times$ 10^{-2}m

H = 2cm = 2 x 10^{-2}m

(a) V = $r\omega$

V = $r.{}^{2\pi}/_{T}$

$= 4 \times {}^{2 \times 3.14}/_{10}$

V = 2.512 m/s

(b) ac = V^2/r = $2.512^2/4$

ac = 1.58m/s²

(c) $F_C = M_{a_c}$

Fc = 200 x 10^{-3} x 1.58

(d) Safe speed V $= \sqrt{{}^{gdr}/_{h}}$

V = $\sqrt{{}^{9.8 \times 0.02 \times 4}/_{0.02}}$

V = 6.26 m/s

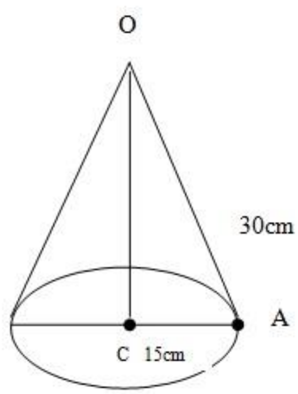
10 . A ball of mass 0.1 kg is suspended by a string 30 cm long, keeping the string always taut, the ball describes a horizontal circle of radius 15cm. Find the angular speed.

Solution

M=0.1 kg

L=30cm = 30 $\times$ 10 $^{-2}$ m

r = 15cm = 15 $\times$ 10 $^{-2}$ m



The angular speed of the ball

$$\omega = \sqrt{g/h}$$

From the right angled triangle

$OC^2 + CA^2 = OA^2$

$OC^2 = OA^2 – CA^2$

h = $OA^2 - CA^2$

h = $0.3^2 + 0.15^2$

h = 0.26m

$\omega = \sqrt{9.8/0.26}$

ω = 6.14 rad/s

11. (a) Explain angular displacement, velocity and acceleration with respect to circular motion.

(i) Whirl an object in a vertical and horizontal plane and observe the motion.
(ii) Deduce expression for tension and period of the vertical and horizontal motion.

(b) Describe application of circular motion.

(i) Centrifuge
(ii) Rotor
(iii) Conical pendulum
(iv) Speed governor
(v) Motion in a vertical circle
(vi) Looping the loop track
(vii) Cyclist
(viii) Motorist
(ix) Banking of roads

-

SIMPLE HARMONIC MOTION

Before discussing simple harmonic motion, it is desirable to discuss periodic motion and oscillatory motion.

1. Periodic motion

Is the motion which repeats itself after a regular interval of time.

The regular interval of time is called time periodic of the periodic motion

Example of periodic motion

(i) The revolution of earth around the sun is a periodic motion. Its period of revolution is 1 year

(ii) The revolution of moon around the earth is a periodic motion. Its period of revolution is 1 year

(iii) The motion of the hands of a clock is a periodic motion.

(iv) Heart beats of person is a periodic motion. It is period of revolution is about 0.83s for a normal person

(v) Motion of Halley's Comet around the sun, period is 76 years.

2. Oscillatory motion (vibratory motion) is the motion which moves along the same path to and fro about an equilibrium position.

3. For a body to oscillate or vibrate three conditions must be satisfied:

(i) The body must have inertia to keep it moving across the midpoint of its path.

(ii) There must be a restoring force (elastic) to accelerate the body towards the midpoint.

(iii) The friction force acting on the body against its motion must be small.

All oscillatory motions are periodic motions but all periodic motions are not oscillatory

Examples for oscillatory motion

(i) Oscillation of a simple pendulum

(ii) Vibration of a mass attached to a spring

(iii) Queering of the strings of musical instruments

Restoring force

Restoring force is one that tries to pull or push a displaced object back to its equilibrium position

Simple harmonic motion is a motion of a particle which moves to and fro about s fixed point under the action of restoring force which is direct proportional to the displacement from the fixed point and always directed towards the fixed point.

This fixed point is called **mean position** or **equilibrium** position.

It is called **mean position** because it lies in the middle of the line of oscillation.

It s called **equilibrium position** because at this point the resultant force acting on the particle is zero

Let the displacement of the particle from the mean position be y and F be the force acting on the particle.

Then

$F \propto -y$

$F = -ky$

-ve sign shows that F and y are oppositely directed.

K is called spring constant because the restoring force F has the property of a spring force.

If the motion takes place under a restoring force it is called liner S.H.M. For restoring torque is called Angular S.H.M.

DAMPING OF S.H.M

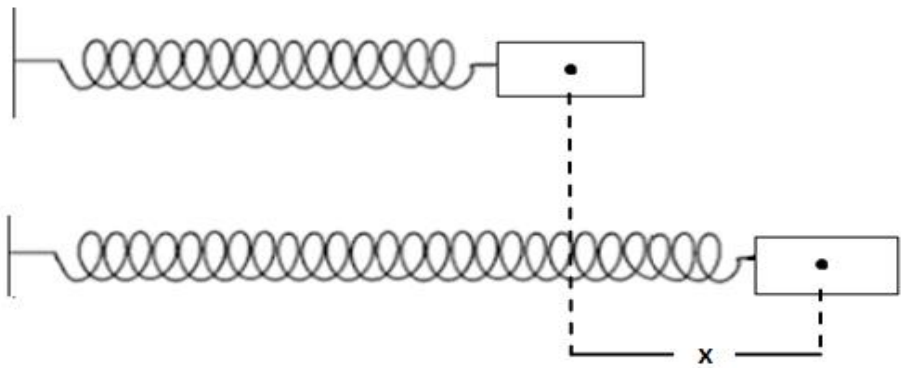
We can now form a definition of simple harmonic motion . It is the motion of particle whose acceleration is always (i)directed toward a fixed point, (ii)directly proportional to its distance from that point

Under the condition when time t=0 the displacement along x-direction is equal to amplitude A.

MECHANICAL OSCILLATIONS

1. **Oscillating system – spring and mass**

The figure below shows a mass connected to spring whose free end is connected to a rigid support. The mass and the spring are laid on a frictionless horizontal surface. The mass of the spring is assumed to be negligible.



When the mass is pulled so that it has a displacement X from (the) its equilibrium position then the spring is extended by X there is restoring force. If the spring obeys Hooke's law then the force is directly proportion to the extension. F acts in opposite direction to so F = -KX is a constant called the spring or force constant.

If M is the mass of the body A, F=Ma so that

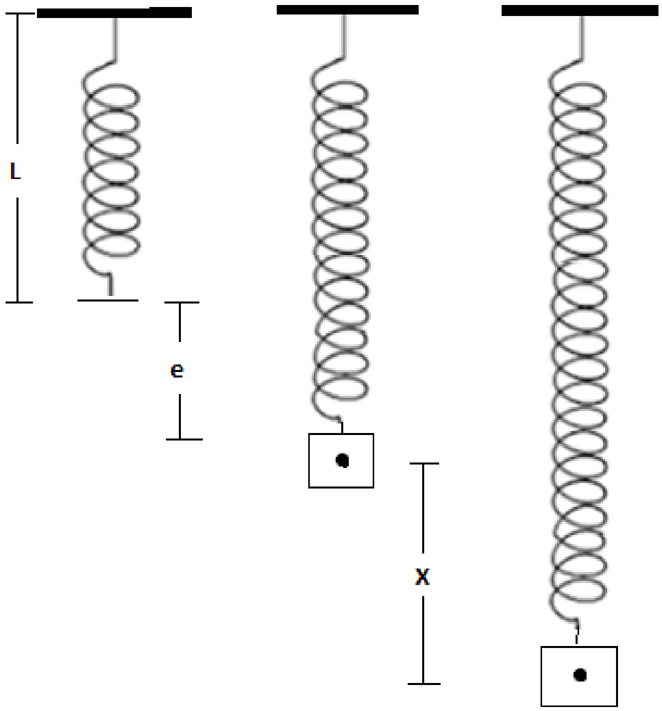
$$ma = -kx$$

$$a = -\frac{k}{m}x \; = m^2 x \text{ where } m^2 = \frac{k}{m}$$

The motion of a simple harmonic motion and the period T is given by $T = 2\pi = 2\pi\sqrt{\frac{M}{k}}$

The acceleration is always directed forwards the equilibrium position.

2. **The spiral spring.**



Consider a spiral spring of natural length 'L' suspended at the upper end

Let a mass M attached to the lower end extends the spring by 'e'

Assuming the spring obeys Hook's law then

mg = Ke Where K is a spring constant.

$K = \frac{mg}{e}$ (Force needed to produce unit extension of the spring)

Suppose 'm' is pulled further down distance 'x' from O and then released from A

The restoring force F = -Kx (Force which produce the extra extension)

Ma = -kx

$a = -\frac{k}{m}x = -w^2 x$ Where $w^2 = \frac{K}{m}$ = Thermos will oscillate e with simple

Harmonic Motion (SHM) about the equilibrium

Angular velocity $W = \sqrt{\frac{K}{M}} \longrightarrow \frac{m}{k} = \frac{e}{g}$

The period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{K}{M}} \text{ Since } mg = ke$$

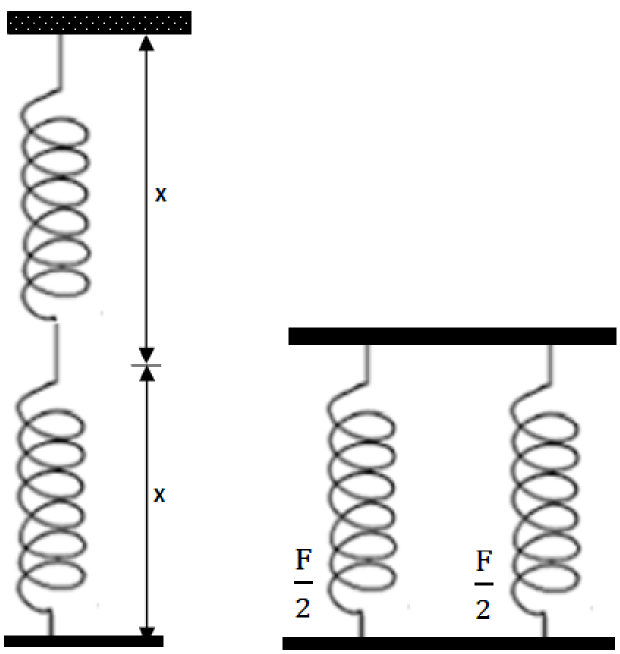
Note

1. Oscillating spring is an exact S.H.M
2. $T^2 = 4\pi^2 \frac{m}{k}$ if the graph of T^2 vs m is plotted it should be a straight lime passing through the origin im practice the graph does not pas through the origin occurring that the mass of the spring was not taken into account

3. **Spring in series and parallel**

Consider a situation shows in Fig (a) below where two

Identical springs are in series.



F = mg fig(a)   F = mg fig (b)

The mass applied to the end of the springs until stretch each spring by the same amount as if it mere applied to each separately. There will be the twice extension that would have occurred with just the single spring. For one spring we have "mg = -kx" but total extension where force constant for the two spring in series

$2xmg = -k_1 2x$ Where K_1 = force constant for the two spring in series

$kx = 2k_1 x \longrightarrow k_1 = {}^{k}/_{2}$

The period of oscillation of the system

$$T = 2\pi \sqrt{M}/_{K_1} \text{ Out } K_1 = {}^{k}/_{2}$$

$$T = 2\pi \sqrt{M}/_{k} = T_1 = 2\pi \sqrt{2m/_{k}} = T_1 = \sqrt{2}\ T$$

Consider parallel identical spring fig(b) force on each spring $= {}^{1}/_{2}\, mg$ the extension is only half (of) as much as would occurred with a single spring or ${}^{X}/_{2}$

$\frac{1}{2}mg = \frac{kx}{2} \longrightarrow mg = k$ (Force on parallel springs)

$$F = mg = k_2 X_2$$
$$k_2 = 2k$$

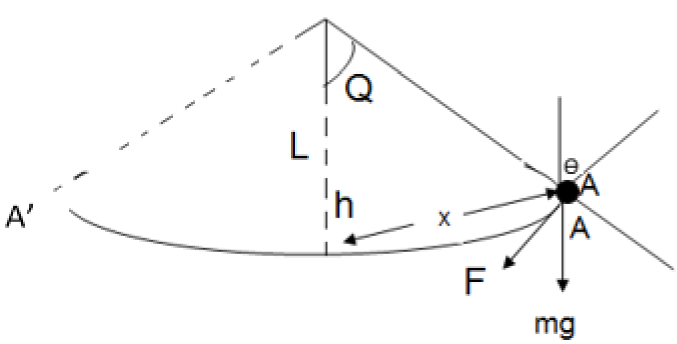
$$T_2 = 2\pi\sqrt{\frac{m}{k_2}} = 2\pi\sqrt{m/_{2k}} = 2\pi\, {}^{1}/_{\sqrt{2}} \sqrt{\frac{m}{k}}$$

$$T_2 = {}^{1}/_{\sqrt{2}}\,T$$

The of the parallel system is less than that of single spring

$$\frac{T_1}{T_2} = \frac{\sqrt{2}\,T}{\frac{1}{\sqrt{2}}T} = \sqrt{4} = 2$$

4. **The simple pendulum**



The bob of length L when it is in equilibrium position at O. suppose the bob is iron displace d through angle θ. The position of A the work done in doing so is stored in the system as

P:E = mgh at A.

If the bob is now released from A it swing's towards O changing all it PE F in mg to KE owing to the KE of the bob at O the bob over shoots from O and swing to A until all its KE change into P:E at A hence the pendulum swings to and from indefinitely assuming no resistance to its motion

Suppose any movements of the right of O are positive suppose also the bob is released from A therefore that tend to restore the bob back to O is F = -mg sinÆŸ (newton's second law of motion)

F is acting along the tangent to the Arc AO

ma = -mg sinÆŸ

a â‰ƒ -gÆŸ (as ÆŸ → 0 sin ÆŸ = ÆŸ)

Suppose the displacement of the bob from equilibrium x = lθ

$\theta = \frac{x}{L}$ LΘ is measured in radians.

$\rightarrow a = -\frac{gx}{L} = -w^2 x.$

$w^2 = \frac{g}{L} = w = \sqrt{\frac{g}{L}}$

NOTE

$^g/_L$ is a position constant…… acceleration is directly proportional to the displacement from a fixed point 0

When X is positive (between O and A) acceleration is negative ie. It is directed towards O

When X is negative (between O and A') the acceleration is positive ie. Directed towards ÆŸ

Thus the bob exciting simple (pendulum) …………… C motion with angular velocity

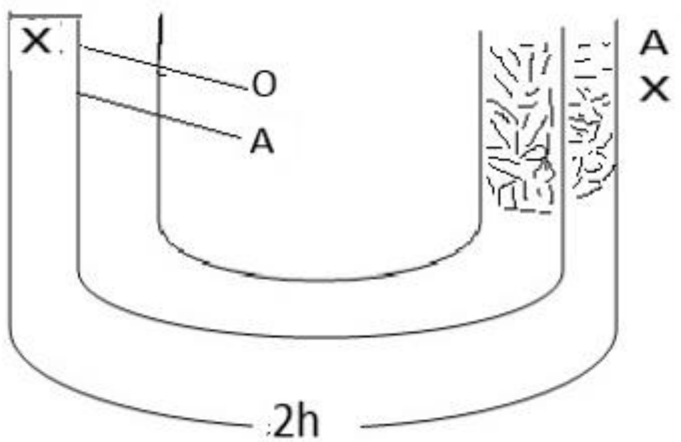
$W = \sqrt{\frac{g}{l}}$

$\therefore$ The period $= T = 2\pi\sqrt{^g/_l}$

This is only on approximate simple harmonic motion which is true only when Ó¨ is very small

If T and L are measured g can be calculated

5. **LIQUID COLUMN**



Consider liquid column of length 2h containing in a U-tube of cross sectional area A

At equilibrium the liquid levels O and O' will be along the same horizontal plane

If the surface is depressed through depth X to the A by blowing into the tube and then left it

The restoring force is =-2xAρg where ρ is the density of liquid.

From the 2nd Newton’s law of motion

$$m = 2hAgs \text{ But } ma = -2AXg\rho$$

$$a = -\frac{2Ax\rho q}{2hA\rho} = \frac{g}{h}x$$

$$a = -{}^{g}/_{h}\,\alpha x = w^2 x.$$

∴ The liquid column oscillates with S H M of angular velocity $w = \sqrt{{}^{g}/_{h}}$ and the period of

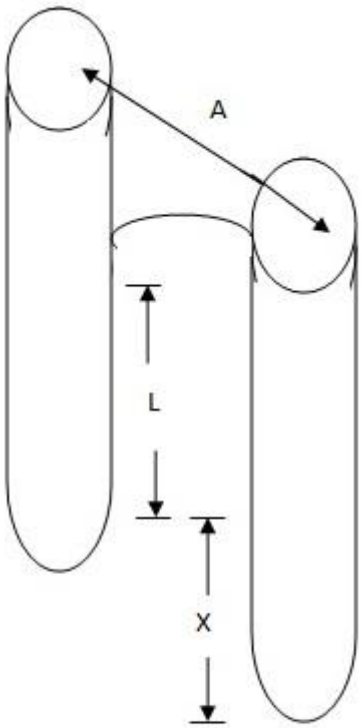
$$T = 2\pi\sqrt{{}^{h}/_{g}}$$

FLOATING CYLINDER

Consider a cylinder of cross section area a floating up right in a liquid of density ρ such that it is submerged to a depth L

The cylinder should be in equilibrium

Of the mass of the cylinder equals to the mass of liquid displaced.



$\therefore m = LA\rho$

Suppose the cylinder is pushed. +VE direction further down through a extra (angle) distance X them released. The extra up thrust will tend to restore the cylinder to equilibrium assuming displacement vector from downwards are positive

The restoring up thrust $= -x\,A\rho g$

Acceleration of the cylinder $W^2 = \frac{-\rho Agx}{m} = a = {}^{F}/_{m}$

The motion of the cylinder is $S\ H\ M$ with angular velocity $w^2 = \frac{A\rho g}{m}$ and the period time is

$$T = 2\pi\sqrt{{}^{m}/_{A\rho g}} \quad \text{But } m = LA\rho$$

$$\therefore T = 2\pi\sqrt{L}/_{g}$$

NOTE

1. This is an example of exact of
2. L is not the long of the cylinder column but depth submerged

Relation between linear S.H.M and uniform circular motion

A particle moving around a circle with constant speed is said to be in uniform circular motion.In uniform circular motion the speed remains constant but the velocity changes due to the change in direction. Hence the particles accelerate.

(i) They are both periodic motions
(ii) Their accelerations are directed towards a fixed point i.e circular motion is directed towards center of a circle while simple harmonic motion is directed towards the mean positions.

Consider a particle P moving along the circumference of a circle of radius A with a uniform angular velocity w as shown in figure 1

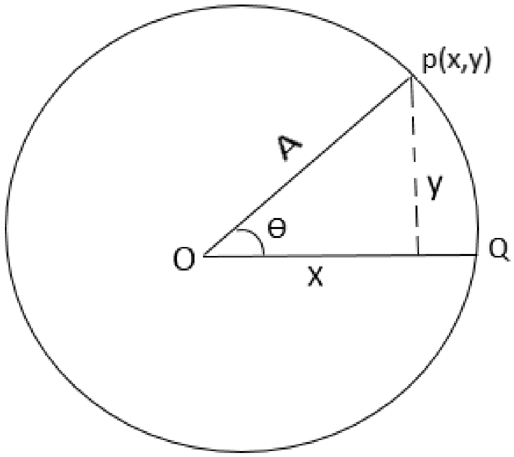


Fig 1. Description of S.H.M

O is called the mean position or equilibrium position

A is the maximum displacement of the particles executing S.H.M on either side of the equilibrium position.

that when the particle reaches point P, the displacement is maximum OP = A = radius of reference circle.

- The amplitude is equal to the radius of the reference circle.
- The displacement of a body which executes simple harmonic motion can be expressed in terms of **x** and **y.**

The displacement of a particle executing simple harmonic motion at any instant is the distance of the particle from the equilibrium position at that instant.

$$\cos\theta = {}^{x}/_{A} \qquad \sin\theta = {}^{y}/_{A}$$

$$\boldsymbol{x = A\cos\theta} \qquad \boldsymbol{y = A\sin\theta}$$

θ is the angular displacement, t is the time taken by the body to oscillate from point O to P and describe an angular displacement θ

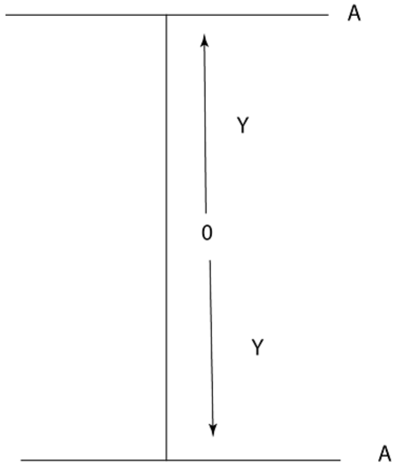
$$\omega = {}^{\theta}/_{t}$$

$$\theta = \omega t$$

- The displacement can be represented by the relation $x = A\cos\omega t$ $y = A\sin\omega t$

Where A is the amplitude of oscillation

Fig. 2



From fig.2

At equilibrium position y =0

The positions where y = $\pm$A are called the extreme positions

Example oscillation of simple pendulum at t = 0, the body is at the maximum reach

$x = A\cos\omega t$

t = 0

$x = A\cos(\omega \times 0)$

$x = A\cos 0$

$x = A$

Example of oscillation of piston rings in engine cylinder

y $= Asin\ \omega t$
t=0

y = $Asin\ (\omega \times 0)$

y = $Asin\ 0$

y = 0

VELOCITY OF A BODY EXECUTING S.H.M

The velocity of a particle executing S.H.M at any instant is the time rate of change of its displacement at that instant

Since the displacement of a S.H.M is a function of time, therefore its velocity will be a function of time (Instantaneous velocity)

V = $dx/_{dt}$

for $x = Acos\ \omega t$

V = $d(Acos\ \omega t)/_{dt}$

V = $Ad(\cos \omega t)/_{dt}$

V = $\boldsymbol{-A\omega sin\ \omega t - - - - - - - - - - - - - - -(i)}$

Also

V = $dy/_{dt}$

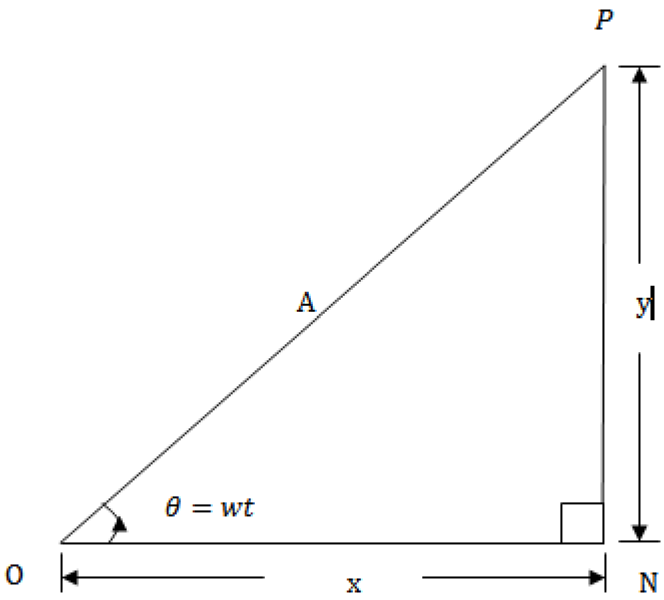
for y = $Asin\ \omega t$

V = $d(Asin\ \omega t)/_{dt}$

$V = {Ad(\sin \omega t)}/{dt}$

$\boldsymbol{V = A\omega \cos \omega t -------------(ii)}$

Consider the right angled triangle OPN in fig. 1



$\sin \omega t = {y}/{A}$

$\cos \omega t = {x}/{A}$

$y = A \sin \omega t$

$x = A \cos \omega t$

From equation (i)

V = $-\omega(Asin\ \omega t)$

V = $-\omega y$

From the triangle OPN

$x^2 + y^2 = A^2$

$y^2 = A^2 - X^2$

$y = \sqrt{A^2 - X^2}$ -------------------------- (iii)

Also

From equation (ii)

V = $A\omega \cos \omega t$

V = $\omega(A cos \omega t)$

V = ωx

From the triangle ONP

$x^2 + y^2 = A^2$

$x^2 = A^2 - y^2$

$V = \pm\omega\sqrt{A^2 - y^2}$-----------------(iv)

Equation (i) and (ii) represents velocity as a function of time

Both equation (iii) and (iv) represents velocity as a function of displacement Velocity of a particle executing S.H.M at any instant is

$$V = \omega\sqrt{A^2 - y^2}$$

At equilibrium position y = 0

$$V_{max} = \omega\sqrt{A^2}$$

$$V_{max} = A\omega$$

ACCELERATION OF A BODY EXECUTING S.H.M

The acceleration of a particle executing S.H.M at any instant is the time rate of change of velocity at that instant.

a = $dv/_{dt} = d^2y/_{dt^2} = d^2x/_{dt^2}$

For $x = A\cos\omega t$

a = $dv/_{dt}$

v = $-A\omega\sin\omega t$

a = $d(-A\omega\sin\omega t)/_{dt}$

a = $-A\omega^2\cos\omega t$

$x = Acos\ \omega t$

a = $-(Acos\ \omega t)\omega^2$

$a = -\omega^2 x$

For y = $Asin\ \omega t$

$$a = \frac{dv}{dt}$$

$$V = \frac{dy}{dt} = A\omega\cos\omega t$$

$$a = \frac{dv}{dt} = \frac{d^2y}{dt^2} = -A\omega^2\sin\omega t$$

$$a = -\omega^2 y$$

At equilibrium position y = 0

a = $-\omega^2 y$

a = 0

At the extreme positions

y = $\pm$ A

a = $-\omega^2 y$

a = $\pm\omega^2 A$ (This is expression for maximum)
The maximum value of acceleration is called acceleration amplitude in S.H.M

The – ve sign means that the acceleration and displacement are directed in opposite direction ensuring that the motion is always directed to the center (fixed point)

GRAPHICAL REPRESENTATION OF SIMPLE HARMONIC MOTION

(i)$x = A\cos\omega t$

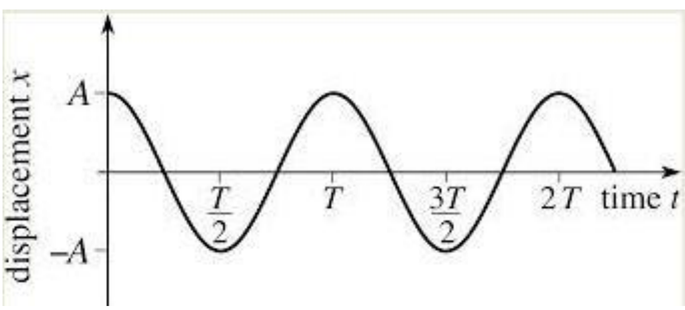


Fig3.(a)

(ii)$y = A\sin\omega t$

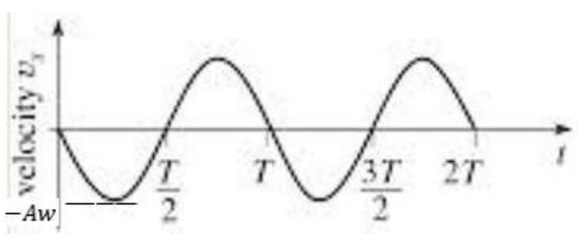


Fig. 3(b)

(iii)$a_x = -\omega^2 x\cos\omega t$

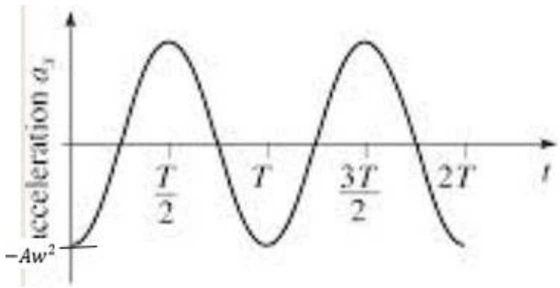


Fig 3.(c)

The displacement velocity and acceleration all vary sinusoidal with time but are not in phase

ENERGY OF A BODY EXECUTING S.H.M

A harmonic oscillation executes S.H.M under the action of a restoring force. This force always opposes the displacement of the particle. So to displace the particle against this force work must be done.

The work done is stored in the particle in form of potential energy (P.E), as the particle is in motion it has kinetic energy (K.E)

The sum of P.E and K.E is always a constant provided that part of this energy is not used to overcome frictional resistance.

Expression for kinetic energy

K.E = ½ mv^2

where $v = \pm\omega\sqrt{A^2 - y^2}$

The kinetic energy as a function of displacement

$$K.E = \frac{1}{2}m\omega^2(A^2 - y^2)$$

The K.E is maximum when its velocity is maximum

$V_{max} = \omega A$ (At midpoint)

K.E = ½ mω²A²

The K.E as a function of time

V = $\omega A\cos\omega t \quad (for \;\; y = A\sin\omega t)$

$V = -\omega A\sin\omega t \quad (for \;\; x = A\cos\omega t)$

K.E = $^1/_2\, m\omega^2 A^2 \cos^2\omega t$

Or

K.E = $^1/_2\, m\omega^2 A^2 \sin^2\omega t$

Expression for potential energy

The P.E is the energy possessed by the body due to its position

By Hooke's law

$F = -kx$

The work done to extend the spring from x = 0 to x = x_o

$$W = \int_0^{x_o} F dx$$

$$W = \int_0^{x} kx dx$$

$$W = {}^1/_2\, k|x^2|_0^{x_o}$$

$$W = {}^1/_2\, kx_0^2$$

The P.E of a string extended by displacement x is given by

P.E = $\boldsymbol{^1/_2\, kx^2}$

F = $-kx$

$ma = -kx$

$a = -\frac{k}{m}x$

ω^2 = $^{k}/_{M}$

k = $M\omega^2$

$PE = {}^{1}/_{2}m\omega^2A^2cos^2\omega t$

Total energy in S.H.M

The total energy E_T of the particle per displacement y is given by

$E_T = K.E$ +P.E

$E_T = {}^{1}/_{2}m\omega^2(A^2 - y^2) + {}^{1}/_{2}m\omega^2y^2$

$E_T = {}^{1}/_{2}m\omega^2(A^2 - y^2) + {}^{1}/_{2}m\omega^2y^2$

Also

$E_T = {}^{1}/_{2}m\omega^2A^2sin^2\omega t + {}^{1}/_{2}m\omega^2A^2cos^2\omega t$

$E_T = {}^{1}/_{2}m\omega^2A^2[sin^2\omega t + cos^2\omega t]$

$E_T = {}^{1}/_{2}m\omega^2A^2$

ω = ${}^{2\pi}/_{T} = 2\pi f$

$E_T = {}^1/_2\, m \times \left({}^{4\pi^2}/_{T^2}\right) A^2$

E_T = $\mathbf{2m\pi^2A^2/_{T^2}}$

E_T = $\mathbf{2m\pi^2A^2f^2}$

<u>GRAPHS OF K.E VS TIME</u>

For

$x = A\cos\,\omega t$

V = $-A\omega\sin\omega t$

Then

K.E = ½ m$\omega^2A^2\sin^2\omega t$

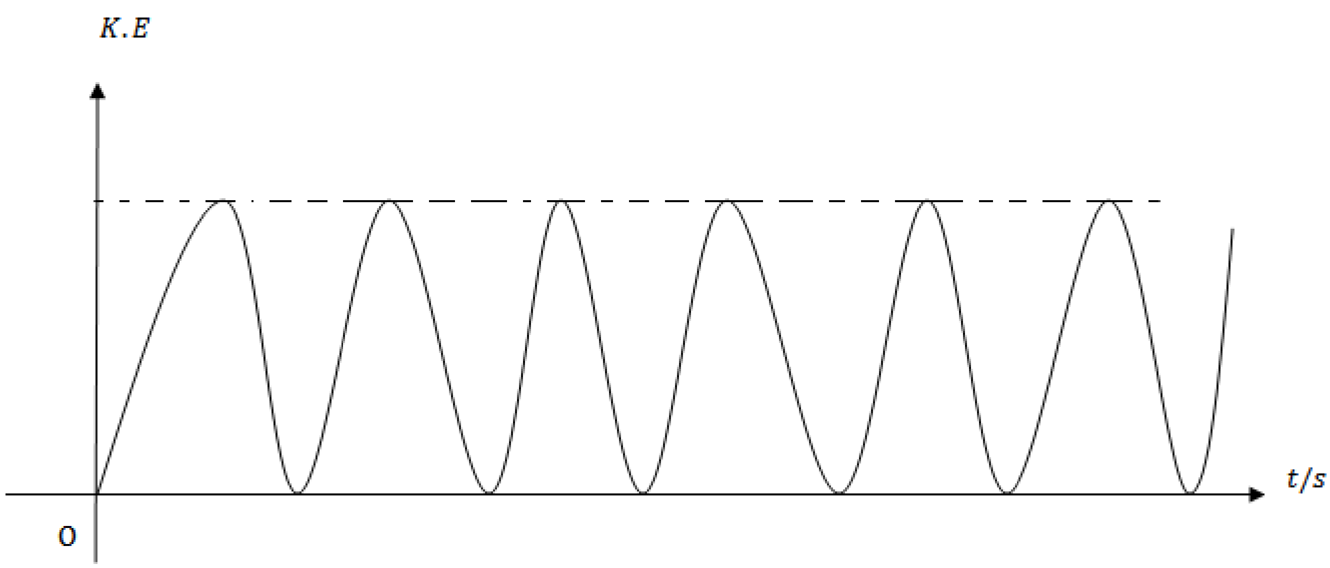


Fig. 4

GRAPH OF P.E Vs TIME

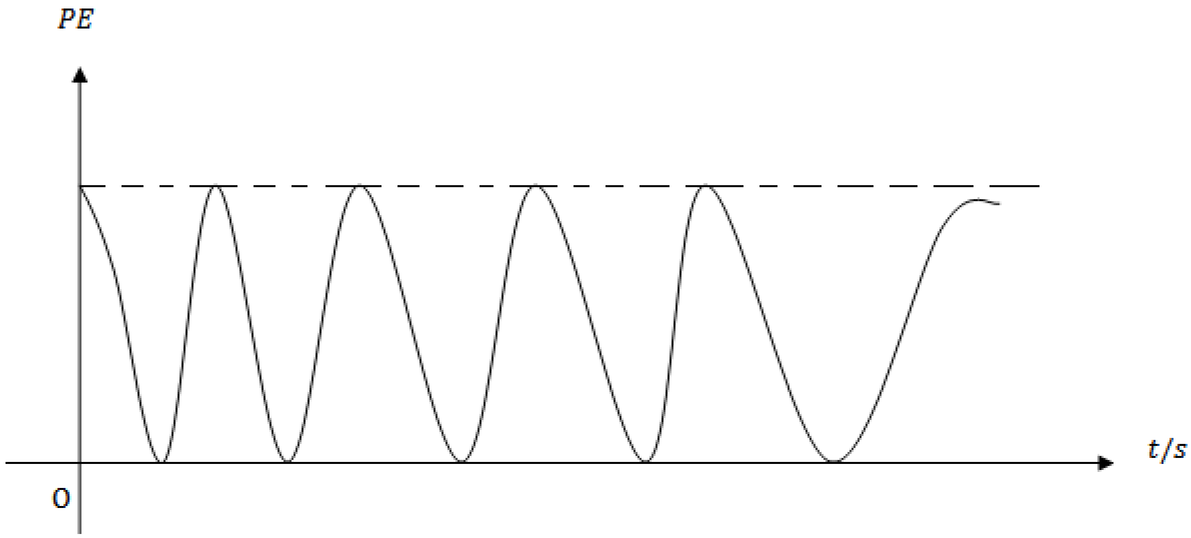


Fig 5.

Since

P.E = $\frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$

ENERGY EXCHANGE

The P.E and K.E for a body oscillating in S.H.M causes the motion of the body.

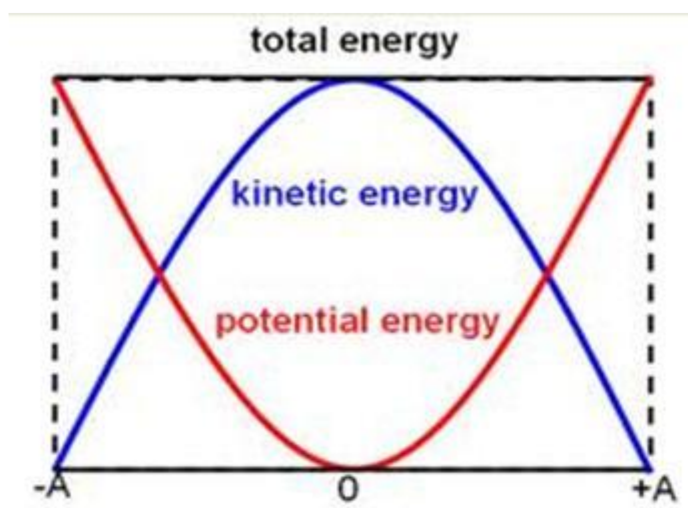


Fig.6 Energy of S.H.M

From the figure 6, the total energy of vibrating system is constant. When the K.E of the mass m is maximum (energy= 1⁄2 $m\omega^2 A^2$ and mass passing through the centre O), the PE of the system is zero (x=0). Conversely, when the P.E of the system is a maximum (energy=1⁄2KA^2=1⁄2 $m\omega^2 A^2$ and mass at end of the oscillation), the K.E of the mass is zero (V=0).

SOLVED PROBLEMS

1.The restoring force acting on a body executes simple harmonic motion is 16N when the body is 4cm away from the equilibrium position. Calculate the spring constant

Solution

Restoring force F = 16N

Displacement y =4 cm = 4×10^{-2}

Spring constant K =?

F = ky

K = $F/y = 16/4 \times 10^{-2}$

K = 400 Nm^{-1}

K = 400 Nm^{-1}

2. A body is executing simple harmonic motion with an amplitude of 0.1m and frequency 4 Hz. Compute

(i) Maximum velocity of the body

(ii) Acceleration when displacement is 0.09m and time required to move from mean position to a point 0.12 away from it

Solution

Amplitude A = 0.15m

Frequency f = 4Hz

Angular velocity = 2πf = 8 π rad/s

(i) Maximum velocity of the body

V = Aω

= 0.15 x 8 π rad/s

V = 3.768 m/s

(ii) Acceleration a = - $\omega^2 y$

= $-(8\pi)^2 \times 0.09$

= - 56.79 m/s^2

The negative sign shows that the acceleration is directed towards the equilibrium position.

$$(iii) \text{time } t = \frac{2x}{a} = \frac{2 \times 0.12}{56.79} = 0.0042 \text{ seconds}$$

3. A particle executes S.H.M with amplitude of 10 cm and a period of 5s. Find the velocity and acceleration of the particle of a distance 5cm from the equilibrium position

Solution

A = 10 cm = 10 $\times$ 10 $^{-2}$

T = 5s y = 5 cm = 5 $\times$ 10 $^{-2}$m

Velocity V = $\omega\sqrt{A^2 - y^2}$

= $^{2\pi}/_{T}\sqrt{A^2 - y^2}$

V = $^{2 \times 2.24}/_{5 \times 10^{-2}}\sqrt{(10 \times 10^{-2})^2 - (5 \times 10^{-2})^2}$

V = 10.88 m/s

Acceleration a = $-\omega y$

$= -\frac{(2\pi)^2}{T^2} \cdot y$

a = $-\frac{4\pi^2}{T^2} y$

$= -\frac{4 \times 3.14^2}{25} \times 5 \times 10^{-2}$

a= - 4 x 3.14^2 x 5 x 10^{-2}/25

a = 0.079m/s²

4. A bob executes simple harmonic of period 20s. Its velocity is found to be 0.05m/s after 2s when it has passed through its mean position. Find the amplitude of the bob.

Solution

T = 20s   v = 0.05 m/s   t = 2s

$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{20} = 0.314 rad/s$

V = $A\omega \cos \omega t$

0.05 = A $\times$ 0.314 $\times \cos$(0.314 $\times$ 2)

A = 0.16m

5. A body describes S.H.M in a line 0.04m long. Its velocity at the center of the line is 0.12 m/s. Find the period also the velocity

Solution

Length of the line $2A = 0.04m$

A = 0.02m

V_{max} = 0.12 m/s

$A\omega$ = 0.12

A $\times {}^{2\pi}/_{T}$ = 0.12

T = $A \times {}^{2\pi}/_{0.12} = {}^{0.02 \times 2 \times 3.14}/_{0.12}$

T = 1.046S

Velocity at a displacement y = 10 $^{-2}$ $\sqrt{3}m$

V = $\omega\sqrt{A^2 - y^2}$

V= ${}^{2\pi}/_{T}\sqrt{A^2 - y^2}$

V = ${}^{2 \times 3.14}/_{1.046}\sqrt{0.02^2 - 3 \times 10^{-4}}$

V = 0.06 m/s

6. In what time after its motion began will a particle oscillating according to the equation $x = 7sin$(0.5 πt) move from the mean position to the maximum displacement?

Solution

$x = 7sin$(0.5πt)

Maximum displacement $A = 7$

Time taken to move from the mean position to the extreme position is to be found out when $x = 7$ t=?

7 = $7sin$0.5πt

$sin 0.5\pi t$ = 1

0.5πt = $sin^{-1} 1$

0.5πt = $\pi/2$

t = 1s

7. A particle with a mass of 0.5 kg has a velocity of 0.3 m/s after 1s starting from the mean position. Calculate the K.E and Total energy if its time period is 6s.

Solution

m = 0.5 kg
T = 6s

V = 0.3 m/s

t = 1s $\omega = 2\pi/T = 2\times\pi/6 = \pi/3\, rad/s$

Velocity v = $A\omega cos\omega t$

V $= A\times\pi/3\, cos\,\pi/3$

$0.3/1.05 = 0.5A$

A = 0.57m

$K.E = 1/2\, mV^2 = 1/2\times 0.5\times 0.3^2$

K.E = 0.0225J

$Total\ energy = 1/2\, m\omega^2 A^2$

$E_T = 1/2\times 0.5\times \pi^2/9\times 0.57^2$

$\mathbf{E_T = 0.089J}$

-

8. A period of a particle executing S.H.M is 0.0786J. After a time π/4 s the displacement is 0.2m. Calculate the amplitude and mass of the particle.

Solution

T =2 π

t = $^{\pi}/_{4}s$

y = 0.2m

E_T = 0.0786J

$\omega = {}^{2\pi}/_{T} = {}^{2\pi}/_{2\pi} = 1rad/s$

Displacement after a time t = $^{\pi}/_{4}$ sec is y = 0.2 m

y = $Asin\omega t$

0.2m = $Asin{}^{\pi}/_{4}$

A = $^{0.2m}/_{\sqrt{2}} \times 2$

A = 0.283m

$E_T = {}^{1}/_{2} m\omega^2 A^2 = 0.5 \times m \times 1 \times 0.283^2$

0.04 m = 0.0786

m = 1.96 kg

9. A simple harmonic oscillation is represented by

x = 0.34cos (3000t + 0.74)

Where x and t are in mm and sec respectively. Determine

(i) Amplitude

(ii) The frequency and angular frequency

(iii) The time period

Solution

$x = 0.34cos$ (3000t + 0.74) ----------------------- (i)

The standard displacement equation of S.H.M

$x = Acos(\omega t + \emptyset)$ ------------------------------- (ii)

Comparing equation (i) and (ii)

(i) Amplitude A = 0.34 m

(ii) Angular frequency w = 3000 rad/s

$\omega = 2\pi f$

$f = \omega/2\pi = 3000/2\pi$

$f = \mathbf{1500/\pi}$ **Hz**

(iii) Time period T = $1/f$

$\mathbf{T = \pi/1500}$ sec

10. An object executes S.H.M with an amplitude of 0.17m and a period of 0.84s.Determine

(i) The frequency

(ii) The angular frequency of the motion

(iii) Write down the expression for the displacement equation

Solution

Amplitude A = 0.17 M

Period T = 0.84 s

(i) $f = 1/T = 1/0.84 = 1.19$ Hz

(ii) $\omega = 2\pi f$

$= 2\pi \times 1.19$

$= 2.38 \times 3.14$

$\omega = 7.48$ Hz

The displacement equation of S.H.M is

$x = A\sin(\omega t + \emptyset)$

$x = 0.17\sin(7.5t + 0)$

$x = 0.17\sin(7.5t)$

11.The equation of S.H.M is given as $x = 6\sin 10\pi t + 8\cos 10\pi t$ where x is in cm and t in second Find

(i) Period

(ii) Amplitude

(iii) Initial phase of motion

Solution

$x = 6\sin 10\pi t + 8\cos 10\pi t$ ---------------------- (i)

The standard displacement equation of S.H.M

$x = A\sin(\omega t + \emptyset)$

$x = A(\sin\omega t \cos\emptyset + \sin\emptyset\cos\omega t)$

$x = A\sin\omega t\cos\emptyset + A\sin\emptyset\cos\omega t$ --------------- (ii)

Comparing equation (i) and equation (ii)

$\omega t = 10\pi t$

$\omega = 10\pi$

$A\sin\omega t\cos\emptyset = 6\sin\omega t$

$A\cos\emptyset = 6$ ------------------------------------ (iii)

$A\sin\emptyset\cos\omega t = 8\cos 10\pi t$

$A\sin\emptyset = 8$ --------------------------------- (iv)

(i) Period

$\omega = {}^{2\pi}/_{T}$

$T = {}^{2\pi}/_{\omega} = {}^{2\pi}/_{10\pi}$

T = 0.2 s

(ii) Amplitude

Squaring equation (iii) and (iv) then add

$A^2sin^2\emptyset + A^2cos^2\emptyset = 8^2 + 6^2$

$A^2(sin^2\emptyset + cos^2\emptyset) = 100$

$\sqrt{A^2} = \sqrt{100}$

A = 10cm

(iii)Initial phase angle

${}^{Asin\emptyset}/_{Acos\emptyset} = {}^{8}/_{6}$ Type equation here.

$tan\,\emptyset = 1.33$

$\emptyset = tan^{-1}(1.33)$

$\therefore \emptyset = 53.13^0$

12.The periodic time of a body executing S.H.M is 2sec. After how much time interval from t = 0 will its displacement be half of its amplitude.

Solution

y = $Asin\ \omega t$

$y = Asin\ {}^{2\pi}/_{T}\, t$

Here,

$y = {}^{A}/_{2}$ $\quad T = 2s$

${}^{A}/_{2} = Asin\ {}^{2\pi}/_{2}\, t$

$\sin 2\pi t = {}^{1}/_{2}$

$2\pi t = sin^{-1}\, {}^{1}/_{2}$

$2\pi t = {}^{\pi}/_{6}$

$t = {}^{1}/_{6}\, seconds$

13. A particle executes S.H.M of amplitude 25cm and time period 3sec. What is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position?

Solution

Let t be the time taken by the particle to move from mean position to a point 12.5cm from it

y = $Asin\ \omega_t$

$y = A\sin({}^{2\pi}/_{T})\, t$

y = 12.5 cm, A = 25 cm, T = 3s

$12.5 = 25\sin\left({}^{2\pi}/_{3}\right)t$

$0.5 = \sin\left({}^{2\pi}/_{3}\right)t$

${}^{2\pi t}/_{3}$ = $\sin^{-1} 0.5$

${}^{2\pi}/_{3} = {}^{\pi}/_{6}$

t = ${}^{1}/_{4}\,s$

Required time = 2t

$= 2 \times {}^{1}/_{4}$

Required time is 0.5 s

14. A particle in S.H.M is described by the displacement function

$x = A\cos(\omega t + \emptyset)$ $\quad \omega = {}^{2\pi}/_{T}$

If the initial (t = 0) position of the particle is 1cm and its initial velocity is π cm/s. What is its amplitude and phase angle? The angular frequency of the particle is π s-1

Solution

$x = A\cos(\omega t + \emptyset)$

At t = 0

x = 1cm
v = π cm/s
w = π/s

$1 = A\cos(0 + \emptyset)$

$1 = A\cos\emptyset$ -------------------------(i)

$V = \frac{dx}{dt} = -A\sin(\omega t + \emptyset)$

$\pi = -A\pi\sin(0 + \emptyset)$

$-1 = A\sin\emptyset$------------------------(ii)

Squaring and adding equation (i) and (ii)

$A^2\cos^2\emptyset + A^2\sin^2\emptyset = 1^2 + (-1)^2$

$A^2 = 2$

$A = \sqrt{2}$

A = 1.41421 m

15.The time period of a particle executing S.H.M is 2 seconds and it can go to and fro from equilibrium position at a maximum distance of 5cm, if at the start of the motion the particle is in the position of maximum displacement towards the right of the equilibrium position, then write the displacement equation of the particle.

Solution

The general equation for the displacement in S.H.M is

$y = A\sin(\omega t + \emptyset)$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2}$

$\omega = \pi\ rad/s$

A = 5 cm, y = 5cm

At, t =0

$5 = 5\sin(\bar{x} \times 0 + \emptyset)$

1 = sin $\emptyset$

$\emptyset = \pi/2$

y = $5\sin(\pi t + \pi/2)$

$y = 5\sin\pi(t + 1/2)$

16 .In what time after its motion begins will a particle oscillating according to the equation y = $7\sin 0.5\pi t$ move from the mean position to maximum displacement?

Solution

y = $7\sin 0.5\pi t$

The standard displacement equation of S.H.M is

y = $A sin\ (\omega t)$

Comparing equation (i) and (ii)

A =7 ω =0.5πt

Let t be the time taken by the particle in moving from mean position to maximum displacement position

y = A =7

y = $A sin\ 0.5\pi t$

y = A = 7

7 = $7\sin 0.5\pi t$

$\sin 0.5\pi t$ = 1

0.5πt = $\sin^{-1} 1$

0.5πt = $\pi/2$

$^{t}/_{2} = {}^{1}/_{2}$

t = 1 s

17.The vertical motion of a huge piston in a machine is approximately simple harmonic with a frequency of 0.5 s^{-1}. A block of 10 kg is placed on the piston. What is the maximum amplitude of the piston's executing S.H.M for the block and piston to remain together?

Solution

The block will remain in contact with the piston if maximum acceleration (a_{max}) of S.H.M does not exceed g $i.e\ a_{max}$ is at most equal to g

$a_{max} = \omega^2$ A

$a = (2\pi f)^2 A$

A = $^{g}/_{4 \times \pi^2 \times f^2}$

A = $^{9.8}/_{4 \times 3.14^2 0.5^2}$

A = 0.994 m

18. A particle executing S.H.M has a maximum displacement of 4cm and its acceleration at a distance of 1 cm from the mean position is 3m/s^2. What will be its velocity when it is at a distance of 2 cm from its mean position

Solution

$a = -\omega^2 y$

a = 3cm/s^2
A = 4cm

y_1 = 1 cm
y_2 = 2 cm

$\omega = \sqrt{^{a}/_{y}} = \sqrt{^{3}/_{1}}$

w = 1.78 rad/s

The velocity of a particle executing SHM is given by

V = $\omega\sqrt{A^2 - y^2}$

V = $1.73 \times \sqrt{4^2 - 2^2}$

V = 6 cm/s

19.A particle executing S.H.M along a straight line has a velocity of 4m/s when its displacement from mean position is 3m and 3m/s when the displacement is 4m. Find the time taken to travel 2.5m from the positive extremity of its oscillation.

Solution

V = $\omega\sqrt{A^2 - y^2}$

For the first case

V = 4m/s

y = 3m

V^2 = $\omega^2(A^2 - y^2)$

16 = $\omega^2(A^2 - 16)$ ----------------- (i)

For the second case

V = 3m/s

y = 4m

V^2 = $\omega^2(A^2 - 16)$

9 = $\omega^2(A^2 - 16)$ -------------------- (ii)

Take equation (i) divide by equation (ii)

$16/9 = \omega^2(A^2-9)/\omega^2(A^2-16)$

$16/9 = A^2-9/A^2-16$

$9A^2 - 81 = 16A^2 - 256$

$7A^2 = 175$

$A^2 = 175/7 = 25$

$A = \sqrt{25}$

A =5m

From equation (i)

$16 = \omega^2(A^2-9)$

$16 = \omega^2(25-9)$

$16 = 16\omega^2$

$\omega^2 = 1$

$\omega = 1\ rad/s$

When the particle is 2.5m from the positive extreme position, its displacement from the mean position is

y = 5 – 2.5

y = 2.5 m

Since the time is measured when the particle is at extreme position

$y = A\cos\omega t$

2.5 = $5\cos t$

Cos t = $^{2.5}/_{5}$

Cos t = 0.5

t = 0.5

t = cos $^{-1}$ (0.5)

t = $^{\pi}/_{3}$

t = 0.1 s

APPLICATION OF S.H.M

We shall consider the following cases of S.H.M

i) Oscillation of a Loaded Spring

ii) Oscillation of a Simple Pendulum

iii Oscillation of a Liquid in a U – tube

iv) Oscillation of a Floating Cylinder

v) Body Dropped in a funnel along earth diameter

vi) Oscillation of a ball placed in the Neck of Chamber Containing air

Oscillations of a Loaded Spring

If load attached to a spring is pulled a little from its mean position and then released the load will execute S.H.M

We shall consider the following two cases

1. Vibrations of a Horizontal spring
2. Vibrations of a Vertical spring

1. VIBRATIONS OF A HORIZONTAL SPRING

Consider a block of mass M attached to one end of a horizontal spring whi9le the other end of the spring is fixed to a rigid support

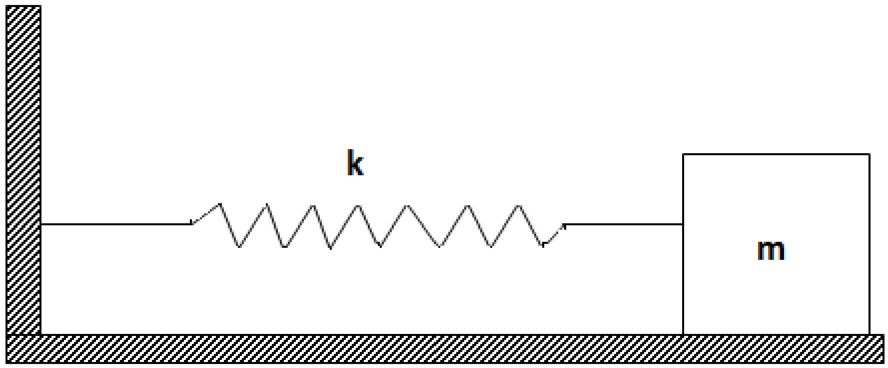


Fig. 7

The Block is at rest but is free to move along a friction less horizontal surface

In figure 7 is displaced through a small distance x to the right, the spring gets stretched

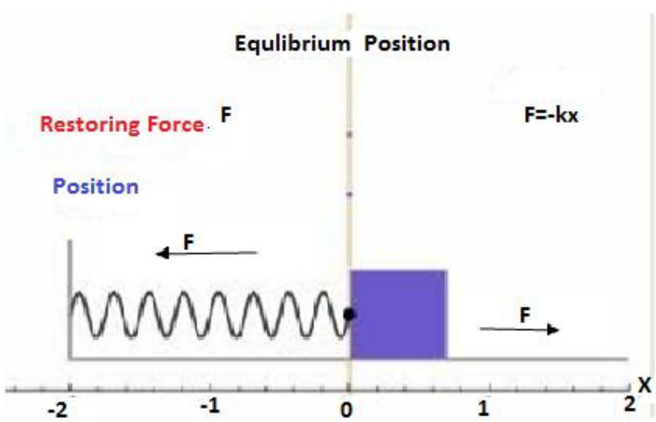


Fig.8

According to Hooke's law, the spring exerts a restoring force F to the left given by

F = $-kx$……………………………… (i)

Here k is the force constant (spring constant) and x is the displacement of mass m from the mean position.

Clearly equation (i) satisfies the condition to produce S.H.M

If the block is released from the displaced position and left , the block will execute S.H.M

The time period (T) and frequency (f) of the vibrations can be obtained from

F = $-kx$

Ma = $-kx$

$a = -{}^{k}/_{m}x$

$\omega^2 = {}^{k}/_{m}$

${}^{4\pi^2}/_{T^2} = {}^{k}/_{m}$

$T^2 = 4\pi^2.{}^{m}/_{k}$

$T = 2\pi\sqrt{{}^{m}/_{k}}$

From

$T = {}^{1}/_{f}$

$f = {}^{1}/_{T}$

2. VIBRATION OF A VERTICAL SPRING

Consider unloaded vertical spring of spring constant k

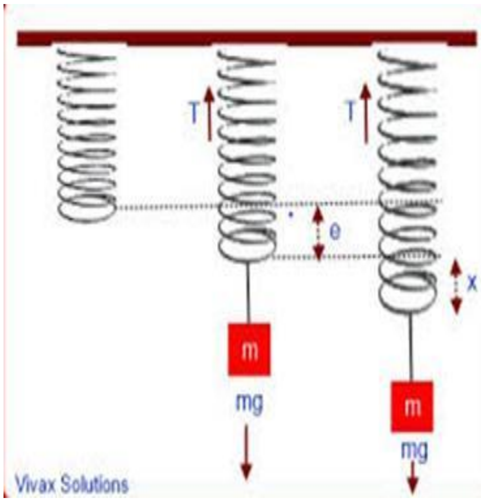


Fig. 9

Suppose the spring is loaded with a body of mass m and extended from its original length to an extension 'e'

By Hookes law

mg = $-ke$

Now suppose the load is displaced down to distance x and then released. The applied force is given by

F = $-k(e + x)$

When realized the applied force is opposed by gravity force (weight)

Net result force = F – W

Ma = $-k(e + x) - -ke$

Ma = $-ke - kx + ke$

Ma = $-kx$

a = $-{}^{k}/_{M} . x$

$\omega_2 = {}^{k}/_{M}$

$4\pi^2/_{T^2} = {}^{k}/_{M}$

$T^2 = 4\pi^2\, {}^{m}/_{k}$

$T = 2\pi\sqrt{{}^{M}/_{k}}$

From

$f = {}^{1}/_{T}$

$f = {}^{1}/_{2\pi}\sqrt{{}^{k}/_{M}}$

The period of oscillation depends on mass of the loaded body and the spring constant.

In many practical situation springs are connected in series as well as in parallel.

SERIES AND PARALLEL CONNECTION OF SPRINGS

1. PARALLEL

Consider two springs of spring constant K_1 and K_2 arranged in parallel and then both loaded with a body of mass m as shown in fig. 10

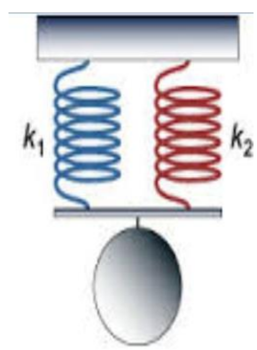


Fig. 10

Suppose this body is displaced from its equilibrium position and the extension is x for the system to remain horizontal

$$F = -Kx$$

$$K = K_1 + K_2$$

$$ma = -(K_1 + K_2)x$$

Let F_1 and F_2 be the restoring forces acting on the springs.

Total force = $F_1 + F_2$

ma = $-K_1 x + (-K_2 x)$

ma = $-(k_1 + k_2)x$

Thus, the springs will execute S.H.M

From

$$a = -\omega^2 x$$

$$4\pi^2 / T_p{}^2 = \frac{K_1 + K_2}{m}$$

$$T_p{}^2 = 4\pi^2 . m / k_1 + k_2$$

$$T_p = 2\pi \sqrt{m / k_1 + k_2}$$

T_P = Periodic time for parallel connection

If $k_1 = k_2 = k$ (for identical spring)

$$T_p = 2\pi \sqrt{m / 2k}$$

From

$$f = {}^{1}/_{T}$$

$$f = {}^{1}/_{2\pi}\sqrt{k_1 + {}^{k_2}/_{m}}$$

OR

For

$k_1 = k_2 = k$ (for identical spring)

$$f = {}^{1}/_{2\pi}\sqrt{{}^{2k}/_{m}}$$

Alternative arrangment of parallel connection formula

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.\sqrt{2}$$

Therefore if two identical sprinngs are arranged in parallel , their frequency increases by the factor of $\sqrt{2}$; since

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ is the frequency for a single spring for n-identical springs their frequency increases by a factor of $\sqrt{n}$.

Consider two springs $S_1$and S_2 of force constants k_1 and k_2 attached to a mass m and two fixed supports as shown in figure 11.

When the mass pulled downward, then length of the spring S_1 will be extended by x while that of spring S_2 will be compressed by x

Since the force constants of the two springs are different the restoring force exerted by each spring

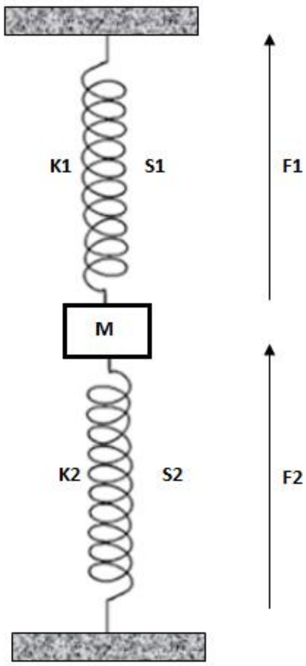
Let F_1 and F_2 be the restoring forces exerted by springs S_1 and S_2 respectively

$$F_1 = -K_1x \qquad and \qquad F_2 = -K_2x$$

Both the restoring forces will be directed upward (opposite to displacement)

The resultant restoring force F

Fig .11



$$F = -k_1 x + (-k_2 x)$$

$$F = -(k_1 + k_2)x$$

Effectively force constant of the system

$$k = k_1 + k_2$$

$$F = -kx$$

$$ma = -kx$$

$$a = \frac{-k}{m}.x$$

$$\omega^2 = {}^{k}/_{m}$$

$$4\pi^2/_{T^2} = {}^{k}/_{m}$$

$$T^2 = 4\pi^2.{}^{m}/_{k}$$

$$T = 2\pi\sqrt{{}^{m}/_{k}}$$

$$T = 2\pi\sqrt{{}^{m}/_{k_1 + k_2}}$$

2. SERIES CONNECTION

Consider two springs S_1 and S_2 of force constant K_1, and K_2 connected in series as shown below

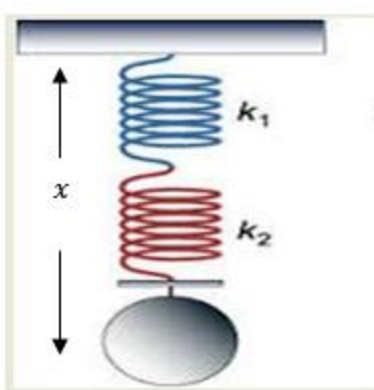


Fig. 12

If x is the total displacement

$$x = x_1 + x_2$$

Where x_1 is extension due to spring of spring constant k_1 and x_2 to that of k_2

$F_1 = -k_1 x_2$

$F_2 = -k_2 k_2$

$F_1 = F_2 = F$

$$x_1 = -F/k_1$$

$$and$$

$$x_2 = -F/k_2$$

$$x = x_1 + x_2$$

$$x = -F/k_1 - F/k_2$$

$$x = -F\left(1/k_1 + 1/k_2\right)$$

$$F = -\left[k_1 k_2/k_1 + k_2\right] x$$

The effective spring constant k_0

$$k_0 = k_1 k_2/k_1 + k_2$$

$$F = -k_0 x$$

$$ma = -k_0 x$$

$$a = -k_0/m x$$

$$4\pi^2/T_s^2 = k_0/m$$

$$T_s = 4\pi^2 m/k_0$$

$$T_s = 2\pi\sqrt{m/k_0}$$

T_s = periodic time for series connection.

$$T_s = 2\pi\sqrt{M(k+k_2)/k_1k_2}$$

For identical spring

$$k_1 = k_2 = k$$

$$T_s = 2\pi\sqrt{m/2k}$$

Also since,

$$T_p = 2\pi\sqrt{m/2k}$$

and $T_s = 2\pi\sqrt{2m/k}$

$$T_p{}^2/T_s{}^2 = {4\pi m}/{2k}\cdot{k}/{8\pi m}$$

$$T_p^2/T_s^2 = 1/4$$

$$\sqrt{T_p^2/T_s^2} = \sqrt{1/4}$$

$$T_p/T_s = 1/2$$

$$T_s = 2T_p$$

OSCILLATION OF SIMPLE PENDULUM

A heavy body suspended by a light inextensible string is called simple pendulum.

The point from which the bob is suspended is called the point of suspension

The center of gravity of the bob is called center of oscillations.

The Distance between point of suspension and center of oscillation is called length of the pendulum.

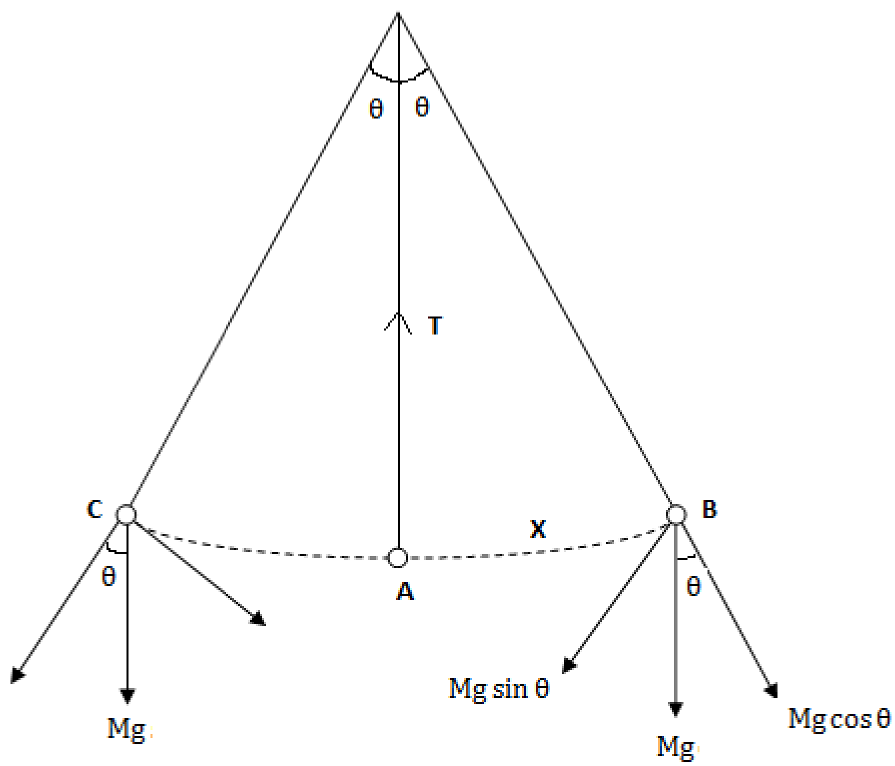


Fig. 13

At A, the weight of the bob acts vertically downwards and the tension in the string acts vertically upwards.

These two force are equal and opposite so A is the equilibrium position

Let the bob be displaced by a small angle θ from the equilibrium position towards B

The weight mg is resolved into two rectangle components. The component $mg\cos\theta$ acts radially along OB

The component $mg\sin\theta$ acts tangentially along BA. This acts towards equilibrium position. So it is called the restoring force F

$F = -mg\sin\theta$

$F = -mg\theta$

$If\ \theta\ is\ small\ angle\ measured\ in\ radian$

$Arc\ AB = l\theta$

$x = l\theta$

$\theta = \frac{x}{l}$

Substitute the value of θ in equation (i)

$F = -mg\theta$

$F = -mg\frac{x}{l}$

$a = -\frac{mg}{l}x$

$\omega^2 = \frac{g}{l}$

$\frac{4\pi^2}{T^2} = \frac{g}{l}$

$T^2 = 4\pi^2\frac{l}{g}$

$T = 2\pi\sqrt{\frac{l}{g}}$

Hence the period of oscillation is independence of the mass of the bob. Such oscillation or vibrations are called Isochronous vibrations.

The period T is directly proportional to the square rout of the length of the pendulum at a place.

Oscillations of a liquid in a tube

One end of a U-tube containing certain liquid is connected to a sunction pump and the other end is open to the atmosphere.

A small pressure different is maintained between the two columns. We can show that when the suction pump is removed the liquid column in the U-tube execute S.H.M

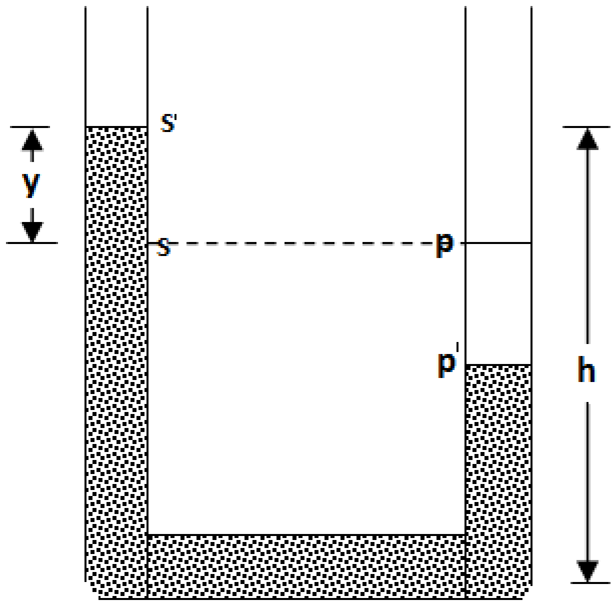


Fig. 14

Let the initial level of liquid in the U-tube to be a height h

When liquid is depressed trough a distance y in one limb it rises by the same amount in the other limb.

The difference between the liquid levels in the two limbs is 2y

Let A be the area of the cross section of the tube. The weight of the liquid column of height 2y provide the restoring force F

F = Weight of liquid column of height 2y

$$F = -A \times 2y \times \rho g$$

$$\rho - Density\ of\ the\ liquid$$

The Negative sign shows that F acts opposite to y

Comparing this with the standard equation of S.H.M

$$F = -2Ay\rho g$$

$$ma = -2A\rho yg$$

$$a = -\frac{2A\rho g}{m}.y$$

$$\omega^2 = \frac{2A\rho g}{m}$$

$$\frac{4\pi^2}{T^2} = \frac{2A\rho g}{m}$$

$$T^2 = \frac{2\pi^2 m}{2A\rho g}$$

$$T = 2\pi\sqrt{\frac{m}{2A\rho g}}$$

Let h be the height of liquid in one of the limbs when liquid is in equilibrium position

Mass of liquid executing S.H.M = M

m = Mass of the liquid of height 2h

$$m = A \times 2h \times \rho$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

Oscillation of a Floating Body

Consider a cylindrical piece of cork of base area A and height h floating in a liquid of density ρ_l

The cork is depressed slightly and released. We can show that the cork oscillates up and down simple harmonically.

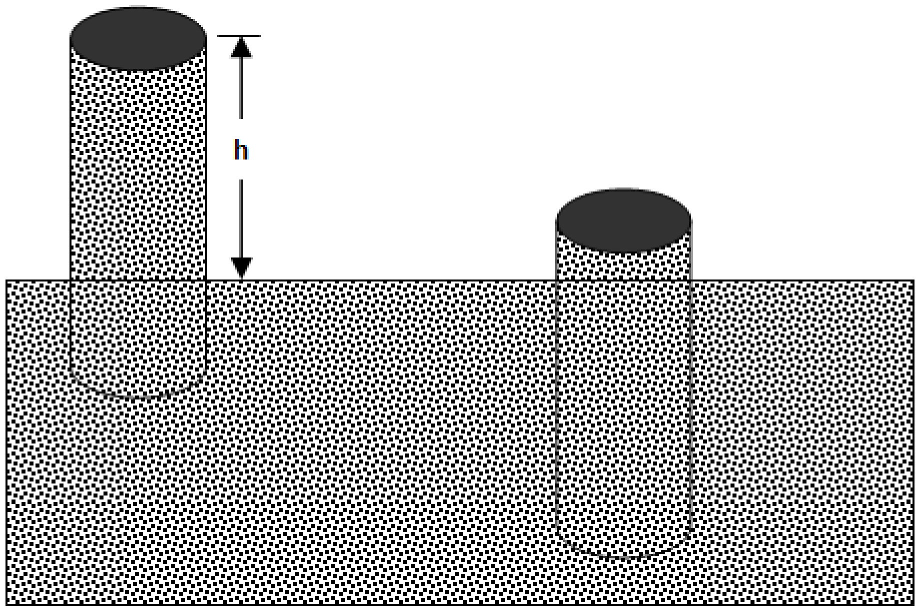


Fig.15

The condition for floatation is

Weight of the cork $= upthrust$

Weight of cork $= mg$

$= V\rho g$

$= Ah\rho g$

Let the cork be depressed vertically downwards by y

$Upthrust =$ Weight of displaced liquid

= mass of displaced liquid $\times g$

$= Ay\rho_l g$

This upthrust provides the restoring force which acts upwards when y is downwards

Restoring force $F = -A\rho_l gy$

$$= -A\rho_l gy$$

$$a = -\frac{A\rho_l gy}{m}.y$$

So the body executes S.H.M

$$\omega^2 = \frac{A\rho_l g}{m}$$

$$\frac{(2\pi)^2}{T^2} = \frac{A\rho_l g}{m}$$

$$T^2 = 4\pi^2 \frac{m}{A\rho_l g}$$

$$T = 2\pi\sqrt{\frac{m}{A\rho_l g}}$$

$$T = 2\pi\sqrt{\frac{Ah\rho}{A\rho_l g}}$$

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

For $\rho = \rho_l = \rho$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

BODY DROPPED IN A TUNNEL ALONG EARTH DIAMETER

Suppose earth to be a sphere of radius R with center O.

Let a tunnel be dig along the diameter of the earth as shown in figure below

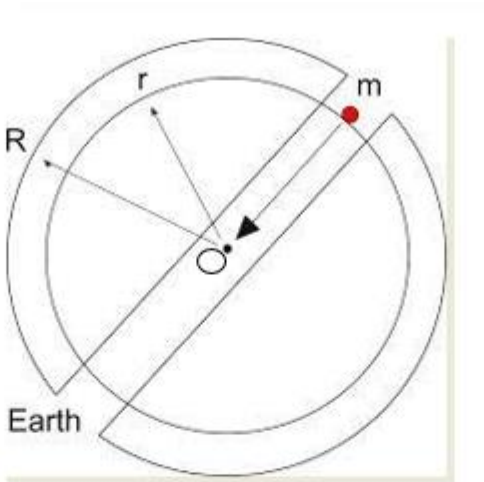


Fig.16

If a body mass m is dropped at one end of the tunnel, the body will execute S.H.M about the center O of the earth.

Suppose at any instant, the body in the tunnel is at a distance y from the center O of the earth

The body is inside the earth, only the inner sphere of radius y will exert gravitation force F on the body

The force F serves as the restoring force that tends to bring the body to the equilibrium position O

Restoring force $$F = -G\frac{(4/_3\pi y^3\rho)m}{y^2}$$

ρ – Density of the earth. The negative sign is assigned because the force is of attraction

$$ma = \frac{-4G\pi y^3\rho m}{3y^2}$$

$$a = -\frac{4G\pi\rho}{3}.y$$

$$\omega^2 = \frac{4G\pi\rho}{3}$$

$$\frac{4\pi^2}{T^2} = \frac{4\pi G\rho}{3}$$

$$T^2 = \frac{3\pi}{G\rho}$$

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

Oscillation of a Ball

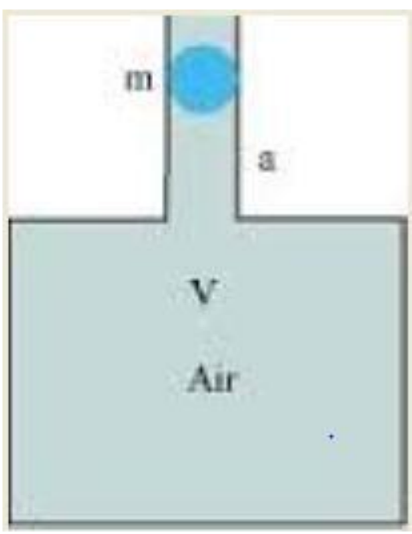
An air chamber of volume V has a neck of area of cross section A into which a ball of mass M can move without friction

We can show that when the ball is pressed down some distance and released, the ball execute $s.h.m$

Let us find the time period assuming the pressure – volume vacations of the air to be

a) Isothermal

b) Adiabatic



Suppose the ball is depressed by a distance y then change in pressure dp produces the restoring force F

Isothermal change

$PV = constant$

$Pdv + Vdp = 0$

$$Vdp = -Pdv$$

$$dp = -P\frac{dv}{V}$$

Change in volume

$$dv = Ay$$

$$dP = -\frac{PAy}{V}$$

Restoring force

$$F = Adp$$

$$F = \frac{-\rho A^2 y}{V}$$

This expression shown that the restoring force is directly proportional to the displacement and is directed towards the equilibrium position

$$F = \frac{-PA^2}{V}.y$$

$$ma = \frac{-PA^2}{mV}.y$$

$$\omega^2 = \frac{PA^2}{mV}$$

$$\frac{4\pi^2}{T^2} = \frac{PA^2}{mV}$$

$$T^2 = 4\pi\frac{mV}{PA^2}$$

$$T = 2\pi\sqrt{\frac{mV}{PA^2}}$$

Adiabatic change

For adiabatic change

$$PV^{\gamma} = constant$$

$$V^{\gamma}dp + \gamma PV^{\gamma-1}dv = 0$$

$$V^{\gamma}dp = -\gamma PV^{\gamma-1}dv$$

$$dp = \frac{-\gamma PV^{\gamma-1}}{V^{\gamma}}.dv$$

$$dp = -\frac{\gamma Pdv}{V}$$

But $dv = Ay$

$$dp = -\frac{\gamma Pdv}{V}$$

$$F = Adp$$

$$F = -\frac{\gamma PA^2}{V}.y$$

$$ma = -\frac{\gamma PA^2}{V}.y$$

$$a = -\frac{\gamma pA^2}{mV}.y$$

$$\omega^2 = \frac{\gamma PA^2}{mV}$$

$$\frac{(2\pi)^2}{T^2} = \frac{\gamma PA^2}{mV}$$

$$T^2 = (2\pi)^2\frac{mV}{\gamma PA^2}$$

$$T^2 = 2\pi\sqrt{\frac{mV}{\gamma PA^2}}$$

WORKED EXAMPLES

1. The resultant force acting on a particle executing S.H.M is 4N when it is 5cm away from Re mean position. Find Re force constant.

Soln

$F = -ky$

$Force\ constant\ k = [F/y] = {}^{4\ N}/_{5 \times 10^{-2}\ m}$

$\therefore k = 80\ N/m$

2. A body of mass 12kg is suspended by a coil spring of natural length 50cm and force constant 2.0 $\times 10^3$ N/M. What is the stretched length of the spring? If Re body is pulled down further stretching the spring to 59cm and then released, what is the frequency of oscillation of Re suspended mass?

Soln

If a force F produces an increase of length l in the length of a spring then force constant of the spring is given

$F = kl$

$l = {}^{F}/_{k} = {}^{mg}/_{k}$

$l = {}^{12 \times 9.8}/_{2 \times 10^{-3}}$

$l = 0.058\ m = 5.88\ cm$

Natural length of the spring $= 50\ cm$

Stretched length of the spring $= 50 + 5.88$

$= 55.88\ cm$

$F = {}^{1}/_{2\pi} \sqrt{{}^{k}/_{m}}$

$F = {}^{1}/_{2\pi} \sqrt{{}^{2 \times 10^3}/_{12}}$

$\therefore F = 2.055\ s^{-1}$

3. For the motion of mass suspended by a coil spring in example 2

(i) What is the net force on the suspended mass at its lower most position?

(ii) What is the elastic restoring force on the mass due to spring at its upper most position?

Solution

(i) The lower most position of the spring corresponding to the position when the it is stretched to a length

Extension produced in this position is

$y = (59 - 50)\ cm$

$y = 9\ cm = 0.09\ m$

Restoring force acting upward $= Ky$

$= 2 \times 10^3 \times 0.09$

$= 180\ N$

Weight of suspended mass $= Mg$

$= 12 \times 9.8$

$= 117.6N$

$\therefore$Net force on the suspended mass at the lower most point is

$F = 180N - 117.6N$

$\therefore F = 62.4\ N\ (Upwards)$

(ii) Original strength of the spring $= 50\ cm$

Stretched length of the spring $= 50 + 5088 = 55.88\ cm$

When the suspended mass is pulled to stretch the spring to a length of 59 cm and released the spring oscillate about its equilibrium length of 55.88 cm

$$Amplitude\ oscillations = 59 - 55.88$$

$$= 3.12\ cm$$

Length of the spring corresponding to upper most position $= 55.88 - 3.12$

$$= 52.76\ cm$$

Extension of the spring corresponding to the upper most position $y' = 52.76 - 50$

$$= 2.76\ cm$$

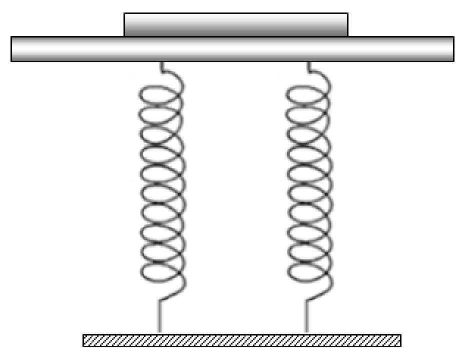
Restoring force at the upper most position

$$F' = Ky'$$

$$= 2 \times 10^3 \times 2.76$$

$$\therefore F' = 55.2\ n$$

5. A tray of mass 12Kg is supported by two identical springs as shown in figure below



When the tray is pressed down slightly aid released, it executes S.H.M with a time period of 1.5s. What is the force constant of each spring? When a block of mass M is placed on the tray, the period of S.H.M changes to 3.0s. What is the mass of the block?

Solution

If K is the force constant of the parallel combination of spring, then period of the tray is given by

$$T = 2\pi\sqrt{m/k}$$

$$T^2 = 4\pi^2.{}^{m}/_{k}$$

$$k = 4\pi^2.{}^{m}/_{T^2}$$

Here

$$m = 12kg$$

$$T = 1.5\ s$$

$$k_T = 4\pi^2.{}^{12}/_{1.5^2}$$

$$k_T = 2.1 \times 10^2\ N/m$$

Since the force constant for the parallel combination is the sum of the force constant for individual springs

Force constant of each spring

$$k = {}^{k_T}/_{2}$$

$$k = {}^{2.1 \times 10^2}/_{2}$$

$$\therefore k = 1.05 \times 10^2\ N/m$$

Again

$$T = 2\pi\sqrt{m/k}$$

$$T^2 = 4\pi^2.{}^{m}/_{k}$$

$$m = {}^{k}/_{4\pi^2}.T^2$$

Here,

$$T = 3\ s$$

$$k = 2.1 \times 10^2\ N/m$$

$$m = {2.1 \times 10^2}/{4\pi^2} \times 3^2$$

$$m = 48\ kg$$

Mass of block m

$$m = (48 - 12)\ kg$$

$$m = 36\ kg$$

6. Two masses m_1 = 1.0 kg and m_2 = 0.5kg are suspended by a weightless spring of force constant K = 12.5N/M. when they are in equilibrium position m_1 is gently removed. Calculate the Angular frequency and Amplitude of m_2. Take g = 10m/s_2.

Solution:

-Suppose y is the extension in the length of the spring when both m_1 and m_2 are suspended, then

$$F = (m_1 + m_2)g = Ky - - - - - - - - -(i)$$

-Suppose the extension is reduced to $y'\ when\ m_1$ is removed. Then

$$m_2 g = ky' - - - - - - - - - - - - - -(ii)$$

Take equation $(i) - equation\ (ii)$

$$ky - ky' = (m_1 + m_2)g - m_2 g$$

$$ky - ky' = m_1 g + m_2 g - m_2 g$$

$$ky - ky' = m_1 g$$

$$k(y - y') = m_1 g$$

$$y - y' = {m_1 g}/{k}$$

This will be amplitude A of the oscillation of m_2

$$A = y - y' = {m_1 g}/_{k}$$

$$A = {1 \times 10}/_{12.5} = 0.8\, m$$

$$\therefore A = 0.8\, m$$

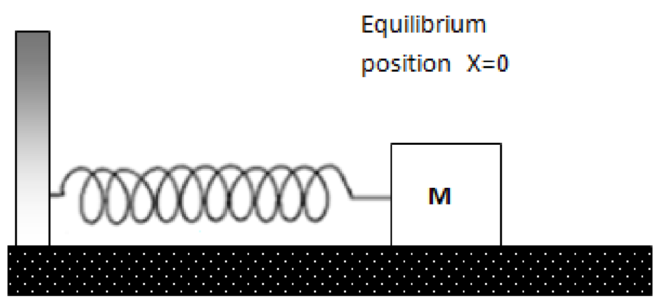
The angular frequency of oscillation of m_2 is

$$\omega = \sqrt{{k}/_{m_2}}$$

$$\omega = \sqrt{{12.5}/_{0.5}}$$

$$\omega = 5\, s^{-1}$$

7. A spring of force constant 1200N/m is mounted on the horizontal table



A mass of 3.0kg attached to the free end of the spring is pulled side way to a distance of 2.0 cm and released. What is the angular frequency of oscillation of the mass? What is

i) The speed of the mass when the spring is compressed by 1.0 cm
ii) Potential energy of the mass when it momentarily comes to rest
iii) Total energy of the oscillating mass?

Solution

$m = 3\, kg$

$K = 1200\, N/m$

$A = 2 \times 10^{-3}\, m$

Frequency of oscillation of mass is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1200}{3}}$$

$$f = 3.18\, s^{-1}$$

Angular frequency of oscillation of the small

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 3.18$$

$$\omega = 20\, rad/s$$

i) Speed of the mass is given by

$$V = \omega\sqrt{A^2 - y^2}$$

$$= 20\sqrt{(0.02)^2 - (0.01)^2}$$

$$\therefore V = 0.3\,\frac{m}{s}$$

ii) The oscillating mass momentarily comes to rest at the extreme position $i.e$ given

$$y = A = 0.02\, m$$

$P.E\ of\ mass\ at\ extrem\ position$

$$P.E = \frac{1}{2}m\omega^2 A^2$$

$$P.E = \frac{1}{2} \times 3 \times 20^2 \times (0.02)^2$$

$$\therefore P.E = 0.24\, J$$

iv) At the extreme position, the P.E of the mass is equal to the total energy

$$E_T = 0.24\, j$$

8. A small bob of mass 50g oscillates as simple pendulum with and amplitude 5cm and time period 25. Find the velocity of the bob and the tension in the supporting thread when the velocity of the bob is maximum

Soln

The velocity of the bob will be maximum when it passes through the mean position

$$m = 50 \times 10^{-3} kg$$

$$A = 5 \times 10^{-2}\ m$$

$$T = 2\ s$$

Angular frequency $\omega = {}^{2\pi}/_{T}$

$$\omega = {}^{2\pi}/_{2}$$

$$\omega = \pi\ rad/s$$

Maximum velocity of bob

$$V_{max} = \omega A$$

$$= \pi \times 5 \times 10^{-2}$$

$$\therefore V_{max} = 0.05\pi\ m/s$$

Suppose F is the tension in the supporting thread then

$$F - mg = {}^{mV_{max}^2}/_{l}$$

$$l = length\ of\ pendulum$$

$$F = {}^{mV_{max}^2}/_{l} + mg - - - - - - - - - - - - - - - (i)$$

Now,

$$T = 2\pi\sqrt{{}^{l}/_{g}}$$

$$T^2 = 4\pi^2 \, {}^{l}/_{g}$$

$$l = {}^{gT^2}/_{4\pi^2} = {}^{9.8 \times 2}/_{4 \times \pi^2}$$

$$l = 0.4965\, m$$

From equation (i)

$$F = {}^{mV_{max}^2}/_{l} + mg$$

$$F = m\left[{}^{V_{max}^2}/_{l} + g\right]$$

$$F = 50 \times 10^{-3}\left[{}^{(0.05 \times \pi)^2}/_{0.4965} + 9.8\right]$$

$$\therefore F = 0.4925\, N$$

9. A pendulum clock shows correct time if the length increased by 0.1%, find the error in time per day.

Soln

Correct no of seconds per day $f = 24 \times 60 \times 60$

$$= 86400$$

-Let the error introduced per day be x seconds incorrect no of seconds per day $f' = 86400 + x$

If l is original length, then its new length is l'

$$l' = l + 0.1\% \;\; 0f\; l$$

$$l' = l + {}^{0.1}/_{100} \times l$$

$l' = l + 0.001l$

$l' = 1.001\, l$

Now,

Frequency $f = {}^{1}/_{2\pi}\sqrt{{}^{g}/_{l}}$

$f \propto {}^{1}/_{\sqrt{l}}$

$f = {}^{k}/_{\sqrt{l}}$

$f\sqrt{l} = k$

$f\sqrt{l} = f'\sqrt{l'}$

${}^{f'}/_{f} = \sqrt{{}^{l}/_{l'}}$

$86400 + {}^{x}/_{86400} = \sqrt{{}^{l}/_{(1.001)l}}$

$86400 + x = 86356.83$

$x = -43.16\, s$

Lose 43.2 seconds per day

10. Two simple pendulums of lengths 1.44m and 1.00m start swinging at the same time. After how much time they will be

i) Out of phase and

ii) In phase gain?

Take g $10 m/s^2$

Soln

Let T_1 and T_2 be the time periods of large (length) and smaller pendulums respectively L_1 and L_2 be the corresponding lengths.

$$T_1 = 2\pi\sqrt{L_1/g}$$

and

$$T_2 = \sqrt{L_2/g}$$

$$T_1/T_2 = \sqrt{L_2/L_2} = \sqrt{1.44/1.00}$$

$$T_1/T_2 = 1.2$$

i) The pendulum of large length (L1) will have smaller frequency and greater time period.

Suppose they get out of phase in time t. The two pendulums will be out of phase if the pendulum of smaller length makes ½ vibration extra compared to the pendulum of larger length.

-If the pendulum of larger length makes f vibration time t then the other

Pendulum will make $(f + 1/2)$ vibrations

$$t = fT_1 = (f + 1/2)T_2$$

$$T_1/T_2 = {f + 1/2}/f$$

$$1.2 = {f + 0.5}/f$$

$$1.2f = f + 0.5$$

$0.2f = 0.5$

$f = 2.5\ vibrations$

$t = 2.5 \times 2\pi \times \sqrt{1.44/_{10}}$

$\therefore t = 5.96\ s$

ii) The two pendulum will be again in phase when the pendulum of larger length complete the f' vibration and pendulum of the smaller length completes $(f' + 1)$ vibrations in time t'

$t' = f'T_1 = (f' + 1)T_2$

$T_1/_{T_1} = {f' + 1}/_{f'}$

$1.2 = {f' + 1}/_{f'}$

$f' = 5\ vibrations$

$t' = f'T_1$

$t' = 11.96\ s$

11. A vertical U-tube of uniform cross-section contains water up to 80cm. find the frequency of oscillations of water when one side is depressed and then released.

The frequency of oscillation is given by

$$f = {}^1/_{2\pi}\sqrt{g/_h}$$

$g = 9.8\ m/s^2$

$h = 80 \times 10^{-2}\ m$

$f = {}^1/_{2\pi}\sqrt{9.8/_{0.8}}$

$\therefore f = 0.56\, HZ$

12. Image a tunnel is dug along a diameter of the earth. Show that a body dropped from one end of the tunnel executes S.H.M. what is the time period of the motion? Assume, the earth to be a sphere of uniform mass density = 5520kg/m^3 and G = 6.67 x 10^{-11} Nm^2 kg-2

Soln

The time period T of the body is

$$T = \sqrt{3\pi / GP}$$

$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

$\rho = 5520\ kg/m^3$

$$T = \sqrt{3 \times \pi / 6.67 \times 10^{-11} \times 5520}$$

$\therefore T =$

13. A cubical body (side 0.1m and mass 0.002kg) floats in water. It is pressed and then released so that it oscillates vertically. Find the time period. The density of water is 1000kg/m3

Solution

Mass of the body m = 0.002kg

$cross - section\ area, A = 0.1 \times 0.1 = 0.01\ m^2$

$Density\ of\ water, \rho = 1000\ kg/m^3$

Suppose at equilibrium, the body is floating with a depth h submerged. According to the principle of floatation

$Weight\ of\ body =$
$weight\ of\ water\ displaced\ by\ the\ immersed\ part\ of\ the\ body$

$$mg = (\rho A h) g$$

$$h = {}^{m}/_{\rho A} = {}^{0.002}/_{1000 \times 0.01}$$

$$h = 2 \times 10^{-4}\ m$$

The time period of oscillation

$$T = 2\pi\sqrt{{}^{h}/_{g}}$$

$$T = 2\pi\sqrt{{}^{2 \times 10^{-4}}/_{9.8}}$$

$$\therefore T = 0.028\ second$$

14. (a) Is the motion of a simple pendulum strictly simple harmonic?

(b) What is the relation between uniform circular motion and S.H.M?

(c) Can simple pendulum experiment be done inside a satellite?

(d) Show that the total energy of a body executing S.H.M is independent of time

(e) If the metal bob of a simple pendulum is replaced by a wooden bob, what will be the change in the time period of the pendulum?

Solution

(a) It is not strictly simple harmonic because we make the assumption that $\sin\theta = \theta$ which is nearly valid only if θ is very small.

(b) Uniform circular motion can be thought of as two simple harmonic motion operation at right angles

(c) The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{{}^{l}/_{g}}$$

Inside a satellite, a body is in a state of weightlessness so that the effective value of g for it is zero. When g = 0, $T \rightarrow \infty$

Therefore, inside the satellite, the pendulum does not, oscillate at all. Hence cannot be performed

(d) Assuming the initial phase to be zero

$$y = A\sin\omega t$$

$$V = {}^{dy}/_{dt} = {}^{d(A\sin\omega t)}/_{dt}$$

$$V = A\omega\cos\omega t$$

$$K.E = {}^{1}/_{2}mV^2 = {}^{1}/_{2}m\omega^2A^2cos^2\,\omega t$$

$$K = m\omega^2$$

$$K.E = {}^{1}/_{2}KA^2cos^2\,\omega t$$

$$P.E = {}^{1}/_{2}Ky^2 = {}^{1}/_{2}KA^2sin^2\,\omega t$$

Total energy E_T

$$E_T = P.E + K.E$$

$$E_T = {}^{1}/_{2}K\omega^2cos^2\,\omega t + {}^{1}/_{2}K\omega^2sin^2\,\omega t$$

$$E_T = {}^{1}/_{2}KA^2(cos^2\,\omega t + sin^2\,\omega t)$$

$$E_T = {}^1/_2 KA^2$$

Thus total energy of a body executing S.H.M is always constant and is independent of time.

(e) No change it is because time period of simple pendulum is independent of the mass of the bob $i.e$

$$T = 2\pi\sqrt{{}^l/_g}$$

15. (a) You have a light spring, a meter scale and a known mass. How will you

Find the time period of oscillation of mass without the use of clock/

(b) Will a pendulum clock gain or lose time when taken to the top of mountain?

(c) Will a simple pendulum vibrate at the center of earth?

(d) What will be the charge in the time period of a loaded spring when taken to moon?

(e) A simple pendulum is executing S.H.M with a time period T. if the length of the pendulum is increased by 21%, what will be the % increase in time period?

(f)The maximum acceleration of a simple harmonic oscillator is a0 and maximum velocity is Vo. What is the displacement amplitude?

Solution

(a) Suspend the spring from a rigid support and attach the mass at its lowest end. Measure the extension (â„“) in the spring with a meter scale.

If K is the force constant of the spring then restoring force F is

$$F = -kl$$

$$-mg = -kl$$

$\therefore {}^{m}/_{k} = {}^{l}/_{g}$

Time period

$T = 2\pi\sqrt{{}^{m}/_{k}}$

$\therefore T = 2\pi\sqrt{{}^{l}/_{g}}$

(b) Time period $T = 2\pi\sqrt{{}^{l}/_{g}}$.

As g is less at the top of mountain, value of T will increase. Therefore, the pendulum will take more time to complete one vibration. As a result pendulum clock will lose time

(c) No it is because g = 0 at the center of the earth.

(d) No change, it is because, the time period of a loaded spring is independent of acceleration due to gravity.

(e) Time period,

$\therefore T = 2\pi\sqrt{{}^{l}/_{g}}$

$T \propto \sqrt{l}$

$T = k\sqrt{l}$

${}^{T}/_{\sqrt{l}} = k$

${}^{T_1}/_{\sqrt{l_1}} = {}^{T_2}/_{\sqrt{l_2}}$

${}^{T_2}/_{T_1} = \sqrt{{}^{l_2}/_{l_1}}$

$$T_2/T_1 = \sqrt{1.21l_1/l_1}$$

$$T_2/T_1 = 1.1$$

$$T_2/T_1 - 1 = 1.1 - 1$$

$$T_2 - T_1/T_1 = 0.1$$

$$T_2 - T_1/T_1 \times 100\% = 0.1 \times 100$$

$$\therefore \%\ increase\ in\ time\ period\ is\ 10\%$$

(f) Let A be the displacement amplitude and w the angular velocity of the oscillator

$$a_0 = \omega^2 A$$

$$V_0 = \omega A$$

$$\omega = V_0/A$$

$$a_0 = \left(V_0/A\right)^2 A$$

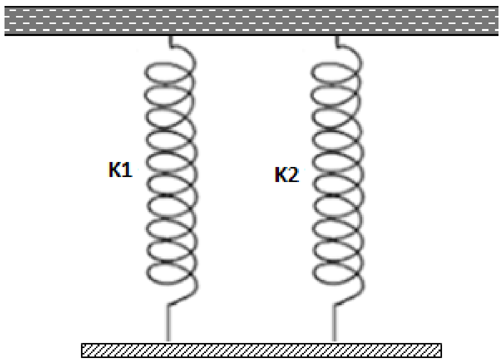
$$a_0 = V_0^2/A^2 \times A$$

$$a_0 = V_0^2/A$$

$$\therefore A = V_0^2/a_0$$

16. (a) Why does a swinging simple pendulum eventually stop?

(b) A mass m suspended separately from two different springs of spring constant K_1 and K_2 given time period T_1 and T_2 respectively.



If the same mass is connected to both the spring as show in figure above, prove that the time period T is given by

$$T^{-2} = T_1^2 + T_2^2$$

(c) A simple pendulum has metallic bob what will be the effect on its time period if metallic bob pendulum is taken to moon?

Solution

(a) Due to friction between air and bob, the amplitude of the pendulum goes on decreasing and eventually it comes to rest.

(b) Time period T

$$T = 2\pi\sqrt{m/k}$$

$$k = 4\pi^2 m/T^2$$

For the first spring

$$k_1 = 4\pi^2 m/T_1^2$$

For the second spring

$$k_2 = {4\pi^2 m}/{T_2^2}$$

When the springs are connected in parallel

$$k = k_1 + k_2$$

$$4\pi^2 m/T^2 = 4\pi^2 m/T_1^2 + 4\pi^2 m/T_2^2$$

$$4\pi^2 m/T^2 = 4\pi^2 m\left[1/T_1^2 + 1/T_2^2\right]$$

$$1/T^2 = 1/T_1^2 + 1/T_2^2$$

$$T^{-2} = T_1^{-2} + T_2^{-2}$$

(c) From time period $T = 2\pi\sqrt{l/g}$

Therefore $T \propto \sqrt{1/g}$

At moon g is less (1/6th that on earth) so that T increases

17. (a) A particle moves such that its acceleration a is given by $a = -bx$ where x is the displacement from equilibrium position and b is constant. Find the period of oscillations

(b) A simple pendulum hangs from the coiling of a car. If the car accelerates with uniform acceleration, will the frequency of the pendulum increase or Decrease?

(c) Is the tension in the string of a simple pendulum constant thought the oscillation?

Solution

(a) Given that $a = -bx$

Since $a \propto x$ and it is directed opposite to x, the particles moves in S.H.M

$a = -\omega^2 x$

$a/_x = -b$

$a/_x = -\omega^2$

$\omega^2 = b$

$4\pi^2/_{T^2} = b$

$T^2 = 4\pi^2/_b$

$\therefore T = 2\pi\sqrt{1/_b}$

(b) Frequency of simple pendulum f

$f = 1/_{2\pi}\sqrt{g/_l}$

$f \propto \sqrt{g}$

As the car accelerates with uniform acceleration a, the resultant acceleration $\sqrt{a^2 + g^2}$. Since $\sqrt{a^2 + g^2}$ is greater than $\sqrt{g}$, frequency of the simple pendulum will increase.

(c) No. the Tension in the string is $T = mg\cos\theta$ therefore, tension in the string T, as θ varies, the tension T in the string also varies.

Note

θ is the angle which the string makes with the vertical

18. (a) Two spring of constants K_1 and K_2 have equal maximum velocities when 3executing simple harmonic motion. Find the ratio of their amplitudes (masses are equal)

(b) Can pendulum clocks be used in artificial satellite? Explain

(c)Draw the velocity – Displacement graph of a body executing S.H.M

(d) Two identical springs of same spring constant of 10NM-1 are corrected in

(i) Series

(ii) Parallel

A mass of 5kg is suspended in each case. What is the effective spring constant and elongation in each case?

Solution

(a) From

$$F = -kx$$

$$ma = -kx$$

$$a = -\, {}^{kx}/_{m}$$

But

$$a \propto x$$

$$a = -\omega^2 x$$

$$\omega^2 = {}^{k}/_{m}$$

$$\omega = \sqrt{{}^{k}/_{m}}$$

$$\omega \propto \sqrt{k}$$

For equal maximum velocity

$$A_1\omega_1 = A_2\omega_2$$

$$A_1/A_2 = \omega_2/\omega_1 ------------ (i)$$

ALSO

From

$$\omega \propto \sqrt{k}$$

$$\omega = k\sqrt{k}$$

$$\omega_1/\sqrt{k_1} = \omega_2/\sqrt{k_2}$$

$$\omega_2/\omega_1 = \sqrt{k_2/k_1} ------------- (ii)$$

By comparing the equation (i) and (ii)

$$A_1/A_2 = \sqrt{k_2/k_1}$$

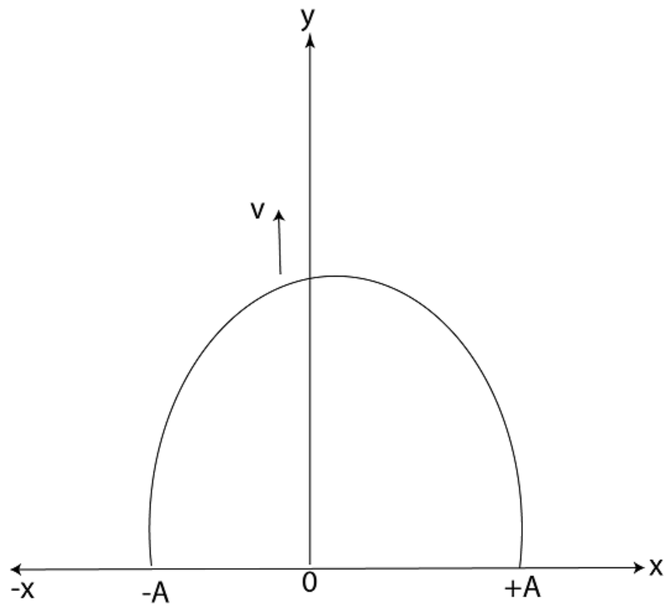
$$\therefore A_1/A_2 = \left(k_2/k_1\right)^{1/2}$$

(b) No, inside the satellite the effect of g is zero. So pendulum will not oscillate

$$T = 2\pi\sqrt{l/g}$$

$$g = 0$$

$\therefore T = 0$



From the graph

- As the Displacement of the particle changes from –A to +A the velocity increases from zero to maximum at the equilibrium position 0 and then decreases to zero at the extreme position +A, where A is the amplitude of the particle
- In the second half of the oscillation the direction of the velocity is charges.

The shape of the graph will be the same but it will below $x'0x$

(c) (i) When spring are connected in series the effective spring constant K

$$k = {k_1 k_2}/{k_1 + k_2}$$

$$k = {10 \times 10}/{10 + 10}$$

$$\therefore k = 5\ N/m$$

$$= {mg}/{k} = {5 \times 9.8}/{5}$$

Elongation

$\therefore Elongation\ is\ 9.8m$

(ii) When springs are connected in parallel, effective spring constant K

$K = K_1 + K_2$

K = 10 + 10

K = 20N/M

$$Elongation = {}^{mg}/_{k} = {}^{5 \times 9.8}/_{20}$$

$\therefore Elongation\ is\ 2.45m$

19. (a) A large horizontal surface moves up and down in S.H.M with an amplitude of 1cm. if mass of 10kg (which is placed on the surface) is to remain continuously in constant with it, what should be the maximum frequency of S.H.M

(b) A girl is swinging a swing in the sitting position. What will be the effect on the time period of the swing if she stands up?

(c) The time period of a mass suspended from a spring (spring constant K) is T if the spring is cut into three equal parts and the same mass is suspended from one piece, what will be the time period?

(d) The Amplitude of an oscillating simple pendulum is double. How will it affect

i) Time period

ii) Total energy

iii) Maximum velocity

Solution

(a) $A = 1 \times 10^{-2} m$

The mass will remain in contact if the restoring force is equal to the weight of 10kg mass

$$F = ma$$

$$mg = m\omega^2 A$$

$$\omega^2 = {}^{g}/_{A}$$

$$\omega = \sqrt{{}^{g}/_{A}} = \sqrt{{}^{9.8}/_{1 \times 10^{-2}}}$$

$$\omega = 31.3\, rad/s$$

$$\omega = 2\pi f$$

$$f = {}^{\omega}/_{2\pi} = {}^{31.3}/_{2\pi} rad/s$$

$$\therefore f = 5HZ$$

(b) The girl swinging a swing is like an oscillating pendulum. Therefore, time period of the swing is

$$T = 2\pi \sqrt{{}^{l}/_{g}}$$

$$T \propto \sqrt{l}$$

Here â„“ is the distance between the point of suspension of the swing and centre of gravity (C.G) of the girl swing system. As the girl stand up, the C.G of the oscillating Time period $T = 2\pi\sqrt{{}^{m}/_{k}} -------------- (i)$

When the spring is cut into 3 equal parts, the effective spring constant of each piece becomes 3K

$$T' = 2\pi\sqrt{{}^{m}/_{3k}} ------------ (ii)$$

Take equation $(ii) \div (i)$

$$T'/_{T} = 2\pi\sqrt{{}^{m}/_{3k}} \cdot {}^{1}/_{2\pi}\sqrt{{}^{k}/_{m}}$$

$$T'/_T = 1/_{\sqrt{3}}$$

$$\therefore T' = T/_{\sqrt{3}}$$

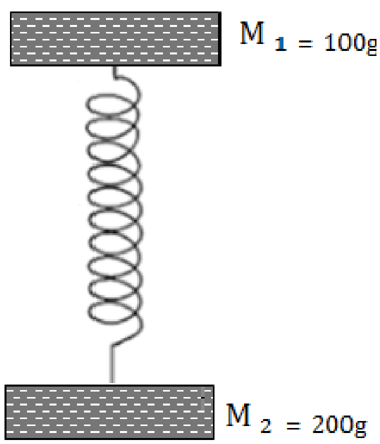
(c) (i) From $T = 2\pi\sqrt{l/_g}$, There will be no effect on time period because it is independent of amplitude (A) of oscillation provided it is small

(ii) Total energy, $E = 1/_2 m\omega^2 A^2$. If amplitude (A) is double, E will become four times

(iii) Maximum velocity $V_{max} = \omega A$. If amplitude (A) is double, V max will become 3 times

20. (a) Show that velocity and displacement of a body executing S.H.M are out of phase by $\pi/_2$ radians.

(b) Consider two discs as shown in figure with a mass less spring of force constant 100N/M



Calculate the frequency of oscillations of the spring

(i) When the system is resting on a table

(ii) When the table is removed and system is falling freely

(c) By using the concept of calculus show that the velocity at any instant in S.H.M is given by $V = \pm\omega\sqrt{r^2 - y^2}$

Hence, show that the displacement for S.H.M is given by y= r Sin wt. where y is the distance from the center of oscillations r is the amplitude of the motion $i.e$ the maximum distance from the center of oscillation ω is the Angular speed and t is the time.

(d) A swinging simple pendulum is placed in a lift which is accelerating

(i) Upwards

(ii) Down wards

How is its time period affected?

Solution

(a) In S.H.M, displacement equation is y = A sin wt …………………. (i)

$$velocity, v = {dy}/{dt} = {d(Asin\ \omega t)}/{dt}$$

$$v = A\omega\cos\omega t$$

$$v = A\omega\sin\left(\omega t + {\pi}/{2}\right) - - - - - - - - - - - (ii)$$

From equation (i) and (ii), it is clear that velocity (V) and displacement (y) are both of phases by ${\pi}/{2}$ radians.

(b) (i) When the system is resting on the table, only the upper disc will vibrate.

Therefore, the frequency of the oscillation of the system is

$$f_1 = {1}/{2\pi}\sqrt{{k}/{m_1}} = {1}/{2\pi}\sqrt{{100}/{0.1}}$$

$$\therefore f_1 = 5/s$$

(ii) When the table is removed and the system is falling freely, both the masses vibrate. Therefore, the frequency of oscillation in this case is

$$f_2 = {1}/{2\pi}\sqrt{{k}/{m}}$$

Reduced mass $m = {m_1 m_2}/{m_1 + m_2} = {100 \times 200}/{100 + 200} = 66.67$

$$f_2 = {1}/{2\pi}\sqrt{{100}/{66.67 \times 10^{-3}}}$$

$$\therefore f_2 = 6.16/s$$

(c) From

$$a = {dv}/{dt}$$

By product rule

$${dv}/{dt} = {dy}/{dt} . {dv}/{dy}$$

But

$${dy}/{dt} = v \qquad and \qquad {dv}/{dt} = a$$

Therefore

$$a = v.{dv}/{dt} ------------------(i)$$

Simple harmonics motion is defined by the equation

$$a = -\omega^2 y$$

$$v\,{dv}/{dy} = -\omega^2 y$$

This is a separable differential equation

$$vdv = -\omega^2 ydy$$

$$\int vdv = \int -\omega^2 ydy$$

$${v^2}/{2} = -{\omega^2 y^2}/{2} + c -----------(ii)$$

Since velocity is equal to zero when the displacement is maximum and velocity to the maximum when the displacement is equal to S.H.M

$$v = 0, when \;\; y = r \qquad and \qquad v = r\omega, when \;\; y = 0$$

By taking the first condition

$$v = 0, \quad when \quad y = r$$

$$0 = -\omega^2 r^2/_2 + C$$

$$\therefore v = \omega^2 r^2/_2$$

By substitute the value of C in equation (ii)

$$v^2/_2 = -\omega^2 y^2/_2 + C$$

$$v^2/_2 = \omega^2 y^2/_2 + \omega^2 r^2/_2$$

$$v^2 = -\omega^2 y^2 + \omega^2 r^2$$

$$v^2 = \omega^2 (r^2 - y^2)$$

$$v = \pm\omega\sqrt{r^2 - y^2}$$

Taking the positive values of v

$$v = \omega\sqrt{r^2 - y^2}$$

$$dy/_{dt} = \omega\sqrt{r^2 - y^2}$$

$$\int dy/_{\sqrt{r^2 - y^2}} = \int \omega dt$$

Let $y = r\sin\theta$

$$dy = r\cos\theta d\theta$$

$$\int r\cos\theta d\theta \Big/ \sqrt{r^2 - r^2 sin^2\theta} = `\omega \int dt$$

$$\int r\cos\theta d\theta \Big/ r\cos\theta = \omega t + C$$

$$\int d\theta = \omega t + C$$

$$\theta = \omega t + C$$

$$sin^{-1}[y/r] = \omega t + C - - - - - - - - - - - - - (iv)$$

$$y/r = \sin \omega t$$

$$\therefore y = r \sin \omega t$$

(d) (i) The time period will decrease because effective acceleration due to gravity increase from g to $(g + a)$

(ii) The time period of a simple pendulum moving downwards with acceleration

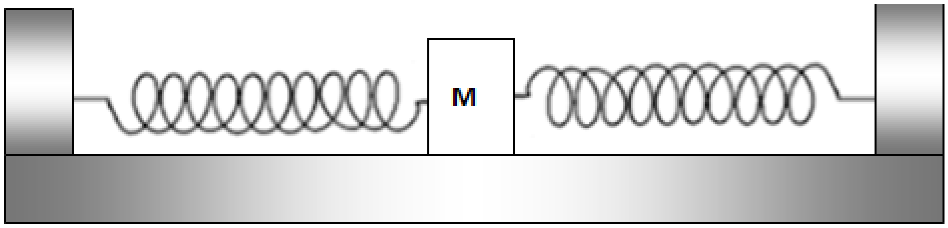
21. (a) A particle has a time period of 1s under the action of a certain force and 2s under the action of another force. Find the time period when the force are acting in the same direction simultaneously.

(b) A particle executes S.H.M with a period 8s. Find the time in which half the total energy is potential

(c) If the length of a second's pendulum is decreased by 2%. Find the gain or loss in time per day

(d) A swinging simple pendulum is placed in a lift which is falling freely what is the frequency of the pendulum

(e) Consider the two spring mass system shown in figure



The horizontal surface is frictionless. Show that the frequency V of horizontal oscillation of the mass m is given by

$$V = \sqrt{V_1^2 + V_2^2}$$

Where V_1 and V_2 are the frequencies at which the block would oscillate if connected only to spring 1 and only to spring 2 …………………

Solution

(a) Let the period of oscillation be T1 under the action of the force F1 and T2 under the action of force F2 when acting separately let F be the resultant force and the period of oscillation under the action of the resultant force

$$F = F_1 + F_2$$

$$m\omega^2 y = m\omega_1^2 y + m\omega_2^2 y$$

$$\left({2\pi}/{T}\right)^2 = \left({2\pi}/{T_1}\right)^2 + \left({2\pi}/{T_2}\right)^2$$

$$1/T^2 = 1/T_1^2 + 1/T_2^2$$

$$T^2 = T_1^2 T_2^2 / T_1^2 + T_2^2$$

$$T = T_1 T_2 / \sqrt{T_1^2 + T_2^2}$$

$$T_1 = 1s$$

$$T_2 = 2s$$

$$T = {}^{1 \times 2}/_{\sqrt{1^2 + 2^2}}$$

$$\therefore T = 0.89s$$

(b) T = 8s

Total energy E1 = ½ mw2A2

Potential energy P.E = 1/2 mw2y2

Where y is the displacement of the particle from the equilibrium position

Given that

$$^1/_2 \times total\ energy = potential\ energy$$

$$^1/_2 \left[^1/_2 m\omega^2 A^2\right] = {}^1/_2 m\omega^2 y^2$$

$$^{m\omega^2 A^2}/_4 = {}^{m\omega^2 y^2}/_2$$

$$^{A^2}/_2 = y^2$$

$$y = {}^A/_{\sqrt{2}}$$

$$\omega = {}^{2\pi}/_T = {}^{2\pi}/_8$$

$$\omega = {}^{\pi}/_4$$

$$y = Asin\ \omega t$$

$$^A/_{\sqrt{2}} = Asin\ {}^{\pi t}/_4$$

$$^1/_{\sqrt{2}} = \sin {}^{\pi t}/_4$$

$$\pi t/_4 = sin^{-1}\ 1/_{\sqrt{2}}$$

$$\pi t/_4 = \pi/_4$$

$$\therefore t = 1s$$

(c) $T = 2\pi\sqrt{l/_g}$

For a second's pendulum

$T = 2s$

$2 = 2\pi\sqrt{l/_g}$ ------------ (i)

The new length when it is decreased by 2% is $98/_{100}\, l$

New time period $t = 2\pi\sqrt{0.98l/_g}$ ------------------------------ (ii)

Take equation $(ii) \div equation\ (i)$

$$t/_2 = \sqrt{0.98l/_g} \times \sqrt{g/_l}$$

$$t/_2 = \sqrt{0.98l/_l}$$

$$t/_2 = \sqrt{0.98}$$

$$t = 2 \times 0.989$$

$$t = 1.98s$$

(d) The time period of a simple pendulum falling with acceleration a is

$$T = 2\pi\sqrt{{}^{l}/_{g - a}}$$

For a freely falling lift a = g so that T = ∞ hence f = 0

(e) From

$$V_1 = {}^{1}/_{2\pi}\sqrt{{}^{K_1}/_{M}}$$

$$K_1 = 4\pi^2 m V_1^2 ------------ (i)$$

Also

$$V_2 = {}^{1}/_{2\pi}\sqrt{{}^{K_2}/_{M}}$$

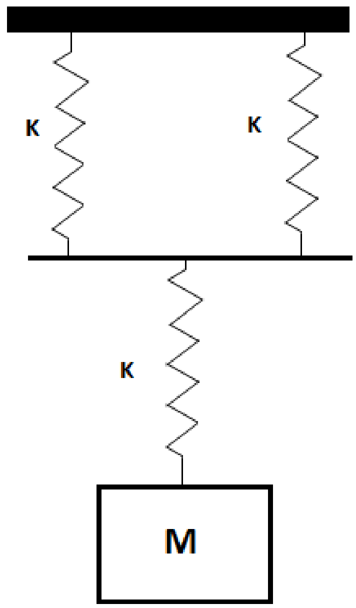
$$K_2 = 4\pi^2 m V_2^2 --------------- (ii)$$

For

$$V = {}^{1}/_{2\pi}\sqrt{{}^{K_{eff}}/_{M}}$$

$$K_{eff} = 4\pi^2 m V^2 -------------- (iii)$$

22.(a) What will be the force constant of the spring system shown in figure?



(b) The acceleration due to the gravity on the surface of moon is 1.7m/s2. What is the time period of a simple pendulum on the moon if its time period on the earth is 3.5s (g on earth = 9.8m/s2)

(c) Two simple pendulums of length o.4m and 0.6m respectively are set oscillating in step.

(i) After what further time will the two pendulums be in step again

(ii) find the number of oscillation made by each pendulum during the time in (i) above

Solution

(a) The two parallel springs each of spring constant K1 given an equivalent spring constant of K_p

$K_p = K_1 + K_1$

$\therefore K_p = 2K_1$

K_p and K_2 are in series

$K_S = {}^{1}/_{K_p} + {}^{1}/_{K_2}$

${}^{1}/_{K} = {}^{1}/_{2K_1} + {}^{1}/_{K_2}$

$\therefore K = {}^{2K_1 + K_2}/_{2K_1 + K_2}$

(b) From

$$T = 2\pi\sqrt{L/g}$$

$$g_m = 1.7\ m/s^2$$

$$T_e = 3.5s$$

$$g_e = 9.8m/s^2$$

$$T_e = 2\pi\sqrt{l/g_e}$$

$$T_m = 2\pi\sqrt{l/g_m}$$

$$T_m/T_e = \sqrt{l/g_m}\cdot\sqrt{g_e/l}$$

$$T_m/T_e = \sqrt{g_e/g_m}$$

$$T_m/T_e = \sqrt{9.8/1.7}$$

$$T_m/T_e = 2.4$$

$$T_m = 2.4 \times T_e$$

$$T_m = 2.4 \times 3.5s$$

$$\therefore T_m = 8.4s$$

(c) Period of longer pendulum T_L

$$T_L = 2\pi\sqrt{L/g}$$

$T_L = 2\pi\sqrt{0.6/_{9.8}}$

$\therefore T_L = 1.547$

Period of shorter pendulum Ts

$T_S = 2\pi\sqrt{L_S/_g}$

$T_S = 2\pi\sqrt{0.4/_{9.8}}$

$T_S = 1.2694$

But every time Ts, the slower pendulum will lag behind the past pendulum by (TL – Ts) sec

The two pendulums will be in phase again when the longer pendulum lags by time equal to its period $i.e$ after n oscillation of the shorter pendulum such that

$n(T_L - T_S) = T_L$

$n = T_L/_{T_L - T_S}$

$n = 1.547/_{1.547 - 1.2696}$

$n = 5.57$

The pendulum will be in step after t seconds such that

$t = n \times T_s$

$= 5.57 \times 1.2696$

$T = 5.57$

(ii) The short pendulum will make 5.5t oscillations, the lag pendulum will make

$= {}^{7.0705}/_{TL}$

$= {}^{7.0705}/_{1.547}$

$= 4.57026 \; oscillations$

GRAVITATION MOTION

GRAVITATIONAL

Is one of the four basic interactions in nature and is the weakest among them.

Universe

Are all the matter, energy and space that exist.

Astronomy

Is the branch of science which deals with the study of the universe.

It is an observational science rather than an experimental science.

Astrophysics

Is the branch of astronomy which deals with the physical bodies and the intervening regions of space.

Cosmology

Is the study of the origin, evolution and nature of the universe

The constituents of the universe are stars galaxy, planets, comets, asteroids, meteoroids meteor etc.

The force which keeps them bound together is called gravitational force

Gravitational force

Is the force or attraction acting between any two bodies of the universe

It is negligible when the masses are small

However, gravitational force is the most important of all the force for large masses

It is the gravitational force that binds us to the earth and the other planets revolve around the sun.

The force of gravitation between two bodies cannot be zero unless r $\infty \longrightarrow$

Earth's Gravitational pull

Is the force of attraction between everybody and earth

Clearly, gravity is a special case of gravitation where one of the bodies is earth and the other body lying on earth or near the surface of earth

- Force of gravity at the center of earth.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

States that "everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of distance between their centers".

The force acts along the line joining the centers of the two bodies

Consider two bodies of masses m1 and m2 that have distance between their centers as shown in figure below.

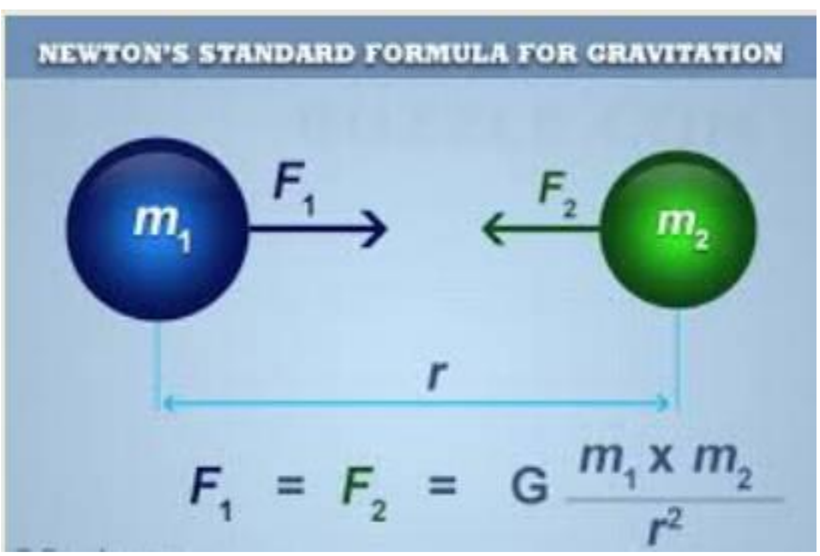


Figure. 1

According to Newton's law of gravitation, the magnitude off attractive force (F) between the two bodies is

$$F \propto M_1M_2 \ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

$$F \propto {}^{1}/_{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Combining equation $(i) and\ (ii)$

$$F \propto {}^{M_1M_2}/_{r^2}$$

$$F = G\,{}^{M_1M_2}/_{r^2}$$

--- --- ---- ---- (iii)

Where G is a constant of proportionality and is called universal gravitational constant

From equation (iii)

$$F = G\,{}^{M_1M_2}/_{r^2}$$

When

$M_1 = 1$, $M_2 = 1$ and $r = 1$. F = G

Gravitational constant (G)

Is the numerically equal to the force of attraction between unit masses separated by unit distance.

In SI unit, the value of G is 6.67×10^{-11} $Nm^2\ kg^{-2}$ This value of G does not depend upon the nature and size of the mass.

It also does not depend upon the nature of the medium between two bodies.

Indeed, G is truly universal both observation and theory suggest that it has the same value through the universe

The gravitational force of attrition force by the presence of others is not altered by the presence of other bodies, thus one mass will experience body independently of whether.

The dimensional formula of G is $M^{-1}L^{3}T^{-2}$

$$F = G\frac{M_1M_2}{r^2}$$

$$G = \frac{Fr^2}{M_1M_2} = \frac{MLT^{-2}L^2}{M_2}$$

$$G = M^{-1}\ L^3\ T^{-2}$$

This law fails if the distance between the objects is less than 10^{-9}m i.e of the order of intermolecular distance

SOLVED EXAMPLES

1. The mass of planet Jupiter is 1.9×10^{27} kg and that of sun is 1.99×10^{30} kg. The mean distance of the sun from gravitational force which sun exerts on Jupiter is 7.8×10^{11}m. Calculate the gravitational force which sun exerts on Jupiter Take G= 6.67×10^{-11}.

Solution

Mass of Jupiter $m_1 = 1.9 \times 10^{27}$ kg

Mass of sun $m_2 = 1.99 \times 10^{30}$ kg

Mean distance between them $r = 7.8 \times 10^{11}$m

The gravitational force exerted by the sun on Jupiter

$$\left[F = G\,{}^{M_1M_2}/_{r^2} \right]$$

$$F = (6.67 \times 10^{-11}) \times \left[{}^{(1.9 \times 10^{27})(1.99 \times 10^{30})}/_{(7.8 \times 10^{11})^2} \right]$$

$$\therefore F = 4.15 \times 10^{23} N$$

2. Assuming the orbit of the earth about the sun to be circulate (it is actually slightly elliptical () with radius 1.5 x 1011 m. Find the mass of the sun. The earth revolves around the sun in 3.15 x 10^{7} seconds

Solution

$$F_c = F_G$$

$$M_c V^2/_{r^2} = G\,{}^{M_e M_s}/_{r^2}$$

v = speed of earth in its orbit around the sun

$$V = \omega r$$

$$v = r.\,{}^{2\pi}/_{T}$$

$$V = {}^{2\pi r}/_{T} = {}^{2\pi \times 1.5 \times 10^{11}}/_{3.15 \times 10^{7}}$$

v = 3 x 10^4 m/s

From

$$M_e V^2/_{r} = G\,{}^{M_e M_s}/_{r^2}$$

$$M_s = {}^{V^2 r}/_{G}$$

$$Ms = {}^{(3 \times 10^4)^2 \times 1.5 \times 10^{11}}/_{6.67 \times 10^{-11}}$$

Ms = 2 x 10^{30}kg

3. A mass M is broken into two parts m and (m – m). How are m and m related so that force of gravitation between the two parts is maximum?

Solution

$$F = {}^{Gm(M-m)}/_{r^2}$$

Let, $${}^{G}/_{r^2} = K$$

$$F = km(M-m)$$

$$F = k[mM - m^2]$$

$$F = k\left[{}^{M^2}/_4 - {}^{m^2}/_4 + mM - m^2\right]$$

$$F = k\left[{}^{M^2}/_4 - \left({}^{M}/_2 - m\right)^2\right]$$

For F to be maximum

$$\left({}^{M}/_2 - m\right)^2 = 0$$

$${}^{M}/_2 - m = 0$$

$$m = {}^{M}/_2$$

4. Two bodies of masses 1 kg and 6 x 10^{24}kg are placed with their centers 6.38 x 10^{6}m apart calculate the force of attraction between the two masses. Also find the initial acceleration of the two masses. (Assume that no other forces act on them.)

Solution

M_1 = 1kg

$M_2 = 6 \times 10^{24}$ kg

$$F = \frac{GM_1M_2}{r^2}$$

$$r = 6.38 \times 10^6 m$$

$$G = 6.67 \times 10^{-11} Nm^2kg^{-2}$$

$$F = \frac{(6.67 \times 10^{-11}) \times (1)(6 \times 10^{24})}{(6.38 \times 10^6)^2}$$

F = 9.83N

Initial acceleration of a_1

$$F = M_1a_1$$

$$a_1 = \frac{F}{M_1} = \frac{9.83}{1}$$

a_1= 9.83m/s^2

Initial acceleration of M_2

$$F = M_2a_2$$

$$a_2 = \frac{F}{M_2} = \frac{9.83}{6 \times 10^{24}}$$

a_2= 1.64 x 10^{-24}m/s

5. Two metal sphere of the same material and of equal radius R are touching each other. Show that force of attraction between them is proportional to R^4.

Solution

$$F = \frac{GM_1M_2}{r^2}$$

If d is the density of the materials of the sphere

But r = 2R

$$M_1 = {}^4/_3\,\pi R^3 d$$

$$M_2 = {}^4/_3\,\pi R^3 d$$

$$F = {(^4/_3\,\pi R^3 d)(^4/_3\,\pi R^3 d)}\Big/_{(2R)^2}$$

Then

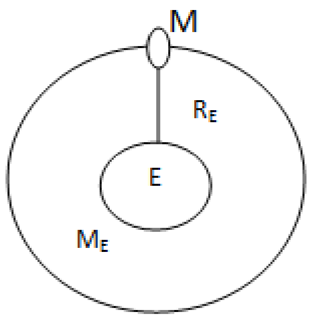
$$F = {}^4/_9\,G\pi^2 d^2 R^4$$

$\therefore F \propto R^4$

EXPRESSION FOR ACCELERATION DUE TO GRAVITY

Consider earth to be a sphere of mass M_E and radius R_E.

Suppose a body of mass M is placed on the surface of the earth as shown in figure below:-



- It is legitimate to consider that the whole mass M_E of the earth is concentrated at its center.
- It is reasonable to assume that the distance between M and M_E is equal to radius RE of the earth

According to the law of gravitation the force of attraction acting on the body due to earth is given by $F = GMM_E/R_E^2$

This attractive force which the earth exerts on the object is simply the weight of the object

F = Mg........ (ii)

By equating equation (i) and (ii)

$$Mg_e = G\,{}^{MM_E}/_{R_E^2}$$

$$g_e = GM_E/R_E^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(iii)}$$

This gives the expression for acceleration due to gravity on the surface of the earth.

i) The equation (iii) shows that the value g does not depend on the mass m of the body.

Thus if two bodies of different masses are allowed to fall freely they will have the same acceleration

If they are allowed to fall from the same height they will reach the earth simultaneous.

ii) Both G and M_{E} are constant and R_E (distance between the centers of body and earth) do not changes appreciably for small variation in height near the surface of the earth.

Therefore the acceleration due to gravity of an object near the surface of earth is approximately constant and does not depend on the mass of the object.

iii) We can easily extend the equation to find the gravitation acceleration at the surface of any planet of mass M_p and Radius R_p.

$$g_p = {}^{GM_p}/_{R_p^2}$$

MASS AND DENSITY OF EARTH

Using the law of universal gravitation and the measured value of the acceleration due to gravity. We can find the mass and density of earth

(i) MASS OF EARTH

The acceleration due to gravity on the surface of the earth is given by

$$g = {}^{GM_E}/_{R_E^2}$$

Mass of earth M_E

$$M_E = {gR_E^2}/{G}$$

Now

G = 9.8m/s^2

RE = 6.37 x 106m

G = 6.67 x 10-11NM^2 kg^{-2}

$$M_E = {(9.8) \times (6.37 \times 10^6)^2}/{6.67 \times 10^{-11}}$$

$\therefore$ M_E = 6 x 10^{24}kg

II) Density of Earth let ρ be the average density of the earth. Earth let ρ be the average density of the earth is a sphere of radius R_E

Mass of Earth = volume x density

$$M_E = {4}/{3}\pi R_E^3 \times \rho$$

$$M_E = {4}/{3}\pi R_E^3 \rho \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (ii)$$

But,

$$M_E = {gR_E^2}/{G}$$

Average density of earth ρ

$$\rho = {3g}/{4\pi R_E G}$$

Now

g = 9.8m/s^2

Re = 6.37 x 106m

G = 6.67 x $10^{-11}Nm^2$ kg^{-2}

Average density of earth

$$\rho = {3 \times 9.8}/{4\pi \times (6.37 \times 10^6) \times (6.67 \times 10^{-11})}$$

ρ = 5.5 x $10^3 kgm^{-3}$

This is the average density the entire earth and is 5.5 times the density of water.

(ii) **MASS OF SUN**

The force of attraction between the earth of mass M_E and the sun of mass separated by a distance R is

$$F = {GM_EM_s}/{R^2}$$

The force of attraction between the two provides the necessary centripetal/force.

$$\frac{GM_EM_S}{R^2} = M_E\omega R^2$$

$$M_s = {\omega^2 R^3}/{G}$$

Mass of the sum Ms

$$M_s = {R^3}/{G} \times [{2\pi}/{T}]^2$$

$$M_s = {4\pi^2 R^3}/{GT^2}$$

T = 365 days

R = 1.5 x 10^{11}m

G = 6.67 x 10^{-11} NM^2 kg^{-2}

Recall

T = 365 days

T = 365 x 24 x 3600

T = 3.15 x $10^7{}_5$

$$M_s = {4\pi^2 \times 1.5 \times 10^{11}}/{(6.67 \times 10^{-11})(3.15 \times 10^7)}$$

SOLVED EXAMPLES

1. What will be the acceleration due to gravity on the surface of the moon if its radius is ¼th the radius of the earth and its mass 1/80th of the mass of the earth.

The acceleration due to gravity on the surface of earth is given by

$g_E = GM_E/R_E^2$...............(i)

If M_m is the mass of the moon and R_m its radius then acceleration due to gravity on the surface of the moon is given by.

$$g_m = \frac{GMm}{R_m^2} \dots\dots\dots\dots\dots\dots\dots (ii)$$

Take equation (ii) $\div$ eqn (i)

$$\frac{g_m}{g_E} = GM_m/R_m^2 \times R_E^2/GM_E$$

$$g_m/g_E = M_m/M_E \times [R_E/R_m]^2$$

Recall; $M_m/M_E = \frac{1}{80}$ and $\frac{R_E}{R_m} = 4$

$$\frac{g_m}{g_E} = \frac{1}{80} \times 4^2$$

$$\frac{g_m}{g_E} = \frac{1}{5}$$

$$g_m = \frac{g_E}{5}$$

2. A man can jump1.5m on the earth, calculate the approximate height he might be able to jump on a plane whose density is one quarter that of the earth and whose radius is one third that of the earth.

Solution

Suppose a man of mass m leaps a height on the earth and height h_1 on the planet. Assumes he can give himself the two planets, the potential energy is the same at the maximum height

$$Mh_1g_1 = Mgh$$

Where g_1 and g are the respective acceleration due to gravity on the planet and earth.

$$h_1 = \frac{g}{g_1} \times h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

$$g = GM_E/R_E^2$$

Recall

$$g = G \times \frac{4}{3}\pi R_E^3 \rho_E / R_E^2$$

$$g_1 = G \times \pi \frac{4}{3} R_E \rho_E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Also $g_1 = G \times \frac{4}{3}\pi R_p \rho_p$..$(iii)$

Take equation $(ii) \div (iii)$

$$\frac{g}{g_1} = G \times \frac{4}{3}\pi R_E \rho_E \times {}^{3}/_{G \times 4\pi R_p \rho_p}$$

$$\frac{g}{g_1} = \left(\frac{R_E}{R_p}\right) \times \left(\frac{\rho_E}{\rho_p}\right)$$

$$\frac{g}{g_1} = 3 \times 4$$

$$\frac{g}{g_1} = 12$$

From equation (i)

$$h_1 = \frac{g}{g_1} \times h$$

$$h_1 = 12 \times 1.5$$

$$\therefore h_1 = 18$$

3. What is the acceleration due to gravity on the surface or a planet that has a radius half that of earth and the same average density as earth?

Solution

The acceleration due to gravity on the surface of earth is

$$g_E = {}^{GM_E}/_{R_E^2} \ldots\ldots\ldots\ldots\ldots\ldots.(i)$$

The acceleration due to gravity on the surface of the planet is

$$g_g = {}^{GM_p}/_{R_p^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

Take equation ii ÷equation i

$$\frac{g_p}{g_E} = {GM_p}/{R_p^2} \times {R_E^2}/{GM_E}$$

$$\frac{g_p}{g_E} = {M_p}/{M_E} \times {R_E^2}/{R_p^2}$$

Since both bodies have the same density, their masses are directly proportional to their volumes ie proportional to the cube of their radii

$$g_{p/g_E} = R_p^3/R_E^3 \times R_E^2/R_P^2$$

4. If the radius of earth shrinks by 15% (mass remaining the same) g then how would the value of accelerated due to gravity change?

$$g = {GM_e}/{R_E^2}$$

$$\ln g = \ln G + \ln M_E - 2\ln R_E$$

Differentiating, we get

$$\frac{\delta g}{g} = -2\frac{\delta R_E}{R_E}$$

$$\frac{\delta g}{g} \times 100 = -2\left(\frac{\delta R_E}{R_E} \times 100\right)$$

$$\frac{\delta g}{g} \times 100 = -2 \times (-1.5)$$

∴ **Acceleration would change it value by 3%**

5. If earth were made of lead of relative density 11.3, what would be the value of acceleration due to gravity on the surface of earth? Radius of earth = 6.4×10^6m

$G = 6.67 \times 10^{-11} Nm^2 \ kg^{-2}$

$$Relative\ density\ of\ earth = {}^{Density\ of\ earth}/_{Density\ of\ water}$$

Density of Earth ρ=relative density of earth x density of water

ρ= 11.3 x 103kg/m^3

p = 11.3 x 1000

Mass of earth

$$M_E = {}^4/_3 \pi R_E^3 \rho$$

M_E =1.24 X 10^{25}kg

$$g = {}^{GM_E}/_{R_E^2}$$

Acceleration due to gravity

$$g = \frac{(6.67\ x\ 10^{-11})\ x\ (1.24\ x\ 10^{25})}{(6.4\ x\ 10^6)^2}$$

$$\therefore g = 20.21 \text{m}/s^2$$

GRAVITATIONAL MASS

It is the mass of a body which determine the gravitational force of attraction due to earth

By Newton's law of gravitation the force acting on a body lying on the surface of the earth.

$$F = {}^{GM_E M_g}/_{R_E^2}$$

$$mg = {FR_E^2}/{GM_E}$$

Where;

M^g = gravitational mass

M = mass of the earth

It can be show that the inertial mass is equal to the gravitational mass

Inertial mass is measured by the acceleration produced by the applied force. The gravitational mass is measured by the gravitational pull on i.

The weight of a body flying on the surface of the earth is

$$F = M_i \times g$$

$$F = {GM_E M_g}/{R_E^2}$$

But,

$$M_i g = {GM_E M_g}/{R_E^2}$$

Then

$$M_i/M_g = {GM_E}/{gR_E^2}$$

M_i = a constant

$M_i \propto M^g$

Thus inertial mass of a body is proportional to its gravitational mass if we take GM_E/GR_E^2 = 1 then the inertial mass and gravitational mass are equal.

Since the two masses are equal the mass appearing in the force equation

F = Ma

And the mass appearing in the definition of weight W are the same

W$= mg$

It is difficult to accelerate or decelerate a heavy football player compared to a light player. This is because his inertial mass is greater

- It is harder to carry off the heavy player out of the court when is hurt. This is due to his greater gravitational mass.

VARIATIONS IN THE VALUE OF g

The value of acceleration due to gravity (g) varies as we go above or below the surface of the earth.

It is also varies from place to place on the surface of the earth we shall discuss the following cases

i) Variation of g with Altitude

ii) Variation of g with Depth

iii) Variation of g due to shape of earth

iv) Variation of g due to rotation of earth

i) **VARIATION OF g WITH ALTITUDE**

Consider earth to be a sphere of radical R and mass M. The acceleration due to gravity on the surface of earth

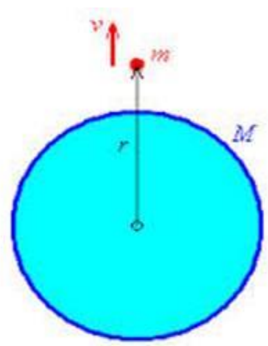


Fig 2

At point Q

$$g = {GM}/{R^2}$$

Consider a point p at a height h above the surface of the earth the acceleration due to gravity at point P is $g' = {GM}/{(R+h)^2} \dots\dots\dots\dots ii$

Dividing equation ii by equation i we have

$$g'/g = {GM}/{(R+h)^2} \times {R^2}/{GM}$$

$$g' = g\frac{R^2}{(R+h)^2}$$

This gives the expression for variation of acceleration due to gravity with height h.

It is clear that $g' < g$. Thus as we go above the surface of earth acceleration due to gravity goes on decreasing.

At a height equal to radius of earth

h = R

From,

$$g' = \frac{gR^2}{(R+h)^2}$$

$$g' = \frac{gp^2}{(2R)^2}$$

$$g' = \frac{gR^2}{4R^2}$$

$$g' = \frac{g}{4}$$

Expression for g' when h<<R

When h is very small as compared to R

$$\frac{g'}{g} = R^2/(h+R)^2$$

$$\frac{g'}{g} = R^2 \Big/ R^2\left(1+\frac{h}{R}\right)^2$$

$$\frac{g'}{g} = \left[1+\frac{h}{R}\right]^{-2}$$

Expanding using Binomial theorem and neglecting higher powers of h/R

$$g'/g = \left[1 - \frac{2h}{R}\right]$$

This gives the expression for g' when h < < R

PERCENTAGE DECREASE IN g (Loss in weight)

We can find the percentage decrease in the value of g with height as follows

$$g' = g\left[1 - {2h}/{R}\right]$$

$$g' = g - {2gh}/{R}$$

$$g - g' = {2gh}/{R}$$

$$g - g'/g = {2h}/{R}$$

% decrease in the value of g with height h

$$g - g'/g \times 100 = {2h}/{R} \times 100$$

ii) **VARIATION OF g WITH DEPTH**

Consider the earth to be a sphere of radius R and mass M. The acceleration due to gravity at point Q on the surface of the earth is

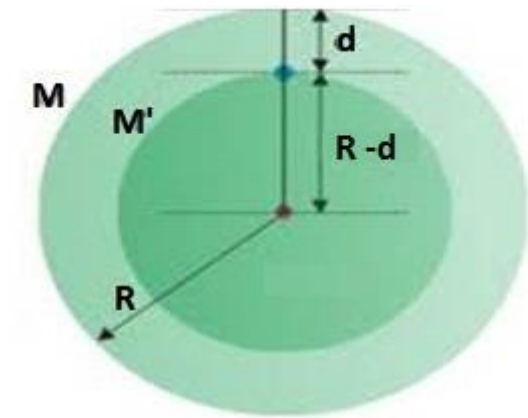


Fig 3

$$g = \frac{GM}{R^2}$$

If P is the density of the earth then,

Mass of earth M

$$M = {}^4/_3 \pi R^3 \rho$$

$$g = G \times {}^4/_3 \pi R^3 \rho / R^2$$

$$g = {}^4/_3 \pi R G \rho$$

Consider a point p which is inside the earth and at a depth below the surface of the earth.

- Its distance from the center O is (R – d)
- Let a sphere be drawn with O as center and (R – d) as radius. The acceleration due to gravity g' point p is only due to the sphere radius (R – d).

$$g' = GM' / (R-d)^2$$

Where;

M' = mass of inner solid sphere (shaded portion)

$$M' = {}^{4}/_{3}\,\pi(R-d)^3\rho$$

$$g' = G \times {}^{4}/_{3}\,\pi\,{}^{(R-d)^3}/_{(R-d)^2}\,\rho$$

$$g' = {}^{4}/_{3}\,\pi(R-d)G\rho \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

Dividing equation ii by equation i

$$g'/_{g} = {}^{4}/_{3}\,\pi(R-d)G\rho \times {}^{3}/_{4\pi RG\rho}$$

This gives the expression for variation of g with depth d

It is clear that g' < g. Therefore as we go below the surface of the earth, the acceleration due to gravity goes on decreasing

At the center of earth, it becomes zero

From ,

$$g' = g\left(1 - {}^{d}/_{R}\right)$$

$$g' = g\left(1 - {}^{R}/_{R}\right)$$

$$g' = g(1-1)$$

g = 0

Therefore, acceleration due to gravity is zero at the center of earth.

Thus the value of acceleration due to gravity is maximum at the surface of the earth and becomes zero at the center of the earth.

- That is why the weight of a body at the center of the earth is zero through its mass is constant

PERCENTAGE DECREASE IN g

We can calculate the % decrease in the value of g with depth inside the earth as follows

$$g' = g\left[1 - {}^{d}/_{R}\right]$$

$$g' = g - {}^{gd}/_{R}$$

$$g - g' = {}^{dg}/_{R}$$

$$g - g'/_{g} = {}^{d}/_{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ fraction\ decrease\ in\ g$$

% decrease in value of g with depth d inside the earth is

$$g - g'/_{g} \times 100 = {}^{d}/_{R} \times 100$$

VARIATION OF "g" DUE TO SHAPE OF EARTH

The earth is not a perfect sphere it flattens at the poles and bulges out at the equator as shown in figure

The value of g is given by $g = {}^{GM}/_{R^2}$

Since G and M are constant g $\propto$ $^1/R^2$ where R is radius of earth

- This the value of g at a place on the surface of the earth varies inversely as the square of the radius of earth at that place.

Where radius of the earth is increase, the value of g is decrease and vice versa.

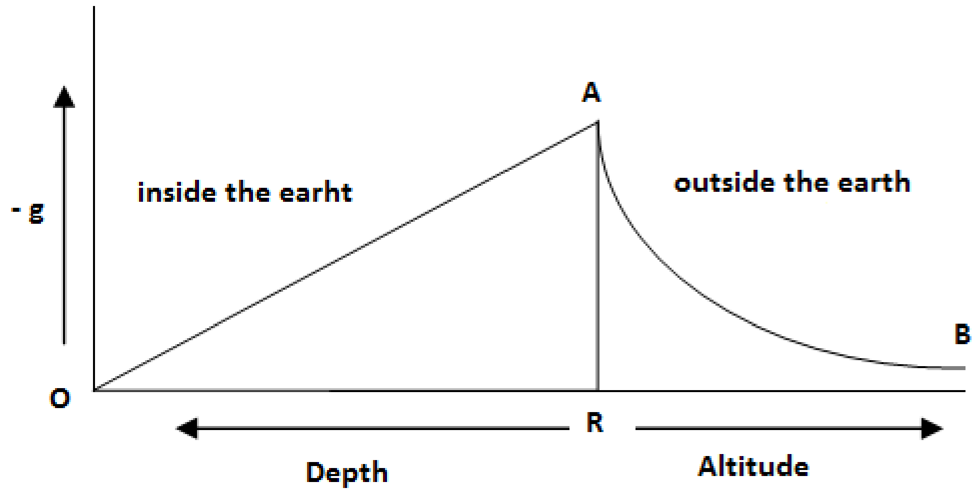
i) The radius of earth is least at the poles R_p and greatest at the equator R_e

Therefore the value of g is maximum at the poles and least at the equator

ii) As we go from the equator toward the poles, the radius of earth goes on decreasing and hence the value of g goes on increasing

iii) The value of g at the equator is $9.78 m/s^2$ where as at the poles it is $9.83 m/s^2$.

The variation of g with height and depth is shown graphically in figure 4



Variation of g due to the earth and the depth from the earht's surface

VARIATION OF g DUE TO ROTATION OF EARTH

The earth rotates about the axis passing through its north and south poles.

- The value of g varies from place to place on the surface of earth due to rotation of the earth.
- Latitude at a place on the surface of earth is the angle which the earth makes with the equatorial plane,demoted by λ
- It is clear that $\lambda = 90^0$ at the poles and $\lambda = 0^0$ at the equation

The equation below gives the expression for variation of acceleration due to rotation of earth

$$g = g - \omega R^2 \cos^2 \lambda$$

It is clear that acceleration due to gravity increases with the increases in the latitude of the place

This means that value of g increase as we go from equator to the poles

AT EQUATOR

At equator λ = 0° so that;

From, $g' = g - \omega R^2 cos^2\lambda$

$$g_e = g - \omega R^2 cos^2 0$$

$$g_e = g - \omega R^2$$

Therefore value of acceleration due to gravity is minimum at the equator.

- This is expected because the particular at the equator executes a circle of maximum radius.
- Therefore, the centrifugal force is maximum.

AT POLES

At poles λ = 90° so that

From,

$$g' = g - \omega R^2 cos^2\lambda$$

$$g' = g - \omega R^2 cos^2 90$$

$$g_p = g$$

Hence the value of g is maximum at the poles.

This is expected because the particle at the pole moves in circle of zero radius.

Therefore no centrifugal force acts on the particle.

As the results the value of g_p remains the same whether the earth is at rest rotating.

i) $g_p = g \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (i)$

$g_e = g - \omega R^2 \dots\dots\dots\dots\dots\dots\dots (ii)$

Take equation $(i) - (ii)$

$g_p - g_e = g - (g - \omega R^2)$

$g_p - g_e = g - g + \omega R^2$

$g_p - g_e = g - g + \omega R^2$

$R = 6.4 \times 10^6 m$

$\omega = 2\pi\ rads^{-1}$

ii) When a body of mass M moved from equator to poles the increase in weight.

$M(g_p - g_e)$ = MRw^2

SOLVED EXAMPLES

1. a) There is no expect of the rotational motion of the on the value of g at the poles why?

b) By how much does the gravitational force between two objects decrease when distance between them is doubled?

c) Gravitational force between two bodies is 1N. If the distance between them is made twice, what will be the value of force.

d) Find the percentage decrease in weight of a body when taken 16km below the surface of the earth. What happens to the weight of the body center of the earth's/ radius of earth = 6400km

Solution

a) At poles λ = 90^o

so that $g' = g - \omega R^2 cos^2\lambda$

$$g_p = g - \omega R^2 cos^2 90$$

$$g_p = g$$

Hence there is no effect of rotational motion of earth on the value of g at the poles.

b) According to Newton's law of gravitation

$$F_1 = G\,{}^{M_1M_2}\!/_{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

If the distance between the objects is double gravitational force between the objects decrease to one – fourth

Also,

$$F_2 = G\,{}^{M_1M_2}\!/_{(2r)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Then, $F_1 r^2 = 4r^2 F_2$

$$\therefore F_2 = {}^{F_1}\!/_4$$

c) **Solution**

The gravitational force between two bodies is

F $\propto$ 1

r²

Fr^2 = constant

$$F_1 r_1^2 = F_2 r_2^2$$

$$1 \times r_1^2 = F_2 \times (2r_1)^2$$

$$1 \times r_1^2 = F_2 \times 4r_1^2$$

$F_2 = 0.25$

d) Solution

d =16km

R = 6400km

The acceleration due to gravity at a depth d is

$$g' = g\left[1 - \frac{d}{R}\right]$$

$$mg' = mg\left[1 - \frac{d}{R}\right]$$

$$mg'/mg = \left[1 - d/R\right]$$

$$1 - mg'/mg = d/R$$

$$mg - mg'/mg = d/R$$

$$mg - mg'/mg = 16/6400 \times 100\%$$

$$\therefore mg - mg'/mg = 0.25\%$$

At the center of the earth,

$d = R$

Weight of the body $= mg' = mg\left(1 - {}^{d}/_{R}\right)$

$$mg' = mg[1 - {}^{R}/_{R}]$$

Weight of body **= 0**

2.a) Explain, why one can jump higher on the surface of the moon than on the earth?

b) What will be the effect on the time period of a simple pendulum on taking it to a mountain?

c) Assuming that the earth is a sphere of radius R at what altitude will be the value of acceleration due to gravity be half its value on the earth's surface.

d) Calculate that imaginary angular velocity of the earth for which the effective acceleration due to gravity at the equator becomes zero. In this condition what will be the length (in hours) of the day? RE = 6400 km

g = 10m/s^2

a) Solution

Let us be the speed of the man while taking a jump and h the height of the jump. Then

$v^2 - u^2 = 2gh$

$$0^2 - U_0^2 = 1(g)h$$

$$-U_0^2 = -2gh$$

Since acceleration due to gravity (g) on the moon is 1/6th of that on the earth, one can jump higher on the surface

b) Time period of simple pendulum T

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Therefore, T $\propto 1/\sqrt{g}$. Since the value of g is less at mountain than at plain the time period of simple pendulum will be more at mountain than at plain.

c) Solution

$$g' = g\left[1 - h/_R\right]$$

Also,

$$g' = {gR^2}/{(R+h)^2}$$

Now,

$$g' = g/_2 = {gR^2}/{(R+h)^2}$$

$(R + h)^2 = 2R^2$

$R + h = \sqrt{2}\,R$

$h = (\sqrt{2} - 1)\,R$

$h = (\sqrt{2} - 1)\,R$

d) Solution

Support g is the acceleration due to gravity in the absence of rotational motion of earth.

The acceleration due to gravity at the equator in the presence of earth's rotation is given by

$$g_e = g - R\omega^2$$

Here w is the angular velocity of the earth for g = o, we have

$$g - R\omega^2 = 0$$

$$g = R\omega^2$$

$$\omega = \sqrt{g/_R} = \sqrt{10/_{6400 \times 10^3}}$$

$$w = 1.25x10^{-3} rads^{-1}$$

Support in this situation, the time period pf rotation of earth about its axis is T

$$T = {2\pi}/_{\omega}$$

$$T = {2\pi}/_{1.25 \times 10^{-3}} = 5024s$$

$$\therefore T = 8.4$$

The earth would complete its rotation in **8.4h** instead of **24h**

3a) (i) What would be the effect on the weight of a body if the earth stops rotating about its axis?

(ii) By what value the acceleration due to gravity at the equator will change?

b) A light body and heavy body are allowed to fall from the same height which will reach the earth earlier?

c) How far above the earth surface does the value of g becomes 16% of its value on the surface? Assume radius of earth to be 6400km

d) Calculate the effect of rotation of earth on the weight of the body at a place at latitude 45^o Take radius of earth = 6.37×10^6m

Solution

Both bodies will reach the earth simultaneously. It is because acceleration due to gravity (g) is independent of mass of the body

$$g = \frac{GM_E}{R_E^2}$$

a) (i) The weight of the body would increase because the effect of centrifugal force will be absent. **Note that;** Centrifugal force comes into picture because of the rotating earth about its axis.

ii) Acceleration due to gravity at the equator is g_e

$$g_e = g - R\omega^2$$

Where R is the radius of earth and ω is the angular velocity of rotation of the earth.

If the earth stops rotating about its axis then acceleration due to gravity will increase by $\omega^2 R$

$$g_e = g - \omega^2 R$$

$$g_e = g - (0)^2 R$$

$$\therefore \; g_e = g$$

c) Solution

R = 6400km

h =?

$g' = \frac{16g}{100}$

$$g'/g = 16/100$$

From, $$g' = g\left[R^2/(R+h)^2\right]$$

$$g'/g = \left[R^2/(R+h)^2\right]$$

$$16/100 = \left[R^2/(R+h)^2\right]$$

Under root both sides, then we have

$$4/10 = R/R+h$$

$$4R + 4h = 10R$$

$$4h = 10R - 4R$$

$$4h = 6R$$

$$h = {}^{3R}/_{2}$$

$$h = 3 \times {}^{6400}/_{2}$$

$$h = 9600$$

d) Solution

$$g' = g - \omega^2 R cos^2\lambda$$

$$\lambda = 45^0$$

$$R_E = 6.37 \times 10^6 m$$

Here

$$cos\lambda = \frac{1}{\sqrt{2}}, \omega = {}^{2\pi}/_{24 \times 60 \times 60} = 7.27 \times 10^{-5}$$

$$g' - g = -\omega^2 R cos^2\lambda$$

$$g - g' = \omega^2 R_E cos^2\lambda$$

$$g - g' = 6.37 \times 10^6 \times (7.27 \times 10^{-5}) \times {}^{1}/_{2}$$

$$\therefore g - g' = 0.0168 ms^{-2}$$

It shows that the value of acceleration due to gravity at the poles is greater than latitude 45° by 0.016m/s? Hence the weight of a body of 1kg mass at the poles will be greater than 45° latitude 0.0168N.

4. a) At what height above earth's surface, value of g is the same as in a mine 100km deep?

b) At what height in km over the earth's pole, the free fall acceleration decreases by 1% radius of earth = 6400km

c) If a person lands on moon, he has to tie a heavy weight at the back. Why?

d) Which pulls with greater force on the oceans of the earth, the sun or moon

Solution

a) Suppose at a height h above the surface of earth's, the acceleration due to gravity g is the same as in a mine at a depth d = 100km into the earth

For a height h above earth's surface

$$g' = g\left[1 - {}^{2h}/_{R}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

A depth d into earth

$$g' = g\left[1 - {}^{d}/_{R}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

As the acceleration due to gravity in the two cases are the same

$$g' = g'$$

$$g\left[1 - {}^{2h}/_{R}\right] = g\left[1 - {}^{d}/_{R}\right]$$

$$1 - {}^{2h}/_{R} = 1 - {}^{d}/_{R}$$

$$-{}^{2h}/_{R} = -{}^{d}/_{R}$$

$$2h/_R = d/_R$$

$$h = d/_2$$

$$h = \frac{100}{2}$$

$$\therefore h = 50km$$

b) Solution

Let g' be the acceleration due to gravity at height h from the surface of earth. Given that

$$g' = 0.99g$$

Now,

$$g' = g\left[1 - 2h/_R\right]$$

$$g'/_g = 1 - 2h/_R$$

$$0.99 = 1 - 2h/_{6400}$$

$$2h/_{6400} = 1 - 0.99$$

$$\therefore h = 32km$$

If we are to find the height (h) over earth's surface at which acceleration due to gravity becomes 1% of that on earth's surface, then use the formula.

$g' =$

GM

(R + h) ^{2}

c) The value of g on the moon is very small so that weight of the person will also be small.

d) The sun but it is so distance that it attracts all parts of the earth with almost equal strength hence its effectiveness in raising tides is less than that of the moon.

5. a) What do you mean by weight of a body

b) i) If there is an attraction force between all objects, why do we not fear ourselves gravitating towards massive buildings in our vanity?

ii) When a body falls towards earth, earth moves towards the body. Why is motion of earth not noticed?

c) Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weight 3/5th as much as at present. Take equatorial radius as 6400km.

Solution

5. a) The weight of a body is the force acting on its mass due to the gravitational attraction of earth

b) i) Gravity pull as to massive buildings and everything else in the universe. The forces between us and the buildings are relative small because masses are small compared to the mass of the earth.

The forces due to starts are small because of their great distance. These tiny forces escape our notes when they are overwhelmed by the over pouring attraction to the earth.

ii) The earth attracts the body of mass m with force F= mg the body also

attracts earth with the equal force mg. Since the mass of the equal is very large acceleration produced in the earth is navigable.

$$a = {}^{F}/_{M_E} = {}^{mg}/_{M_E}$$

Solution

c) At present, the weight of the person on the equator is nearly the same which would have been if earth were stationary.

Suppose for weight to remain 3/5th, the angular speed of earth is ω

$$g_e = g - \omega^2 R$$

$$^{3}/_{5}\,mg = mg - m\omega^2 R$$

$$m\omega^2 R = -^{3}/_{5}\,mg + mg$$

$$m\omega^2 R = {}^{2mg}/_{5}$$

$$\omega^2 R = {}^{2g}/_{5}$$

$$\omega^2 = {}^{2g}/_{5R}$$

$$\omega = \sqrt{^{2g}/_{5R}} = \sqrt{^{2 \times 9.8}/_{5 \times 6400}}$$

$$w = 7.8X10^{-4} rad/s$$

d) Solution

Weight of the body at depth d below earth's surface

$$mg' = mg\left[1 - {}^{d}/_{R}\right]$$

$$mg' = mg - {}^{mgd}/_{R}$$

$$^{mgd}/_{R} = mg - mg'$$

$$^{d}/_{R} = {}^{mg - mg'}/_{mg}$$

% decrease in weight

$$mg - mg'/_{mg} \times 100 = {}^{d}/_{R} \times 100$$

$$= {}^{32}/_{6400} \times 100$$

% decrease in weight 0.5%

EXERCISE

1.A body of mass m is raised through a height h from a point on the surface of earth where the acceleration due to gravity is g prove that loss of weight due to prove that loss of weight due to variation of g is approximately $2mgh/_{R}$ where R is the radius of earth.

2. A man weighs 60kg at earth's the earth's surface, his weight becomes 30kg? Radius of earth R= 6400km, h = 2625 km

3. The earth's radius is about 6370km.An object of mass 20kg is taken to a height of 160km above the earth's surface

i)What is the objects mass of this height?

ii) How much does the object weight at this height?

iii) SAME

iv) 186.5N

Ø SATELLITE AND PLANETARY MOTION

Universe

Is the totality of all matter, space and time. The smallest basic unit of universe is the star just as the smallest unit in the army to the platoon.

A star

Is a luminous gaseous heavenly body that generates energy by means of nuclear fusion reactions.

The sun is the nearest star to earth. Around some stars are planets

Planet

Is a heavenly body that orbits around the sun or another star and shines only by the light that it reflects.

Natural satellite

Is heavenly body that revolves around the planet in the stable orbits.

For examples, moon is the natural satellite of the earth. It goes round the earth in about 27.3 days in a nearly circular orbit of radius 3.84 x 10^{s} km.

Artificial satellite

Are man made satellites that orbit around the earth or some other heavenly bodies.

Artificial satellites circling the earth are now quite common. They are called earth satellites.

For example, communication satellites are used routinely to transit information around the globe.

USES OF ARTIFICIAL

SATELLITES

1. They are used to learn about the atmosphere near the earth.
2. They are used to forecast weather.
3. Space flights are possible due to artificial satellites.
4. They are used to receive and transmit various radio and television signals
5. They are used to know the exact shape and dimensions of the earth.
6. To study the details of ozone layer in the atmosphere.
7. To view and take photograph of every place on the Earth.

POLAR SATELLITES

Is the satellite that revolves around the earth in polar orbit

The polar satellite passes over both geographical north and south poles in one revolution.

Since the polar satellite passes over different parts or the earth during its each revolution, it can survey and scan the entire surface of the earth.

USES OF POLAR SATELLITES

1. They are used to record the land and sea temperature.
2. They are used to take picture of clouds and make forecasting of climatic changes.

ORBITAL VELOCITY OF SATELLITE

Is the velocity required to put a satellite in to a given orbit around the earth

The direction of the orbital velocity of the satellite at any instant is along the tangent to the orbit of satellite at that instant.

The centripetal force required by the satellite to move in the circular orbit is provided by the gravitational pull of earth acting on the satellite

Centripetal force required by satellite, $F_C = {mV^2}/{(R+h)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

Gravitation pull of the earth on the satellite, $F_G = {GMm}/{(R+h)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Equating equation $(i) and\ (ii)$

$$F_C = F_G$$

$$mV^2/R+h = GMm/(R+h)^2$$

$$mV^2 = GMm/(R+h)$$

$$V^2 = {GM}/{R+h}$$

$$V = \sqrt{{GM}/{R+h}}$$

I. The orbital velocity of the satellite is independent of the mass of the satellite.

II. The orbital velocity of the satellite decreases as the height of the satellite increases.

Therefore, the value of orbital velocity is different from orbits around the earth.

III. The orbital velocity of a satellite depends upon the mass (M) and radius (R) of the earth/ planet around which it revolves.

FOR SATELLITE CLOSE TO EARTH

When the satellite orbits are very close to the earth's surface, then h can be neglected as compared to R in equation above.

From,

$$V = \sqrt{{GM}/{R+h}}$$

$$h = 0$$

$$V = \sqrt{{GM}/{R}}$$

Alternative Expression

For orbital velocity

From,

$$V = \sqrt{{GM}/{(R+h)}}$$

The acceleration due to gravity on earth's surface is

$$g = {GM}/{R^2}$$

$$gR^2 = GM$$

Putting the value of $GM = gR^2$ in the above expression

Orbital velocity V,

$$V = \sqrt{{gR^2}/{(R+h)}}$$

$$V = R\sqrt{{g}/{(R+h)}}$$

For satellite close to earth neglecting h compared to R we have.

$$V = R\sqrt{{g}/{R}}$$

$$V = \sqrt{{gR^2}/{R}}$$

Now,

$$R = 6.4 \times 10^6 m$$

$$g = 9.8ms^{-1}$$

$$V = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$V = 8ms^{-1}$$

$$\therefore v = 8km/s(Approx)$$

Thus the orbital speed of the satellite close to earth's surface is about 8km/s

In practice, the satellite is carried by a rocket to the height of the orbit and then given an impulse, by firing sets, to deflect it in a direction parallel to the tangent of the orbit.

Its velocity is boosted to 8km/s so that it stays in the orbit since this motion may continue indefinitely; we may say that the orbit is STABLE.

The orbital velocity of a satellite revolving around any planet is

$$V_p = \sqrt{{GM_p}/{R_p + h}}$$

$M_p =$ Mass of the planet

$R_p =$ Radius of the planet

TIME PERIOD OF SATELLITE

Is the time taken by the satellite to complete one revolution around the earth.

It is denoted by T

Suppose a satellite of mass m is to be put into circular orbit around the earth at a height h above its surface.

The mass and radius of earth are M and R. Then the radius of the orbit of the satellite is (R + h).

Let V be the orbital velocity of the satellite in this orbit.

Time period of satellite T

From,

$$V = \omega(R + h)$$

$$V = {}^{2\pi(R+h)}/_{R}$$

$$T = {}^{2\pi(R+h)}/_{V}$$

We shall derive three equivalent expressions for the time period T of the satellite.

I. FIRST EXPRESSION FOR T

Now, From

$$V = \sqrt{{}^{GM}/_{R+h}}$$

Then,

$$T = {}^{2\pi(R+h)}\Big/_{\sqrt{GM}/\sqrt{R+m}}$$

$$T = 2\pi(R+h) \times {}^{\sqrt{R+h}}\Big/_{\sqrt{GM}}$$

$$T = 2\pi\sqrt{{}^{(R+h)^3}/_{GM}}$$

It is clear that the time period (T) for the satellite depends only upon its height (h) above the earth's surface.

The greater the height of a satellite above the earth's surface the greater is its period of revolution.

It is for this reason that the moon, which is at a height of 3.84 x 10^{s} km above earth, completes one revolution in about 27 days while an artificial satellite circling close to the earth's surface completes 10 to 20 revolutions in a day.

II. SECOND EXPRESSION FOR "T"

Also $V = R\sqrt{{}^{g}/_{R+h}}$

$$T = {2\pi(R+h)}/{V}$$

From,

$$T = {2\pi(R+h)}/{R} \times \sqrt{{R+h}/{g}}$$

$$T = {2\pi}/{R}\sqrt{{(R+h)^3}/{g}}$$

Note:

Again that time period (T) of the satellite depends only upon its height (h) above the earth's surface.

III. THIRD EXPRESSION FOR (T)

$$T = 2\pi\sqrt{{(R+h)^3}/{GM}}$$

Now,

Mass or earth M

M = volume x Density

$$M = {4}/{3}\pi R^3 \rho$$

$$T = 2\pi\sqrt{{(R+h)^3}/{G \times {4}/{3}\pi R^3 \rho}}$$

$$T = \sqrt{{3\pi(R+h)^3}/{G\rho R^3}}$$

Again, the time period (T) of the satellite depends only upon its height (h) above the earth's surface.

Note:

If the satellite orbits very close to the surface of the earth then h can be neglected as compared to R.

Therefore equations (i) , (ii) and (iii) for the time period (T) of the satellite become

$$T = 2\pi\sqrt{R^3/GM}$$

$$T = 2\pi\sqrt{R/g}$$

$$T = \sqrt{3\pi/Gp}$$

Now, $T = 2\pi\sqrt{R/g}$

Here

R = 6.4 x 10^6m

g = 9.8m/s^2

$$T = 2\pi\sqrt{6.4\times10^6 m/9.8}$$

T = 5075 Seconds

∴ T = 84.58 minutes

Thus the orbital speed of a satellite revolving very near to the earth's surface is about 8km/s and its period of revolution is nearly 84 minutes.

HEIGHT OF SATELLITE ABOVE EARTH'S SURFACE

Let us now see how the height or the satellite above earth's surface can be determined

Consider a satellite of mass m orbiting the earth at a height h above its surface. The mass and radius of earth are M and R. Then the radius or the orbit of the satellite is (R + h)

The time period (T) of the satellite is given by

$$T = {2\pi}/{R}\sqrt{{(R+h)^3}/{g}}$$

Squaring both sides,

$$T^2 = {4\pi^2(R+h)^3}/{R^2 g}$$

$$(R+h)^3 = {gR^2T^2}/{4\pi^2}$$

$$(R+h) = \left[{gR^2T^2}/{4\pi^2}\right]^{1/3}$$

$$h = \left[{gR^2T^2}/{4\pi^2}\right]^{1/3} - R$$

If we know the time period T of the satellite, Radius R or earth and acceleration due to gravity (g) at the earth's surface, the height h of the satellite above earth's surface can be calculated.

ENERGY OF SATELLITE

A satellite orbiting the earth has both kinetic energy (K.E) and potential energy (P.E)

Therefore, the total mechanical energy of the satellite is the sum of its K.E and P.E.

Consider earth to be a sphere of radius R and mass M. Suppose a satellite or mass M revolves around the earth in a circular orbit at a height h above the surface of earth.

The radius of the orbit of the satellite is (R + h). Let V be the Orbital velocity of the satellite in this orbit.

$$K.E\ of\ satellite = {1}/{2}\, mV^2$$

The gravitational force exerted by the earth on the satellite is

$$F = {}^{GMm}/_{(R+h)^2} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots .. (i)$$

This gravitational force provides the necessary centripetal force Fc to the satellite to move in the circular orbit of Radius.

$$F = {}^{MV^2}/_{(R+h)}$$

$$F_C = F$$

$${}^{mV^2}/_{R+h} = {}^{GMm}/_{(R+h)^2}$$

$$mV^2 = {}^{GMm}/_{R+h}$$

$${}^{1}/_{2}mV^2 = {}^{GMm}/_{2(R+h)}$$

Kinetic Energy of the satellite

$$K.E = {}^{GMm}/_{2(R+h)}$$

The potential Energy of the satellite at a height h above the earth's surface is P.E

$$P.E = -{}^{GMm}/_{(R+h)}$$

Total energy of satellite E_T

$$E_T = K.E + P.E$$

$$E_T = {}^{GMm}/_{2(R+h)} - {}^{GMm}/_{(R+h)}$$

$$E_T = {}^{GMm - 2GMm}/_{2(R+h)}$$

$$E_T = -{}^{GMm}/_{2(R+h)}$$

The total energy of the satellite in the orbit is **NEGATIVE.**

At infinity (R + h = ∞), the potential energy as well as kinetic energy is zero. Therefore Total energy is zero.

The kinetic energy can never be negative. Therefore a negative total energy means that in order to send the satellite to infinity, we have to give energy to the satellite.

Unless a revolving satellite gets extra energy it would not leave its orbit i.e it will go on revolving in a closed orbit.

We say that the satellite is bound to the earth.

BINDING ENERGY OF SATELLITE

Is the energy required to remove the satellite from its orbit around the earth to infinity.

$$Binding\ energy\ of\ satellite = 0 - Total\ energy\ of\ satellite$$

$$= 0 - \left[-{}^{GMm}/_{2(R+h)}\right]$$

$$Binding\ energy\ of\ satellite = {}^{GMm}/_{2(R+h)}$$

`

ANGULAR MOMENTUM OF SATELLITE.

Consider a satellite of mass m orbiting the earth at a height h above its surface.

The radius of the radius of the orbit of the satellite is (R + h)

Let v be the orbital velocity of the satellite in this orbit. Therefore, the magnitude of Angular momentum (L) is given by

L = MV X Radius of satellite orbit

L = MV x (R + h)

Then

Orbital velocity V

$$V = \sqrt{{GM}/{R+h}}$$

$$L = m \times \sqrt{{GM}/{R+h}} \times (R+h)$$

$$L = m\sqrt{Gm(R+h)}$$

$$\therefore L = m\sqrt{GM(R+h)}$$

The Angular momentum of a satellite depends on

(i) The mass (m) of the satellite and the mass (m) of the earth or planet.

(ii) The radius of the orbit (R + h) of the satellite.

GEOSTATIONARY SATELLITES

Is the satellite that appears to be at a fixed position in the sky to an observer on earth. It also called stationery satellite.

A geostationary satellite revolve around the earth with the same angular speed in the same direction (west to east) as is done by the earth.

Therefore, the velocity of such a satellite relative to earth is zero. For this reason, the satellite appears to be stationery to an observer.

Since the Angular speed of geostationary satellite is the same as that of earth, the period of revolution of this satellite is 24 hours (T = 24 hours.)

The geostationary satellite revolves around the earth from west to east in a close circular orbit and coplanar with the equation plane.

Most of communication satellites are geostationary satellites so that information can be transmitted from one part of the world to another.

Parking orbit;

Is a orbit in which the period of revolution of a satellite is equal to the period of earth revolution (24Hrs) about its axis.

The orbit of the geostationary satellite is sometimes called parking orbit.

When in a parking orbit, a satellite stays at the same place with respect to Earth.

HEIGHT OF GEOSTATIONARY ABOVE EARTH'S SURFACE

We know that the time period T of an earth satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{R^2 g}}$$

Here,

$R =$ Radius of Earth $=$ 6400km

$T =$ Time period $=$ 24hrs $=$ 24 x 60 x 60

$h =$ Height of the artificial satellite above earth's surface, corresponding to T= 24hrs

$g =$ 9.8m/$1s^2 =$ 0.0098km/s^2

$$24 \times 60 \times 60 = {}^{2\pi}/_{6400}\sqrt{{}^{(6400+h)^3}/_{0.0098}}$$

$\therefore$ **h = 36000km**

The Geostationary satellite orbits around the earth at a height of 36000km above the surface of the earth.

ORBITAL SPEED OR GEOSTATIONARY SATELLITE

The radius of the parking orbit is R_P

$R_P =$ R + h

$R_P =$ 6400 + 36000

$R_P =$ 42400km

Therefore, orbital speed v or the satellite in the parking orbit is

$$V = {circumference\ of\ parking\ orbit}/{Time\ period\ of\ satellite}$$

$$V = {2\pi R_P}/{24hrs}$$

$$V = {2\pi \times 42400}/{24 \times 60 \times 60}$$

$$\therefore V = 3.1km/s$$

Thus a geostationary satellite revolves around the earth at a height of 36000km above earth's surface with an orbital speed of 3.1 km/s.

PLACING SATELLITE IN THE PARKING ORBIT

When a satellite is to be placed in the parking orbit, it is first carried to a height of 36000km above earth's surface.

It is then given the necessary tangential velocity v = 3.1km/^{1s} by firing rocket engines which are aligned parallel to earth's surface.

In order to put a satellite in the parking orbit, following condition must be satisfied.

(i) It should rotate in the same direction as the earth is rotating.
(ii) The height of the parking orbit should be 36000km above the earth's surface.
(iii) The time period of the satellite should be 24hrs.
(iv) It should revolve in an orbit concentric and coplanar with the equatorial plane.

WEIGHTLESSNESS

Is the phenomenon when the effective weight of a body becomes zero.

The weight Mg of a body becomes zero where $g = 0$. Under this condition the body is in a weightlessness state.

The state of weightlessness can be observed in the following situations

(i) BODY AT THE CENTER OF EARTH

At the center of earth, g = 0.

Therefore, if a body is taken to the center of the earth, its effective weight becomes zero.

From $$g' = g\left[1 - {}^{d}/_{R}\right]$$

$$mg' = mg\left[1 - {}^{d}/_{R}\right]$$

At the center of the earth

$$d = R$$

$$mg' = mg[1 - {}^{R}/_{R}]$$

$$mg' = 0$$

∴ The effective weight, when a body is taken to the center of earth becomes zero

(ii) BODY IN A FREELY FALLING LIFT

When a body is lying in a lift and fall freely, the effective acceleration is g^1

$$g' = g - a$$

But, $$a = g$$

$$g' = g - g$$

Therefore the effective weight of a body in the free falling **lift is zero**.

(iii) BODY AT NULL POINTS IN SPACE

As we go up, the gravitational pull of earth goes on decreasing but the gravitational pull of moon goes on increasing.

At a particular position in space, the two gravitational pulls are equal and opposite and effective value of g.

$$g = 0$$

Such points in space are called null points. If a body is taken to null points, its effective weight becomes zero.

(iv) BODY IN A SATELLITE ORBITING EARTH

The effective weight or body inside the satellite orbiting around the earth becomes zero.

This condition or weightlessness in a satellite poses many problems to astronauts including

* Difficult in controlling movements
* Difficulty in eating and drinking
* Adverse effects on human organism.

WORKED EXAMPLE

1. The period of moon around the earth is 27.3 days and the radius of its orbit is 3.9 x 10^s km. if G = 6.67 x 10^{-11} Nm^2 kg^{-2}, Find the mass of the earth.

Solution

Time period of moon T = 27.3days = 27.3 x 60 x 60 x 24 Radius of moon's orbit (R + h) = 3.9 x 10^8m.the time period of a satellite is given by,

$$T = 2\pi\sqrt{{(R+h)^3}/{GM_e}}$$

$$M_e = {4\pi^2(R+h)^3}/{GT^2}$$

$$M_e = {4\pi^2 \times (3.9 \times 10^8)^3}/{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$\therefore M_{e=6.31x10^{24}kg}$$

2. A satellite is revolving in a circular orbit at a height of 3400km above earth's surface. Calculate the orbital velocity and the period of revolution radius of earth = 6400km and g = 9.8mls

Solution

R = 6400km

H = 3400km

R + h = 9800km = 9.8 x10^6m

$$V = R\sqrt{g/_{R+h}}$$

$$V = 6.4 \times 10^3 m/s$$

$$T = {2\pi(R+h)}/_V$$

$$T = 9616s = 2.67hrs$$

3. A small satellite revolves around a planet in an orbit just above the surface of the planet. Taking G = 6.67 x 10-11Nm^2 and near density of the planet as 8 x 10^3kg.

Find the time period of a satellite orbit very close to the surface of a planet is given by

$$T = \sqrt{3\pi/_{GP}}$$

$$T = \sqrt{3\pi/_{(6.67 \times 10^{-11})(6.67 \times 10^3)}}$$

$$\therefore T = 4202S$$

4. An artificial satellite revolving coplanar with the equator ground the earth appears stationary to an observer on the earth.

Calculate the height of the satellite above earth's surface. Given that g = 9.8 m/s^2 and radius of earth R = 6.37 x 10^6m

Solution

Let h be the height of the satellite above the surface of the earth. The period of revolution of a satellite is given by

$$T = {}^{2\pi}/_{R} \sqrt{{}^{(R+h)^3}/_{g}}$$

$$T = {}^{2\pi(R+h)^{3/2}}/_{R\sqrt{g}}$$

Since the satellite appears to be stationary to an observer on the earth its period of revolution is equal to the period of axial rotation 24hrs of the earth (also the satellite is revolving from west to east.

$$T = 24hrs = 24 \times 60 \times 60$$

$$T = 24 \times 3600s$$

$$24 \times 3600 = {}^{2\pi(R+h)^{3/2}}/_{R\sqrt{g}}$$

$$(R+h)^3 = {}^{(24 \times 3600)^2 \times R^2 \times g}/_{4\pi^2}$$

$$(R+h)^3 = {}^{(24 \times 3600)^2 \times (6.37 \times 10^6)^2 \times 9.8}/_{4\pi^2}$$

$$(R+h)^3 = 75 \times 10^{21} m^3$$

$$R + h = \sqrt[3]{75 \times 10^{21}}$$

$$R + h = 4.2 \times 10^7 m$$

$R + h = 42 \times 10^6$ m

$R - R + h = (42 \times 10^6) - R$

$h = (42 \times 10^6) - (6.37 \times 10^6)$

$h = 36 \times 10^6 m$

$\therefore h = 36000 km$

5. An artificial satellite of mass 100kg is in circular orbit of 500 km above the earth's surface. Radius of earth = 6.5 x 106m. find

(i) The acceleration due to gravity at any point along the satellite orbit

(ii) The centripetal acceleration of the satellite

Solution

Mass of satellite m = 100kg

Radius of earth R = 6.5 x 10^6m

Height of satellite h = 500km

Radius of satellite orbit v

But,

$r = R + h$

$r = (6.5 \times 10^6) + (0.5 \times 10^6)$

$r = 7 \times 10^6 m$

(i) $g' = g\left[1 + {}^{h}/_{R}\right]^{-2}$

$$g' = 9.8\left[1 - {0.5 \times 10^6}/{6.5 \times 10^6}\right]^{-2}$$

$$g' = 9.8[1 + 0.0769]^{-2}$$

$$g' = 9.8[0.08622]^{-2}$$

$$\therefore g' = 8.45 m/s^2$$

6. Prove that the angular momentum of a satellite of mass Ms revolving round the earth of mass Me in orbit of radius r is equal to

$$L = \sqrt{GM_e R_S^2 r}$$

Solution

The gravitational force between the earth and satellite provides the necessary centripetal force to move the satellite in the circular orbit

If V is the orbital velocity of the satellite, then

$$F_C = {M_S V^2}/{r} \dots\dots\dots\dots\dots\dots\dots\dots\dots..(i)$$

$$F_G = {GM_S M_e}/{r^2} \dots\dots\dots\dots\dots\dots\dots(ii)$$

By equating the two equation

$$F_C = F_G$$

$${M_S V^2}/{r} = {GM_e M_S}/{r^2}$$

$$V = \sqrt{{GM_S}/{r}}$$

Angular momentum of satellite L

$$L = M_S V_r$$

$$L = M_S \times \sqrt{GM_E/_r} \times r$$

L = $\sqrt{GM_s^2 \, M_e \, r}$

7. How much work must be done to lift an artificial satellite of mass m from the surface of earth (mass M, Radius R) and put in a circular orbit with a radius equal to twice the earth's radius?

Solution

Initial K.E of the satellite $= k_0 = 0$

Total P.E of satellite $= U_0 = -{}^{GMm}/_R$

Total Energy of satellite E_0

$$E_o = k_0 + U_0$$

$$E_0 = 0 + [-{}^{GMm}/_R]$$

$$E_0 = -{}^{GMm}/_R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Total final energy of the satellite

$$E = -{}^{GMm}/_{2(R+R)}$$

$$E = -{}^{GMm}/_{4R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Work done W

$$W = E - E_0$$

$$W = -{}^{GMm}/_{4R} - [-{}^{GMm}/_{R}]$$

$$\therefore W = {}^{3GMm}/_{4R}$$

8. A satellite orbits the earth at a height of 500km from its surface. Compute its

(i) Kinetic Energy = 8.7×10^9J

(ii) Potential Energy = -17.4×10^9J

(iii) Total Energy = -8.7×10^9J

Mass of the satellite = 300kg;

Mass of the earth = 6.0×10^{24}kg

Radius of earth = 64×10^6 and the

Value of G = $6.6\ 7 \times 10^{-11}$Nm^2 kg $^{-2}$

Solution

(i) $K.E = {}^{GMm}/_{2(R+h)}$

(ii) P.E = -2 x K.E

(iii) E.T = K.E + P.E

The total energy is Negative for the satellite in the orbit. This indicates that the satellite is held to the orbit.

Positive work must be done to force the satellite from gravitation force.

9. A satellite orbits the earth at a height of 400km above earth's surface how much energy must be ended to rocket satellite out of the earth's gravitational influence.

Mass of the satellite = 200kg

Mass of the earth = 6×10^{24}kg

Solution

The total Energy of the satellite in the orbit is

$$E = -\,{}^{GMm}/_{2(R+h)}$$

$$E = -\,{}^{(6.67\times10^{-11})\times(6\times10^{24})\times(200)}/_{2(6.4\times10^{6}+4\times10^{5})}$$

$$E = -5.89\times10^{9}J$$

The total energy of the satellite at infinity is zero. Therefore, the energy required for the satellite to leave its orbit around the earth and escape to infinity.

$$0 - E = 0 — (5.89\times10^{9})$$

$$E = 5.89\times10^{9}J$$

10. Two stars, masses 10^{20}kg and 2 x 10^{20}kg respectively, rotate about their common center of mass with an angular speed W. Assuming that the only force on a star is the mutual gravitational force between them, calculate W Assume that the distance between the stars is 10^{6}Km and that G is 6.67 x 10^{-11} Nm^{2} kg^{-2}

Solution

$$F_G = {}^{GMm}/_{r^2}$$

$$F_C = {}^{mV^2}/_{r}$$

Also,

$$F_C = F_G$$

$$^{GM}/_{r^2} = {}^{mV^2}/_{r}$$

$$^{GM}/_{r} = V^2$$

But, $V = r\omega$

$$GM/_r = r^2\omega^2$$

$$\omega^2 = GM/_{r^3} = (6.67 \times 10^{-11})(2 \times 10^{20} + 10^{20})/_{(10^9)^3}$$

$$\omega^2 = 2.001 \times 10^{-17}$$

$$\omega = 4.47 \times 10^{-17}$$

ESCAPE VELOCITY OF A BODY

Is the minimum velocity with which it is to be projected so that it just overcomes the gravitational pull of earth or any other planet

Since Earth's gravitational field extends to infinity (however weak it may be at large distance) escape velocity is obviously the velocity that must be given to an object for it to escape all the way to infinity.

If a ball is thrown upwards from the surface of the earth its speed decreases from the moment it is projected due to retarding effect of earth gravitational field.

It would simply rise to a certain height, reverse direction and then fall back to earth.

The height which the ball ultimately attains depends upon the speed with which it is projected- the greater the speed, the greater the height.

Ultimately at a certain velocity of projection, the body will go out of the gravitational field and will never return to the earth.

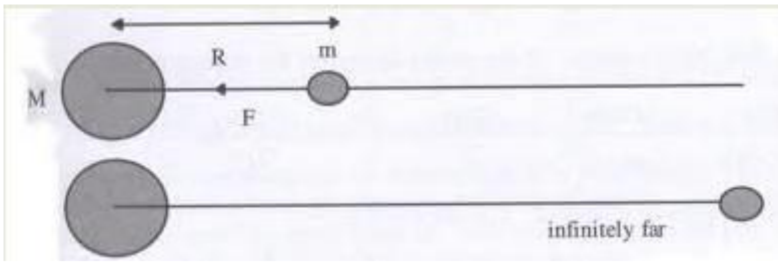
The minimum projection velocity that achieves this is known as the escape velocity.

EXPRESSION FOR ESCAPE VELOCITY

Consider earth to be a sphere of mass M and Radius R.

Suppose a body of mass M is projected upward with escape velocity Ve, when the body is at point P at a distance X from the center of the earth, the gravitational force of attach exerted by the earth on the body is

$F = {}^{GMm}/_{x^2}$



In moving a small distance DX against this gravitational force, the small work done at the expense of the kinetic energy of the body is given by

$$\Delta W = F \times \Delta x$$

$$\Delta W = {}^{GMm}/_{x^2} \times \Delta x$$

Therefore, total work done (W) in moving the body from earth's surface (where X = R) to infinity (where $\propto = \infty$) is given by

$$W = \int_R^\infty {}^{GMm}/_{x^2}\,\delta x$$

$$W = GMm \int_R^\infty {}^{\delta x}/_{x^2}$$

$$W = GMm \int_R^\infty x^2\,\delta x$$

$$W = GMm\left[-{}^{1}/_{x}\right]_R^\infty$$

$$W = GMm\left[-{}^{1}/_{\infty} - {}^{1}/_{R}\right]$$

$$W = GMm\left[0 - {}^{1}/_{R}\right]$$

$$W = {}^{GMm}/_{R}$$

If the body is to be able to do this amount of work (and so escape), it needs to have at least this amount of K.E at the moment it is projected.

Therefore, the escape velocity Ve is given by

$$^1/_2 mV^2 = {}^{GMm}/_R$$

$$V_e = \sqrt{^{2GM}/_R}$$

NOTE

(i) The escape velocity does not depend on the mass (m) of the body. It is the same for all masses for a given planet.

(ii) The escape velocity does not depend upon the direction of projection from the surface of earth/ planet

- It is because the kinetic energy of a body loses in reaching any particulars height depends only on the height concerned and not the path taken to reach it.

(iii)The escape velocity depends upon the mass (m) and radius *R* of the planet from the surface of which the body is projected.

Ve in terms of g and R

From $V_e = \sqrt{^{2GMm}/_R}$

Now $g = {}^{GM}/_{R^2}$

$$gV^2 = GM$$

$$V_e = \sqrt{^{2GR^2}/_R}$$

$$V_e = \sqrt{2gR}$$

(i) The orbital velocity of a satellite close to earth surface is V

$$V = \sqrt{gR}$$

$$Ve = \sqrt{2gR}$$

$$Ve = \sqrt{2} \cdot \sqrt{gR}$$

$$Ve = \sqrt{2} \cdot V$$

Thus the escape velocity is only 41% greater than the orbital velocity of a satellite close to the surface of earth.

(ii) Since escape velocity depends upon mass and Radius of the planet, it is different for different planets.

(iii) The escape velocity from the earth's surface is

$$Ve = \sqrt{2gR}$$

Now,

g = 9.8mls^2

R = 6.37 x 10^{6}m

$$Ve = \sqrt{2 \, x \, 9.8 \, x \, 6.37 \, x \, 10^6}$$

Ve = 11.2 x 10^{3}m/s

The escape velocity from the earths surface is 11.2km/s

Therefore, with an initial velocity of about 11.2km/s a rocket will completely escape from gravitational attraction of the earth.

- For example, it can be made to travel towards the moon so that eventually it comes under the gravitation attraction of the moon.

- At present, soft landings on the moon have been made by firing retarding retro rocket

V_e in terms of ρ and R

From, $V_e = \sqrt{2GM/_R}$

Now, $m = {}^4/_3 \pi R^3 \rho$

$$V_e = \sqrt{2 \times G \times 4 \times R^3 \rho\pi /_{3 \times R}}$$

$$V_e = \sqrt{{}^8/_3 \pi\rho G R^2}$$

$$V_e = \sqrt{2\pi\rho G 4R^2 /_3}$$

Diameter D of the earth

$$D = 2R$$

$$D^2 = 4R^2$$

$$V_e = \sqrt{2\pi\rho G D^2 /_3}$$

$$V_e = D\sqrt{2\pi\rho G /_3}$$

Hence, denser planet has greater escape velocities.

(i) If the velocity of projection (V) of the body from the surface of a planet is greater than the escape velocity (Ve) of the planet, the body will escape out from the gravitational.
- Field of that planet and will move in the interstellar space with velocity V^1

- We can find the value of V1 by applying law of conservation of energy

$$\frac{1}{2}mV'^2 = \frac{1}{2}mV^2 - \frac{1}{2}mV_e^2$$

$$V'^2 = V^2 - V_e^2$$

$$V' = \sqrt{V^2 - V_e^2}$$

(ii) The escape velocity of the sun is 618km/s

(iii) If the body is at height h above the surface of earth, then escape velocity of the body is

$$V_e = \sqrt{2GM/_{R+h}}$$

WORKED EXAMPLES

1. What is the escape velocity for a rocket on the surface of mars? Mass of mars = 6.58 x 10^{23}kg and Radius of Mars = 3.38 x 10^6m

Solution

The escape velocity on the surface of mars is

$$V_e = \sqrt{2GMm/_{R_m}}$$

Now,

G = 6.67 x 10^{-11} $NMkg^{-2}$

m_m = 6.58 x 10^{23}kg

R_m = 3.38 x 10^6m

$$V_e = \sqrt{2 \times 6.65 \times 10^{-11} \times 6.58 \times 10^{23}/_{3.38 \times 10^2}}$$

$V_e = 5.1 \times 10^3 m/s^2$

$V_e = 5.1 km/s$

The escape velocity from the surface of Mars is less than one half of the escape velocity (11.2km/s) from the earth's surface.

2. Find the velocity of escape at the moon given that its radius is 1.7 x 10^6m and the value of g at its surface is 1.63m/s^2

Solution

The escape velocity on the surface of moon is

Ve = $\sqrt{2gR}$

g = 1.63m/s^2

R = 1.7 x 10^6m

Ve = $\sqrt{2 \, x \, 1.63 \, x \, 1.7 \, x \, 10^6}$

$\therefore$ Ve $= 2.354X10^3 m/s$

3. Jupiter has a mass 318 times that of the earth and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given the escape velocity from earth's surface is 11.2KM/s

Solution

Escape velocity on the surface of earth is

$V_E = \sqrt{2GM_E/R_E}$ ....(i)

Escape velocity on the surface of Jupiter is

$V_J \quad = \quad \sqrt{2GM_J/R_J}$ .... (ii)

Take equation (ii) ÷ (i)

$$V_J/V_E = \sqrt{M_E/M_J} \times \sqrt{R_E/R_J}$$

$$V_J/V_E = \sqrt{R_E/R_J \times M_J/M_E}$$

Now,

$$R_J/R_E = 11.2\,, M_J/M_E = 318\,, V_E = 11.2km/s$$

$$V_J/11.2 = \sqrt{1/11.2 \times 318}$$

$$V_J = 11.2\sqrt{1/11.2 \times 318}$$

$$V_J = 59.68km/s$$

4. Show that moon would depart forever if its speed were increased by 42%

Solution

Mass of earth $=$ M_E

Mass of moon $=$ M_m

If the distance between the earth and moon is r, then necessary centripetal force F_C is provided by the gravitational attraction F_G between the earth and moon.

$F_C = F_G$

$M_m V^2/_r = GM_m M_E/_{R^2}$

$V^2 = GM_E/_r$

$gr^2 = GM_E$

$V^2 = gr^2/_r$

$V = \sqrt{gr}$

Escape velocity for the moon is

$V_e = \sqrt{2gr}$

Percentage increase in the velocity of the moon

$= V_e - V/_V \times 100$

$= \sqrt{2gr} \times \sqrt{gr}/_{\sqrt{gr}} \times 100$

$= 42\%$

5. Calculate the escape velocity on an atmospheric particle 1600km above the earth's surface, given that, the radius of earth is 6400km and acceleration due to gravity on earth's surface is $9.8m/s^2$

Solution

If g1 is the acceleration due to gravity at height h above the earth's surface, then

$g' = gR^2/_{(R+h)^2}$

Here

$R = 6400km = 6.4 \times 10^6 m$

$h = 1600km = 1.6 \times 10^2 m$

$R + h = 6400 + 1600$

$= 8000km$

$=8 \times 10^6 m$

$g' = 9.8 \times (6.4 \times 10^6)^2 / (8 \times 10^6)^2$

$\therefore g' = 6.27 m/s^2$

Also

Escape velocity V_e

$V_e = \sqrt{2g'R}$

$V_e = \sqrt{2 \times 6.27 \times (6.4) \times 10^6 + 1.6 \times 10^6})$

$V_e = \sqrt{2 \times 6.27 \times 8 \times 10^6}$

$\therefore V_e = 10.02 \times 10^3 m/s$

PLANETARY MOTION AND KEPLE'S LAWS

The German astronomer Johannes Kepler made a detailed study of the motion of planets about the sun.

He worked out three empirical laws that govern the motion of planets about the sun. These are now known as Kepler's laws of planetary motion.

KEPLER'S FIRST LAW

All planets move in elliptical path having the sun as one focus (as frame of reference).

By using the law of universal gravitation, Newton showed that the general path of a planet under the influence of an inverse square law of force is an ellipse with the center of force at one focus if the path is closed.

Kepler's first law is also known as the law of orbits or orbital rule.

KEPLER'S SECOND LAW

The line joining the sun and the planet sweeps out equal areas in equal times.

" Each planet moves in such a way that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal periods of time."

The second law can be used to predict the speed of a planet in one part of its orbit if we know the speed in another part.

KEPLER'S THIRD LAW

The squares of the periods of revolution of the planets about the sun are proportional to the cubes of their mean distances from the sun.

If T is the period of the planet and r is its average distance from the sun, then

$$T^2 \propto r^3$$

$$\frac{T^2}{r^3} = constant$$

Thus T^2/r^3 is the same for all planets. Hence smaller the orbit of the planet around the sun, the shorter the time it takes to complete one revolution.

If T_1 and T_2 are the periods of two planets and r_1 and r_2 are their respective average distances from the sun, then

From

$$\frac{T^2}{r^3} = constant$$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{r_1}{r_2}\right]^3$$

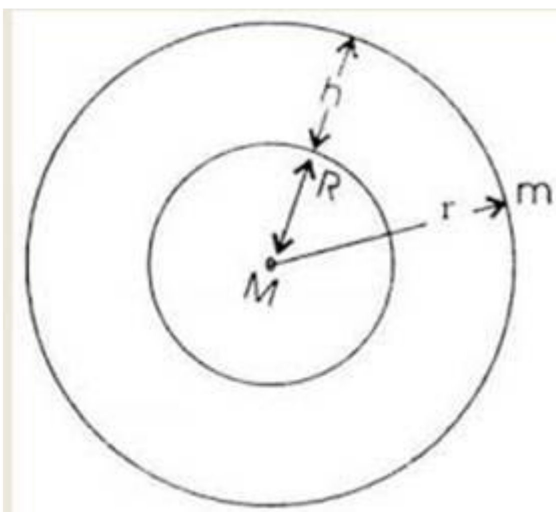
Kepler's third law is also known as the law of periods or period rule.

DEDUCTION OF KEPLER'S THIRD LAW

We now show that Kepler's third law follows from the law of universal gravitation.

We make the approximation the orbits of the planets are circles (most of the planetary orbit are pearly close to a circle, which is a special case of an ellipse). It is also approximately true that the orbital speed is constant.

Consider a planet of mass m revolving around the sun of mass M in a circle as orbit of radius r as shown in figure below.



Suppose V is the orbital speed of the planet and T is its time period. The sun's gravitational force F on the planet is:-

$$F_G = \frac{GmM}{r^2}$$

Because the mass of the sun is so much large than the mass of the planet, we can assume that the sun lies at the centre of the orbit. Therefore, the central petal force Fc acting on the planet is:-

This centripetal force is provided by the sun's gravitational force on the planet.

$$F_c = \frac{MV^2}{r}$$

$$V = r\omega$$

$$F_c = \frac{M}{r}(r\omega)^2$$

Also

$$\omega = \frac{2\pi}{T}$$

$$F_c = \frac{M}{r}r^2\omega^2$$

$$F_c = \frac{M}{r}r^2.\frac{4\pi^2}{T^2}$$

$$F_c = M.\frac{4\pi^2 r}{T^2}$$

$$F_c = \frac{4\pi^2 Mr}{T^2}$$

This is centripetal force is provided by the sun's gravitational force on the planet

$$F_c = F_G$$

$$\frac{GmM}{r^2} = \frac{4\pi^2 Mr}{T^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$$T^2 = \frac{4\pi^2}{GM}.r^3$$

Since the quantities $4\pi^2$, G, and M are constant.

$$T^2 = r^3$$

This is Kepler's third law, it has been derived on the basic of Newton's law of universal gravitational and therefore the two laws are consistent.

Therefore Kepler's third law can be derived only for uniform circular motion of the planet the result is true for elliptical orbits if we use the average distances from the sun for r.

ASSUMPTION USED TO DERIVE KEPLER'S THIRD LAW OF GRAVITATION

Newton was able to show Kepler's laws could be derived of universal gravitation and his law of motion.

Indeed, he used Kepler's third law as evidence in favor of his law of universal gravitation.

We now show that Newton's law of universal gravitational follows from Kepler's third law.

We make approximation that orbits of the planets are circles.

Consider a planet of mass m revolving around the sun of mass M in a circular orbit of radius (r).

Suppose V is the orbital speed of the planet and T is its period in time T the planet travels a distance

$$V = r\omega$$

$$V = r.\frac{2\pi}{T}$$

$$T = \frac{2\pi r}{V}$$

$$V = \frac{2\pi r}{T}$$

The centripetal force F required to keep the planet in the circular orbit is.

$$F = \frac{MV^2}{r}$$

$$F = \frac{M}{r}\left[\frac{4\pi^2 r^2}{T^2}\right]$$

$$F = \frac{4\pi^2 Mr}{T^2} - - - - - - - (i)$$

This centripetal force is provided by the sun's gravitational force on the planet.

According to Kepler's third law $T^2 \propto r^3$

$$T^2 = Kr^3$$

Substitute the value of T^2 on equation (i)

$$F = \frac{4\pi^2 Mr}{Kr^3}$$

$$F = \frac{4\pi^2 M}{Kr^2}$$

$$F = \frac{4\pi^2}{K} . \frac{M}{r^2} - - - - - - - - - (ii)$$

F is the force existed on the planet by the sun. We can see that force on the planet is directly proportional to the mass of the planet and inversely proportional to the severe of distance of the planet from the sun.

By the law of action and reaction this equals in magnitude to the force exerted on the sun by the planet.

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The force exerted by the planet on the sun is proportional to the mass of the planet.

$$\frac{4\pi^2}{K} \propto M$$

$$\frac{4\pi^2}{K} = GM - - - - - - - - (iii)$$

Where G is a constant of proportional independent of the mass of the sun or planet it is called universal gravitational constant. From equation (ii) and equation(iii).

$$F = G\frac{mM}{r^2}$$

WORKED EXAMPLE

1.The distances of the two planets from the sun are 10^{13}M and 10^{12}M respectively. Find the ratio of the time periods and speeds of the two planets.

Solution

$$\frac{T^2}{r^3} = constant$$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

When;

$$r_1 = 10^{13}\ m$$

$$r_2 = 10^{12}\ m$$

$$\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{r_1}{r_2}\right]^3$$

$$\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{10^{13}}{10^{12}}\right]^3$$

$$\frac{T_1}{T_2} = \left[\frac{10^{13}}{10^{12}}\right]^{3/2}$$

$$\therefore \frac{T_1}{T_2} = 10\sqrt{10}$$

Now

$$V_1 = \frac{2\pi r_1}{T_1}$$

$$and \qquad V_2 = \frac{2\pi r_2}{T_2}$$

$$\frac{V_2}{V_1} = \frac{2\pi r_2}{T_2} . \frac{T_1}{2\pi r_1}$$

$$\frac{V_2}{V_1} = \frac{r_2}{r_1} . \frac{T_1}{T_2}$$

$$\frac{V_2}{V_1} = \frac{10^{12}}{10^{13}} . 10\sqrt{10}$$

$$\frac{V_2}{V_1} = \sqrt{10}$$

$$\therefore \frac{V_1}{V_1} = \frac{1}{\sqrt{10}}$$

2. The moon has a period of 28days and an orbital radius 3.8 x 10^5 km satellite that has a period of one day?

Solution

According to Kepler's third law

$$\frac{T^2}{r^3} = constant$$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Here

$$r_1 = 3.8 \times 10^5 \ km$$

$$T_1 = 28 \ days$$

$$r_2 =$$

$$T_2 = 1 \ day$$

$$r_2^3 = R_1^3 \times \frac{T_2^2}{T_1^2}$$

$$r_2^3 = (2.8 \times 10^5)^3 \times \frac{1}{302}$$

$$r_2^3 = 6.999 \times 10^{13}$$

$$\therefore r_2 = 4.121 \times 10^4\ km$$

3. An earth satellite 5 has an orbit radius which is 4times that of a communication satellite C. what the time period of 5

Solution

According to Kepler's third law

$$\left[\frac{T_s}{T_c}\right]^2 = \left[\frac{R_s}{R_c}\right]^3$$

$$T_s = T_c \times \left[\frac{R_s}{R_c}\right]^{3/2}$$

$$T_s = 1 \times \left[\frac{4}{1}\right]^{3/2}$$

$$\therefore T_s = 8\ days$$

Time period Tc of communication satellite is 1day.

4. The revolution period of earth is 365.3 days and its mean distance from the sun is 1.5 $\times$ 10^8km.the revolution periods of Venus and mars are 224.7 days and 687days respectively Estimate the mean distances of these planets from the sun. Neglect the effect of other planets.

Solution

Revolution period of earth, $T = 365.3\ days$

Revolution period of Venus, $T_1 = 224.7\ days$

Revolution period of mars, $T_2 = 687\ days$

Distance of earth from the sun $R = 1.5 \times 10^8\ km$

According to Kepler's third law

$$\frac{T^2}{R^3} = constant\ (K)$$

$$K = \frac{(224.7)^2}{(1.5 \times 10^8)^3}$$

$$K = 3.95 \times 10^{-20} days^2 km^{-3}$$

For Venus

$$\frac{T_1^2}{R_1^3} = K$$

$$R_1^3 = \frac{T_1^2}{K} = \frac{(224.7)^2}{3.95 \times 10^{-20}}$$

$$\therefore R_1 = 1.085 \times 10^8\ km$$

For Mars

$$\frac{T_1^2}{R_1^3} = K$$

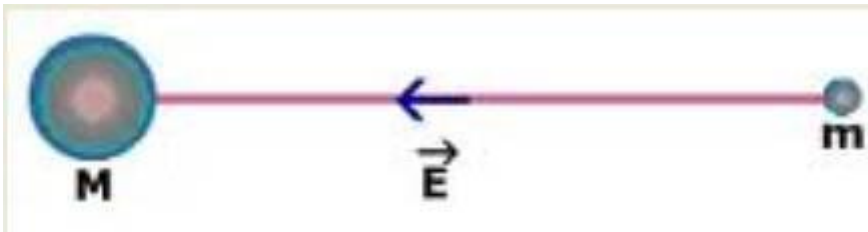
$$R_2^3 = \frac{T_2^2}{K} = \frac{(687)^2}{3.95 \times 10^{-20}}$$

$$\therefore R_2 = 2.28 \times 10^8 km$$

GRAVITATIONAL FIELD

Is the space around the body in which any other mass experiences a force of attraction?

Consider an isolated body of mass M as shown in figure below



If a small test mass Mo is placed near it (say at point P) the mass attraction. The gravitational field of M exerts a force on Mo placed at point P.

Theoretically, the gravitational field due to a material body extends up to infinity.

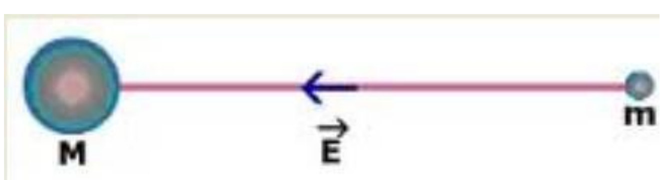
However, the effect of gravitation field decreases as the distance from the body is increased.

GRAVITATIONAL FIELD STRENGTH

Gravitational field strength (intensity of Gravitational field) is the force per unit mass acting on a test mass placed at that point.
It is a vector quantity. It is denoted by E and has SI unit of m/s^2 or N/Kg.

-It is also known as intensity of Gravitational field.

Consider an isolated body of mass M as shown in figure



The mass M sets up gravitational field in the space surrounding it.

If a small test mass Mo placed at point P experiences an attractive force F, then gravitational field strength at point P is

$$E = \frac{F}{M_0}$$

Intensity of gravitationally field E at point is just the acceleration that a unit mass would experience when placed at that point.

Its SI unit is that of acceleration that is M/s^2 or N/kg
The direction of E is that in which the test mass Mo would move under the influences of the field.

Dimensional formula of E

$$E = \frac{F}{M_0} = \frac{[MLT^{-2}]}{[M]}$$

$$\therefore E = [M^0LT^{-2}]$$

Intensity of gravitational field is directed towards the centre of gravity of the body.

Clearly, for a spherical body it will be directed towards the Geometric centre.

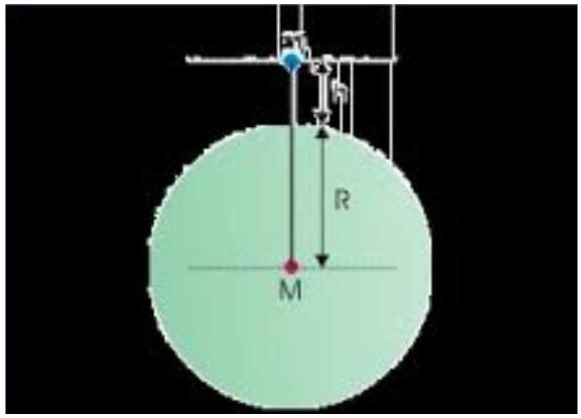
The test mass should be small that its gravitational field does not affect the gravitational field of the body under consider ration.

INTENSITY OF GRAVITATION FIELD DUE TO EARTH

The intensity of earth's gravitational field at a point is equal to the acceleration that a unit mass would experience when placed at that point.

Thus the intensity of earth's gravitational field on the surface of earth is $g = 9.8\ m/s^2$.

Consider earth to be a sphere of radius R and mass M as shown in figure below



Suppose we want to find the intensity of gravitational field due to earth at a point P located at a height h above the surface of earth.

If a test mass M_0 is placed at point P, then gravitational force exerted by earth on M_0 is F_G. $F_G = \frac{GMM_0}{(R+h)^2}$

By definition the magnitude of the intensity of gravitational field due to earth at point P

$$E = \frac{F_G}{M_0}$$

$$E = \frac{GMM_0}{(R+h)^2} \times \frac{1}{M_0}$$

$$E = \frac{GM}{(R+h)^2}$$

The direction of E is along PO i.e. gravitational field due to earth is directed toward the centre of the earth.

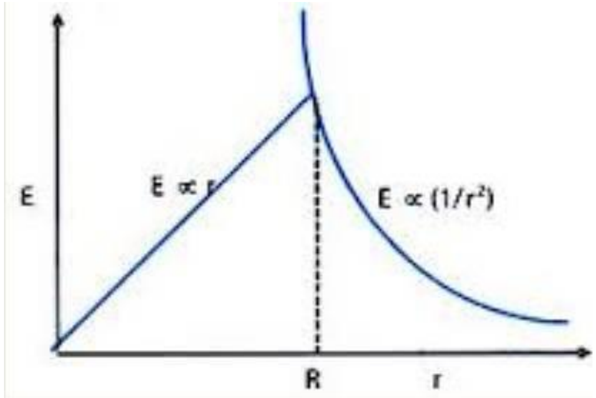
We have seen that acceleration due to earth's gravity g^1 (g is acceleration due to gravity on earth surface) at any point P at a height h above the surface of earth is

$$g' = \frac{GM}{(R+h)^2}$$

Then

$$E = g' = \frac{GM}{(R+h)^2}$$

Figure below shows the graph between gravitational field strength of earth versus distance from the center of the earth



Now

$$g' = E = \frac{GM}{(R+h)^2}$$

As $h \to \infty;\ g' \to 0$. Clearly the intensity of earth's gravitational field is zero at infinity.

Acceleration due to gravity is zero at infinity.

GRAVITATIONAL POTENTIAL ENERGY

Is the amount of work done in bringing the body from infinity to that point?

Because work is obtained (not done) in bringing the body from infinity to the desired point, the gravitational potential energy is always **NEGATIVE.**

It was proved that gravitational potential energy of a body of mass M raised through a height h is given as;

$$Gravitation\ P.E = mgh$$

This expression might for gravitational potential energy is an approximation based on the assumption that g is constant, an assumption that is approximately valid near the surface of earth.

If we wish to determine the gravitational potential energy of an object that is far removed from the earth's surface, the equation above is in Adequate.

In that equation, we arbitrarily assume that gravitational potential energy of a body is zero at the earth's surface.

Since the gravitational force is attractive, the gravitational potential energy will be zero at infinity.

In other words, gravitational potential energy of a body near the earth surface is Negative and become less negative (Greater) as it moves away from the earth.

The maximum potential energy is zero, the value obtained at infinity distance from the earth's centre.

Consider the earth to a sphere of radius R and mass M. supposed a body of mass m is situated outside the earth at point A at a distance r from the centre of earth.

It is desired to find the potential energy of the body at point A.

OA = r

By definition, the gravitational potential energy U_A of the body at point A is equal to work done in bringing the body from infinity to point A.

Suppose at any instant the body is at point B at a distance x from the centre of the earth.

OB = x

The gravitational force exerted by earth on the body at B is

$$F = {GMm}/{x^2}$$

Small amount of work done when body moves from B to C,

$$dw = Fdx$$

Where

$$BC = dx$$

$$dw = \frac{GMm}{x^2}dx$$

Total work done by the gravitational force when the body of mass moves from infinity to point is

$$W = \int_{\infty}^{r} dw$$

$$W = \int_{\infty}^{r} \frac{GMm}{x^2} dx$$

$$W = GMm \int_{\infty}^{r} \frac{dx}{x^2}$$

$$W = GMm \int_{\infty}^{r} x^{-2} dx$$

$$W = GMm \left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$W = GMm \left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$W = -GMm \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = -\frac{GMm}{r}$$

Therefore, gravitational potential energy UA of a body of mass m at a distance r from the centre of the earth is

$$U_A = -{GMm}/{r}$$

i. The above equation reveals that a body's gravitational potential energy is Negative near the earth's surface and becomes less negative as it moves away from the earth.

The maximum gravitational potential energy of a body is zero at r = ∞.

This is in agreement without previous observation that objects decreases have move gravitational potential energy as they move away from the earth.

ii. Since the force between the body and earth is attractive, an external agent must do positive work to increase the separation of the body from the earth.

The external work done increases the potential energy as the body is taken away from the earth.

iii. The gravitational potential of a body of mass M at a distance r1 from the centre of earth is

The gravitational potential energy of the body at a distance r_2 from the centre of the earth ($r_2 > r_1$)

$$U_2 = -\frac{GMm}{r_2}$$

Change in potential energy is

$$\Delta P.E = U_2 - U_1$$

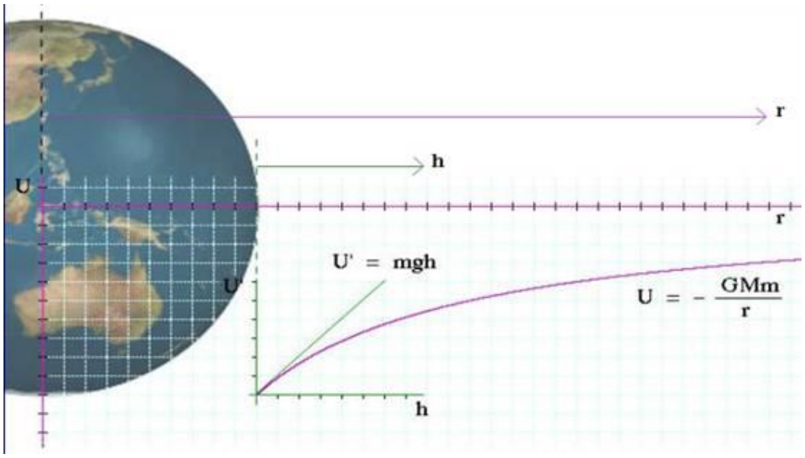
$$= -\frac{GMm}{r_2} - \left[\frac{GMm}{r_1}\right]$$

$$\Delta P.E = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Since $r_1 < r_2$, the change in potential energy is positive. It means that if the body is moved away from the earth, the gravitational potential energy of the body increases.

GRAVITATIONAL P.E NEAR EARTH'S SURFACE

Consider a body of mass M at rest on the surface of earth at point A in figure below.



The gravitational potential energy of the body at the surface of earth is

$$U_A = -\frac{GMm}{R}$$

We the lift it to height h at point B above the surface of the earth. The gravitational potential energy of the body now is

$$U_B = -\frac{GMm}{(R+h)}$$

Increase in potential energy of the body is

$$\Delta P.E = U_B - U_A$$

$$P.E = \left(-\frac{GMm}{(R+h)}\right) - \left[-\frac{GMm}{R}\right]$$

$$\Delta P.E = \left[\frac{GMm}{R}\right] - \left[\frac{GMm}{(R+h)}\right]$$

$$\Delta P.E = \frac{GMm(R+h)}{R(R+h)} - GMmR$$

$$\Delta P.E = \frac{GM\,mR + GMmh - GMmR}{R(R+h)}$$

$$\Delta P.E = GMm\left[\frac{h}{R(R+h)}\right] - - - - - - - - (i)$$

If R>>> h i.e. body is close the surface of earth h can be neglected as compared to R

Therefore, the above expression becomes

$$\Delta P.E = GMm\left[\frac{h}{R(R+0)}\right]$$

$$\Delta P.E = \frac{GMmh}{R^2}$$

$$\Delta P.E = \frac{GMmh}{R^2}$$

$$\Delta P.E = \frac{GM}{R^2}.mh$$

Recall;

$$g = \frac{GM}{R^2}$$

Therefore, the above expression becomes

$$\Delta P.E = mgh - - - - - - - - - - - (ii)$$

Thus for changes in position near earth's surface equations (i) and (ii) both give the same answer.

For motion over distances that are not small compared with the earth's radius, you must use eqn (i)

GRAVITATIONAL POTENTIAL

Is the amount of work done in bringing a body of unit mass from infinity (where the potential is zero) to that point.

$$V = {}^{W}/_{m}$$

Where

V = Gravitational potential

W = Work done in bringing mass m from infinity to that point.

Gravitational potential is a scalar quantity and has SI units of J/Kg

The dimensional formula is [$M^0 L^2 T^{-2}$].

Suppose we want to find the gravitational potential at a point P. outside the earth at a distance r from the centre of the earth.

We know that the amount of work done in bringing a body of mass M from infinity to point P is

$$W = -\frac{GMm}{r}$$

Gravitational potential at point P in earth's gravitational field

$$V_p = -\frac{GMm}{r}.\frac{1}{m}$$

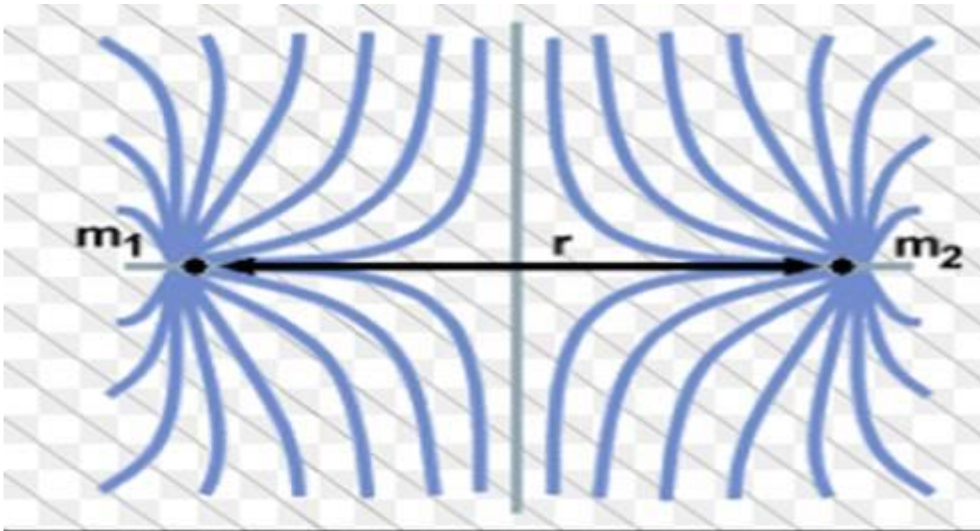
$$V_p = -\frac{GM}{r}$$

Gravitational potential is always negative, it is zero at infinity.

It follows that a body at infinity would fall towards the earth a body on the earth does not fall to infinity.

RELATION BETWEEN GRAVITATIONAL FIELD AND POTENTIAL

Suppose that a particle of mass m is moved by a force F from A to B in the gravitational field.



Suppose that AB = where is so small that F can be consider constant between A and B

Work done in going from A to B is

$$\Delta W = F\Delta r$$

But,

$$E = -\frac{F}{m}$$

The minus sign is necessary because E and F are oppositely directed

$$\Delta W = Em\Delta r$$

Also

$$\Delta V = \frac{\Delta W}{m}$$

$$\Delta V = -\frac{Em\Delta r}{m}$$

$$\frac{\Delta V}{\Delta r} = E$$

In the limit $$E = -\frac{dV}{dr}$$

The quantity dV/dr is called potential gradient i.e. rate of change of gravitational potential with distance.

ROTATION OF RIGID BODIES

Linear inertia

Is the tendency of a body to resist change in its linear velocity.

In other words; objects do not change their state of linear motion unless acted upon by some not external force.

Rotational inertia

Is the tendency of a body to resist change in its angular velocity.

- In other words, objects do not change their rotational motion unless acted upon by some not external torque.
- It is also called moment of inertia.

CONCEPT OF MOMENT OF INERTIA

Moment of inertia of a body about an axis is a measure of the difficulty in starting, stopping or changing rotation of the body about that axis.

- It is denoted by I
- The greater the difficulty in starting or stopping , the greater is the moment of inertia of the body about that axis and vice-versa.
- A body rotates under the action of a net external torque.

The Greater the moment of inertia of a body about an axis of rotation, the greater is the torque required to rotate or stop or change rotation of the body that axis and vice-versa.

MOMENT OF INERTIA OF A RIGID BODY

Consider a rigid body rotating about the axis yy^{-1} with an angular speed ω as shown in figure 1 below.

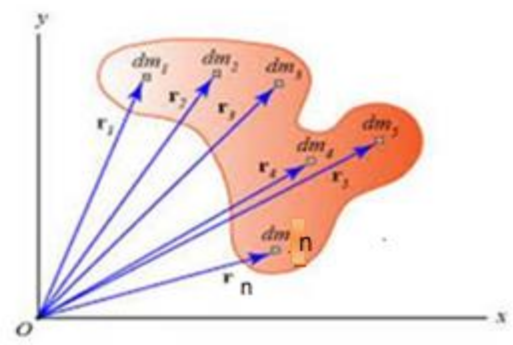


Figure. 1

Suppose the body is made up of a large number (n) of small particles of masses m_1,m_2,m_3 ………..m_n situated at perpendicular distances r_1, r_2, r_3…………..r_n respectively from the axis of rotation yy'.

As the body rotates, each particle of the body follows a circular path around the axis.

Although each particle of the body has the same angular speed ω, the linear velocity (v) of each particle depends upon particles distance from the axis of rotation.

Thus particle of mass m_1 follows a circular path of radius r_1. The linear velocity of this particle is v_1

$$V_1 = \omega r_1$$

Rotational kinetic energy of the particles of mass m_1

$$R.K.E = \frac{1}{2} m_1 V_1^2$$

$$R.K.E = \frac{1}{2}m_1\omega^2 r_1^2$$

$$\therefore R.K.E = \frac{1}{2}m_1 r_1^2\omega^2$$

The total kinetic energy K_r of the rotating body is the sum of the kinetic energies of all the particles of which the body is composed.

$$K_r = \frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega^2 + \frac{1}{2}m_3 r_3^2\omega^2 ----$$

$$K_r = \frac{1}{2}\omega^2\left[\sum_{i=1}^{i=n} m_i r_i^2\right]$$

$m_i = mass\ of\ i^{th}\ particle$ and $r_i = its\ perpendicular\ distance\ from\ axis\ of\ ro$

$$\sum_{i=1}^{i=n} m_i r_i^2 = moment\ of\ inertia\ I$$

Hence the total K.E of rotating body

$$K_r = \frac{1}{2}I\omega^2$$

Moment of inertia of a rigid body about a given axis of rotation

Is the sum of the products of the masses of its particles and the squares of their respective perpendicular distance from the axis of rotation.

$$I = \sum_{i=1}^{i=n} m_i r_i^2$$

The moment of inertia of a body about an axis of rotation is directly proportional to the total mass of the body.

The more massive the body, the more difficult will be to start its rotational motion or stop it from rotating.

For a given mass, the moment of inertia of a body depends upon the distribution of the mass from the axis of rotation. The larger the distance of the mass from the axis rotation the larger will be its moment of inertia. The moment of inertia plays the same role in rotational motion as mass plays in translational motion.

RADIUS OF GYRATION

Radius of gyration Is the distance from the given axis of rotation at which if whole mass of the object were supposed to be concentrated the moment of inertia would be the same as with the actual distribution of mass.

The radius of gyration is denoted by the symbol K.

T The moment of inertia of a body of mass M and radius of gyration K is given by,

$$I = MK^2$$

For example, the moment of inertia of a thin rod of mass M and length L about an axis through its centre and perpendicular to its length is

$$I = \frac{ML^2}{12}$$

$$\frac{ML^2}{12} = MK^2$$

$$K = \frac{L}{\sqrt{12}}$$

The radius of gyration is a measure of the distribution of mass of a body relative to a given axis of rotation.

A large radius of gyration means that, on the average, the mass is relatively far from the given axis of rotation.

The SI unit of radius of gyration is the metre (m)

Suppose a body consist of n particles each of mass m, then total mass of the body is M.

$$M = mn$$

$$I = \sum mr^2 = m[r_1^2 + r_2^2 - - - - + r_n^2]$$

$$I = MK^2$$

$$MK^2 = mn\left[\frac{r_1^2 + r_2^2 - - - - - - + r_n^2}{n}\right]$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 - - - - - - + r_n^2}{n}}$$

Therefore r**adius of gyration** is the root mean square distance of the various particles of the body from the axis of rotation.

EQUATIONS OR UNIFORMLY ACCELERATED ROTATIONAL MOTION

Consider a rigid body rotating about a given axis with uniform angular acceleration.

Let

$\omega_0 =$ Initial angular velocity.

$\omega =$ Final angular velocity after time t.

$\propto =$ Uniform angular acceleration.

$\theta =$ Angular displacement after time t.

i) To derive $\omega = \omega_0 + \propto t$

From

$$\propto = \frac{d\omega}{dt}$$

$$d\omega = \propto dt$$

At $t = 0, \omega = \omega_0 \; and \; at \; t = t, \omega = \omega$

$$\int_{\omega_0}^{\omega} d\omega = \int_{0}^{t} \propto dt$$

$$[\omega]_{\omega_0}^{\omega} = [t]_0^t . \propto$$

$$\omega - \omega_0 = \propto t$$

$$\omega = \omega_0 + \propto t$$

For linear motion

v = u + at

ii) To derive $\theta = \omega_0 t + \frac{1}{2} \propto t^2$

From $\frac{d\theta}{dt} = \omega$

$$d\theta = \omega dt$$

At $t = 0, \theta = 0, at \; t = t, \theta = \theta$

$$\int_0^\theta d\theta = \int_0^t \omega dt$$

$$\int_0^\theta d\theta = \int_0^t (\omega_0 + \propto t) dt$$

$$\int_0^\theta d\theta = \int_0^t \omega_0 dt + \int_0^t \propto t dt$$

$$[\theta]_0^\theta = \omega_0 [t]_0^t + \propto \left[\frac{t^2}{2}\right]_0^t$$

$$\theta = \omega_0 t + \frac{1}{2} \propto t^2$$

For the linear motion

$$s = ut + \frac{1}{2} at^2$$

iii) To derive $\omega^2 = \omega_0^2 + 2 \propto \theta$

$$\omega = {d\theta}/{dt}$$

From

$$\propto = {d\omega}/{dt}$$

Also

$$\propto = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\propto = \omega \frac{d\omega}{d\theta}$$

$$\omega d\omega = \propto d\theta$$

When $\theta = 0, \omega = \omega_0$ *and when* $\theta = \theta, \omega = \omega$

$$\int_{\omega_0}^{\omega} \omega d\omega = \int_{0}^{\theta} \propto d\theta$$

$$\left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \propto [\theta]_0^{\theta}$$

$$\frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \propto \theta$$

$$\omega^2 - \omega_0^2 = 2 \propto \theta$$

$$\omega^2 = \omega_0^2 + 2 \propto \theta$$

For linear motion

$v^2 = u^2 + 2as$

MOTION OF A RIGID BODY AND MOMENT OF INERTIA

Consider a rigid body rotating about the axis yy' with an angular speed ω as shown in figure.2 below

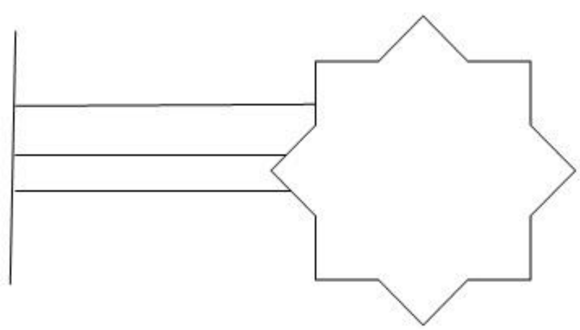


Figure. 2

Suppose the body is made up of a large number of particles of masses m_1, m_2,m_3 situated at perpendicular distances r_1, r_2, r_3……… respectively from the axis of rotation.

As the body rotates, each particle within the body follows a circular path around the axis. Although each particle within the body has the same angular speed ω, the velocity of each particle depends upon particles position with respect to axis of rotation.

1. Relation between rotational kinetic energy and moment of inertia.

When a rigid body rotates about an axis, possesses K.E called the kinetic energy of a rotating body, and its rotational kinetic energy denoted by (K_r).

Let us now find the K.E of a rotating body.

The particle of mass m_1 follows a circular path of radius r_1.

The magnitude of the linear or tangential velocity of the particle on this circle is v_1

$$K_r = \frac{1}{2} m_1 v_1^2$$

$$K_r = \frac{1}{2} m_1 r_1^2 \omega^2$$

Similarly, the rotation kinetic energy of particles of masses $m_2, m_3 \dots\dots\dots\dots\dots\ are\ \frac{1}{2} m_2 r_2^2 \omega^2, \frac{1}{2} m_3 r_3^2 \omega^2 \dots\dots\dots\dots respectively$

The rotational kinetic energy K_r of the body is equal to the sum of rotational kinetic energies of all particles

$$K_r = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 \dots\dots$$

$$= \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \dots\dots\dots)\omega^2$$

$$K_r = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

But, $\sum m_i r_i^2$ is the moment of inertia of the body about the given axis of rotation

$$K_r = \frac{1}{2} I \omega^2$$

Thus the rotational K.E of a body is equal to half of the product of the moment of inertia of the body and the square of the angular velocity of the body about a given axis of rotation.

We also know that kinetic energy K of a body for linear motion is,

$$K = \frac{1}{2} mv^2$$

i) Note that both expressions for K.E are one half the products of property of body and square of velocity term.
We know that linear velocity v is an analogue of angular velocity ω in rotational motion
Therefore, mass (m) of the body is an analogue of moment of inertia I of the body in rotation motion.
Hence we arrive at a very important conclusion that moment of inertia plays the same role in rotational as the mass (m) plays in linear motion.

ii) If $\omega = 1$

From $K_r = \frac{1}{2} I\omega^2$

$$K_r = \frac{I}{2}$$

$$I = 2K_r$$

Thus, the moment of inertia of a rigid body about a given axis of rotation is numerically equal to twice the rotational K.E of the body when rotating with unit angular velocity about that axis.

2. Relation between torque and moment of inertia

Suppose a body rotates about an axis under the action of a constant to torque τ .

Let the constant angular acceleration produced by the body be ∝.
The angular acceleration (∝) of all the particles of the body will be the same but the linear acceleration (a) of each particle will depend upon the particles position with respect to the axis of rotation.
The particle of mass m_1 follows a circular path of radius r_1. The magnitude of the linear acceleration of this particle $a_1 = r_1 \propto$.

If F_1 is the net external force acting on this particle

$F_1 = m_1 a_1$

$F_1 = m_1(r_1 \propto)$

$F_1 = m_1 r_1 \propto$

The magnitude of torque due to this force on this particle

$\tau_1 = F_1 r_1$

$\tau_1 = m_1 r_1 \propto . r_1$

$\tau_1 = m_1 r_1^2 \propto$

Similarly the magnitude of torque on the particles of masses m_2, m_3 .... are $m_2 r_2^2 \propto$, $m_3 r_3^2 \propto$........... respectively.

By right hand rule, the torques on all the particles act in the same direction. The magnitude of the total torque τ on the body is just the sum of individual torques on the particles.

$torque\ on\ the\ body = \tau$

$\tau = m_1 r_1^2 \propto + m_2 r_2^2 \propto + m_3 r_3^2 \alpha \dots \dots$

$\tau = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \dots \dots.) \propto$

$\tau = \left(\sum m_i r_i^2\right) \propto$

But $\sum m_i r_i^2$ is the moment of inertia of the body about the axis of rotation

$\tau = I \propto$

□□□ This is basic relation for rotational motion

$\tau = I \propto$

It is analogous to Newton's second law of linear motion

$F = ma$

The torque is analogous to force F, the moment of inertia I is analogous to mass m and the angular acceleration ∝ is analogous to the linear acceleration

If $\propto = 1$

$\tau = I \propto$

$\tau = I$

Hence the moment of inertia of a body about a given axis is equal to the torque required to produce unit angular acceleration in the body about that axis.

3. Relation between angular momentum and moment of inertia

The particle of mass m_1 follows a circular path of radius r_1 about the axis of rotation.

Therefore, the magnitude v_1 of linear velocity of this particle is

$$V_1 = r_1 \omega$$

The magnitude of the angular momentum of this particle about the axis of rotation

= linear momentum x radius r_1

= $m_1 v_1 \times r_1$

Similarly, the magnitude of angular momentum of particles of masses m_2,m_3 .... are $m_2 r_2^2 \omega$, $m_3 r_3^2 \omega$ .... respectively.

By right hand rule, the angular momentum of all the particles L of the rigid body point in the same direction, parallel to the angular velocity vector.

Therefore, the magnitude of total angular momentum L of the body about the axis of rotation is just the sum of individual momentum of the particles.

$$L = m_1 r_1^2\omega + m_2 r_2^2\omega + m_{31} r_3^2\omega + ...$$

$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_{32})\omega$$

$$= (\sum m_1 r_1^2)\,\omega$$

But $\sum m_1 r_1^2$ is the moment of the body about the axis of rotation

$$L = I\omega$$

The SI units of L is kgm^2s^{-1}

4 .Relation between torque and angular momentum.

The magnitude of angular momentum of a body about an axis is

$$L = I\omega$$

Differentiating both sides with respect to t we get

$$\frac{dL}{dt} = I\frac{d\omega}{dt}$$

$$\propto = \frac{d\omega}{dt}$$

$$\frac{dL}{dt} = I \propto$$

But Torque $\tau = I \propto$

Then

$$\tau = \frac{dL}{dt}$$

Thus the torque acting on a body is equal to time rate of change of angular momentum of the body.

Power in Rotational Motion

Consider a rigid body rotating about an axis due to a constant applied $torque\ \tau$.

If the body rotates through a small change angle $\Delta\theta$ in small time Δt then small work ΔW done on the body is

$$\Delta W = \tau\Delta\theta$$

$$power = \frac{\Delta\omega}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t}$$

$$power\ P = \tau\omega$$

$$P = \tau\omega$$

Thus power of a rotating body is equal to the product of torque and angular velocity.

Work-energy theorem in rotational motion

In linear motion, the work-energy theorem states that the net work done on a body by the external force is equal to the change in body's linear kinetic energy.

Similar, in rotational motion, the work done by a torque in rotating the body about an axis is equal to the change in body's rotation kinetic energy.

Therefore, work-energy theorem in rotational motion may be stated as under "The net work-done by the external torque in rotating a rigid body about an axis is equal to the change in body's rotational K.E".

$$Work\ done\ on\ body = change\ in\ body's\ rotational\ K.E$$

$$Workdone\ on\ body = \frac{1}{2}I\omega_F^2 - \frac{1}{2}I\omega_0^2$$

LAW OF CONSERVATION OF ANGULAR MOMENTUM

If not net external torque acts on a body or system, its angular momentum remains constant in magnitude and direction.

$$I\omega = constant \qquad If \sum \tau_{ext} = 0$$

It is very important to keep in mind that it is $I\omega$ - the product of moment of inertia and angular velocity that remains constant and not the angular velocity ω.

$$I_1\omega_1 = I_2\omega_2 = constant$$

MOMENT OF INERTIA AND RADIUS OF GYRATION OF DIFFERENT BODIES WITH REGULAR GEOMETRIC SHAPES

1. Moment of inertia of a uniform rod
   (a) About an axis through its centre
   Consider a uniform rod of total mass M and length L
   Imagine, a small infinitesimally element of mass "dm" and length "dx" at a distance "x" from the axis of rotation passing through the middle of the uniform rod.

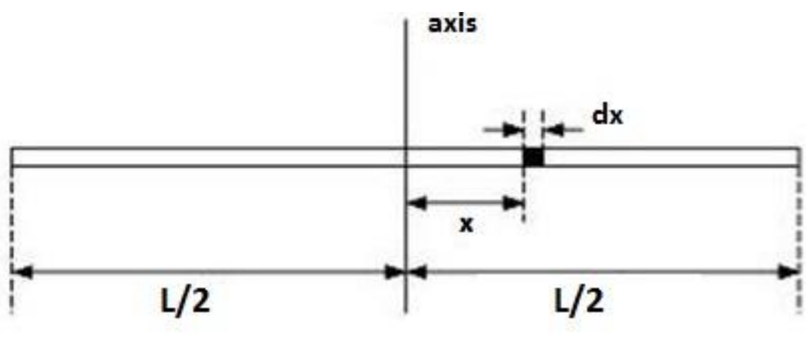


Figure. 3

By similarity

$$\frac{dm}{m} = \frac{dx}{L}$$

$$dm = m.\frac{dx}{L}$$

The moment of inertia of the imaginary small infinitesimally element a distance x from the axis of rotation is given by

$$dI = \left[m.\frac{dx}{L}\right]x^2$$

The total moment of inertia for the whole uniform rod is obtained by integrating the above expression

$$\int_{O}^{\frac{I}{2}} dI = \int_{O}^{\frac{L}{2}} \frac{m}{L}.x^2 dx$$

$$I_{I/2} = \frac{m}{L}\int_{0}^{L/2} x^2 dx$$

$$I_{1/2} = \frac{m}{L}\left[\frac{X^3}{3}\right]_0^{\frac{L}{2}}$$

$$I_{\frac{I}{2}} = \frac{m}{L}\left[\frac{\left({}^{L}/_{2}\right)^3}{3} - \frac{0}{3}\right]$$

$$I_{\frac{I}{2}} = \frac{m}{L}\left[\frac{L^3}{24} - 0\right]$$

$$I_{\frac{l}{2}} = \frac{mL^2}{24}$$

Total moment of inertia for the whole rod is

$$I = 2I_{l/_2}$$

$$I = 2 \times \frac{ML^2}{24}$$

$$I = \frac{ML^2}{12}$$

Its Radius of gyration

$$I = MK^2$$

$$K^2 = \frac{I}{M}$$

$$K = \sqrt{\frac{I}{M}}$$

But is the moment of inertia of a uniform rod about an axis through its centre

$$I = \frac{ML^2}{12}$$

Then

$$K = \sqrt{\frac{ML^2}{12M}}$$

$$K = \sqrt{\frac{L^2}{12}}$$

$$K = \sqrt{\frac{1}{12}}.L$$

$$K = \frac{L}{\sqrt{12}}$$

$$K = 0.2887L$$

(b) Moment of inertia about an axis at one end

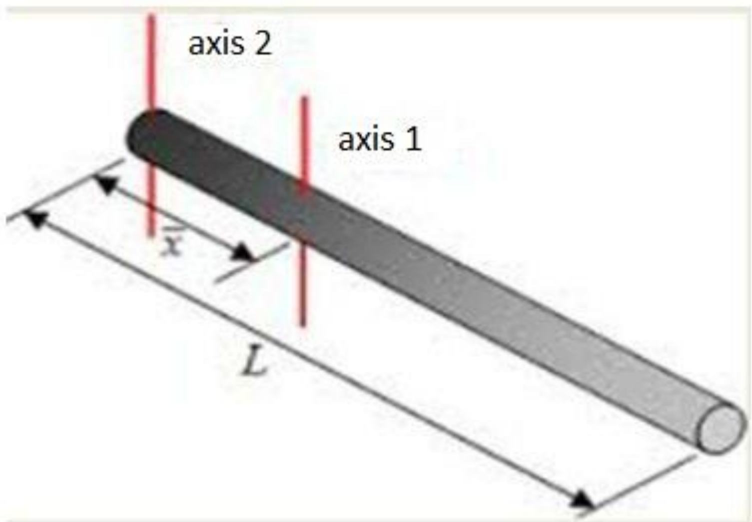


Figure 4

By similarity

$$\frac{dm}{M} = \frac{dx}{L}$$

$$dm = M.\frac{dx}{L}$$

Recall

$dI = dmx^2$

$$dI = \left[M.\frac{dx}{L}\right]x^2$$

$$dI = \frac{M}{L}.x^2dx$$

By integrating both sides

$$\int_0^I dI = \int_0^L \frac{M}{L}.x^2 dx$$

$$[I]_0^I = \frac{M}{L}\int_0^L x^2 dx$$

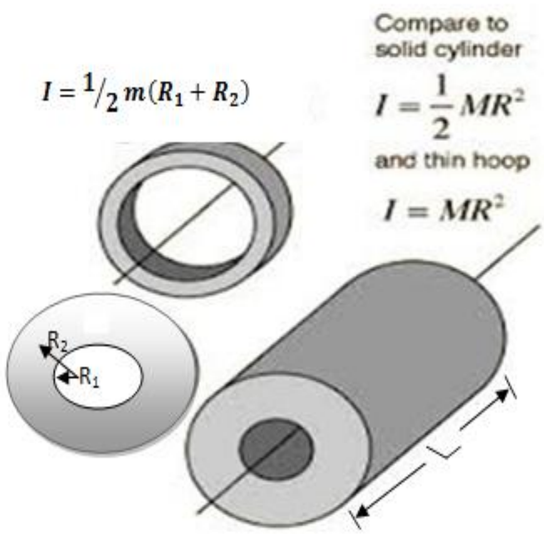
$$I = \frac{M}{L}\left[\frac{x^3}{3}\right]_0^L$$

$$I = \frac{M}{L}.\frac{L^3}{3}$$

$$I = \frac{ML^2}{3}$$

2. Moment of inertia of hollow Solid cylinder (ring)

Consider a hollow solid cylinder of uniform density is related about an axis passing through the center along the length L of the cylinder.



Consider a small infinitesimal element (shell) at a distance *r* and having thickness, dr

then dV = area x thickness

$= 2\pi rl \times dr$

$dV = 2\pi rldr$

The mass of shell is given by

$dm = \rho.dV$

$dm = \rho.2\pi lrdr$

The moment of inertia for a shell

$dI = dmr^2$

$dI = \rho.2\pi lr^3dr$

$dI = \rho.2\pi lr^3dr$

$I = \int dI$

$$I = \int_{R_1}^{R_2} 2\pi l \rho r^3 dr$$

$$I = 2\pi l \rho \int_{R_1}^{R_2} r^3 dr$$

$$I = 2\pi l \rho \left[{r^4}/{4} \right]_{R_1}^{R_2}$$

$$I = 2\pi l \rho \left[{R_2^4}/{4} - {R_1^4}/{4} \right]$$

$$I = \frac{2\pi l \rho}{4}(R_2^4 - R_1^4)$$

$$I = \frac{2\pi l \rho}{4}[(R_2^2)^2 - (R_1^2)^2]$$

$$I = \frac{2\pi l \rho}{4}[(R_2^2 + R_1^2)(R_2^2 - R_1^2)]$$

But the mass of the cylinder is given

$$M = \pi l \rho (R_2^2 - R_1^2)$$

Then

$$I = \frac{1}{2} M (R_2^2 + R_1^2)$$

Hence for hollow cylinder (ring) the moment of inertia is given by

$$I = \frac{1}{2} M (R_2^2 + R_1^2)$$

But if the thickness of the cylinder is very small

$$R = R_1 = R_2$$

$$I = \frac{1}{2} M (R_2^2 + R_1^2)$$

$$I = \frac{1}{2}M(R_2^2 + R_2^2)$$

$$I = \frac{1}{2}M(2R^2)$$

$I = MR^2$ is the moment of inertia for thin hoop.

Its radius of gyration

From

$$I = MK^2$$

$MR_2 = MK_2$

$K_2 = R_2$

Therefore K = R

3. Moment inertia of a solid cylinder (Disc)

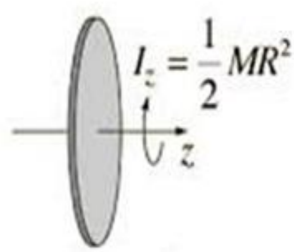


Figure. 6

Area at any radius x from the centre of the circle

$$A = \pi x^2$$

Differentiate

$dA = 2\pi x dx$

By similarity

$\frac{dm}{dA} = \frac{M}{A}$

dm = $\frac{M}{A} . dA$

dm $= 2\pi x \frac{M}{A}\ dx$

$But\ A = \pi R^2$

dm $= 2\pi x \frac{M}{\pi R^2}\ dx$

dm $= \frac{2M}{R^2}\ x dx$

dm $= \frac{2M}{R^2}\ . x dx$

But dI = x^2dm

$$\int_0^I dI = \int_0^R \frac{2M}{R^2}\ x^3 dx$$

$$I = \frac{2M}{R^2} \int_0^R x^3 dx$$

$$I = \frac{2M}{R^2} \left[\frac{x^4}{4}\right]_0^R$$

$$I = \frac{2M}{R^2} \cdot \frac{R^4}{4}$$

$$I = \frac{1}{2} MR^2$$

Its radius of gyration

From

$$I = MK^2$$

$$\frac{1}{2} MR^2 = MK^2$$

$$K^2 = \frac{R^2}{2}$$

$$\therefore K = \frac{R}{\sqrt{2}}$$

4. Moment of inertia of a sphere

Consider a spherical rigid body and a particle at a distance dr from the centre

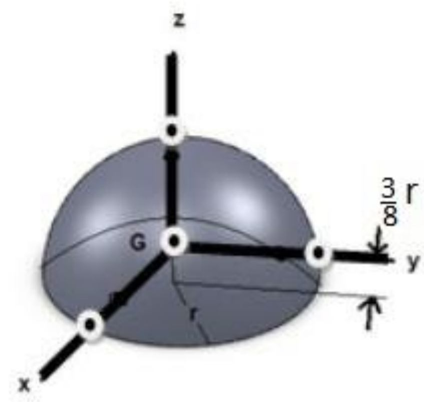


Figure. 7

Volume of a small element

$$dV = \frac{4}{3}\pi r^3 - dr$$

Mass of a sphere

$$dm = \rho.dV$$

$$= \rho.\frac{4}{3}\pi r^3 dr$$

$$dm = \frac{4}{3}\pi\rho r^2 dV$$

Moment of inertia of a shell

$$dI_{1/2} = dM.r^2$$

$$dI_{1/2} = \frac{4}{3}\pi\rho r^2 drV^2$$

$$I = \frac{2}{5}MR^2$$

PARALLEL AXES THEOREM

The parallel axes theorem state that moment of inertia of a rigid body about any given axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.

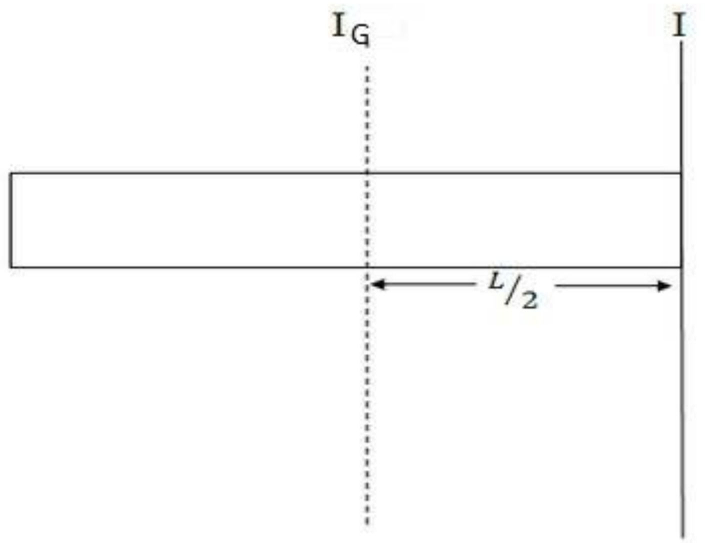
$I = I_G + Mh^2$

This is a very simple and useful theorem for objects of regular shapes e.g. rod, sphere, disc.

Application of parallel axes theorem

1. For a uniform rod the moment of inertia about an axis through the centre is $I_G = MK^2/12$

Therefore the moment of inertia about an axis through one end can be obtained



From

$$I = I_G + Mh^2$$

$$I = \frac{ML^2}{12} + M\left({}^{L}/_{2}\right)^2$$

$$I = {}^{4ML^2}/_{12}$$

$$I = {}^{ML^2}/_{3}$$

Moment of inertia of a sphere of radius R and mass M about an axis through the point on its circumference can be obtained.

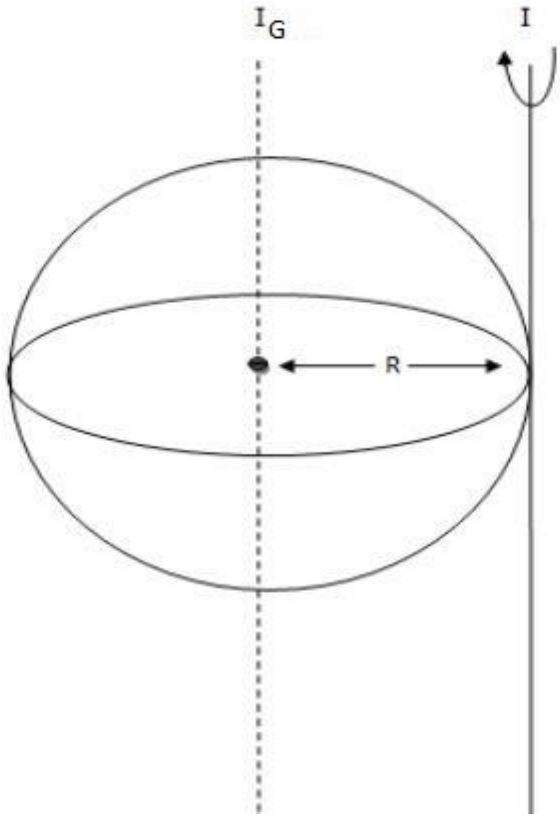


Figure 9

$I_G = \frac{2MR^2}{5}$

From parallel axes theorem

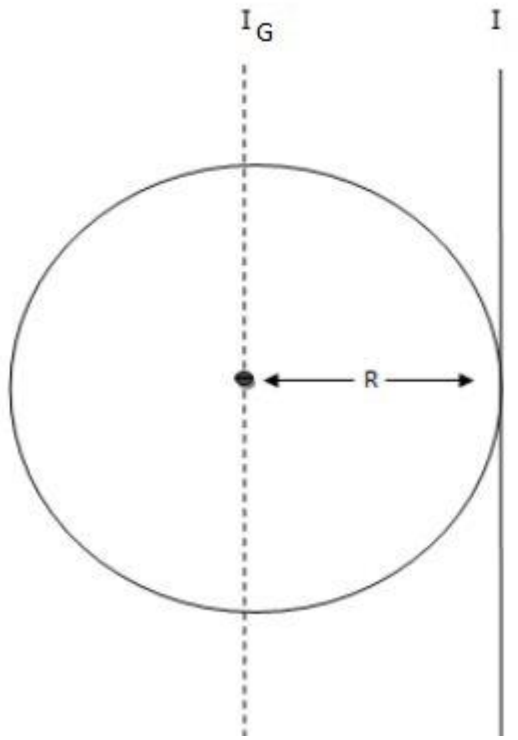
$I = I_G + Mh^2$ h = R

$$I = \frac{2MR^2}{5} + MR^2$$

$$I = \frac{2MR^2 + 5\,MR^2}{5}$$

$$I = \frac{7MR^2}{5}$$

2. 3 . Moment of inertia of a disc of radius r and mass m about an axis through a point on the circumference can be obtained.



$$I_G = \frac{2MR^2}{5}$$

From parallel axes theorem

$$I = I_G + Mh^2$$

$$I = \frac{1}{2}MR^2 + MR^2 \quad \text{But } h = R$$

$$I = \frac{MR^2 + 2MR^2}{2}$$

$$I = \frac{3MR^2}{2}$$

PERPENDICULAR AXES THEOREM

The perpendicular axis theorem states that "the moment of inertia of plane body about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the plane body about any two mutually perpendicular axes in the plane of the body which intersect the first axis"

$$I_z = I + I_y$$

COMBINED ROTATIONAL AND TRANSLATIONAL MOTION

Consider a wheel rolling on a flat surface without slipping. Each particle of the wheel is undergoing two types of motion at the same time.

The centre of the wheel, which is wheel's centre of mass, is moving horizontally with speed V_G.

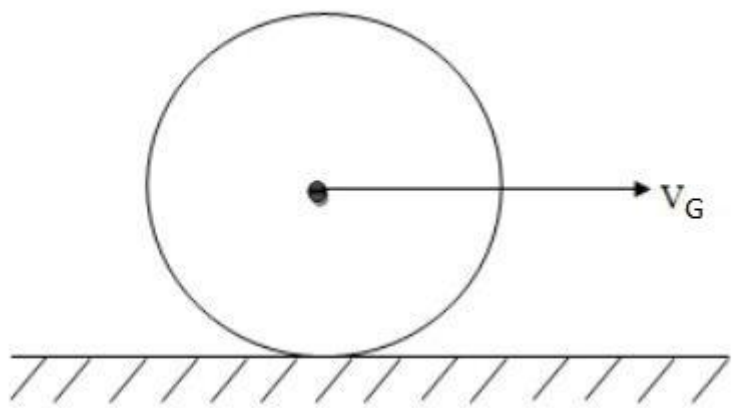


Figure 10 (a)

At the same time, the wheel is rotating about its centre of mass with angular speed ω.

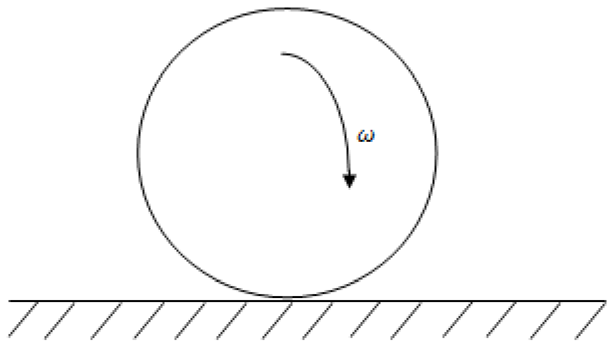


Figure 10 (b)

Thus rolling is the superposition of two motions

(i) Translation of objects centre of mass at velocity V_G along a straight line

(ii) Rotation about the center of mass at angular velocity ω.

Therefore the wheel possesses both translational and rotational K.E.

$$K.E_{\text{total}} = K_t + K_t$$

$$K.E_r = \frac{1}{2} M.\ v_G^2 \quad + \frac{1}{2} I\omega^2$$

M = Mass of the object

V_G = Velocity of object's centre of mass

I = Moment of inertia of the object about an axis through the centre of mass.

The total K.E of an object undergoing both translational and rotational motion about its centre of mass axis is equal to the sum of translation K.E of the centre of mass and the rotational K.E about the centre of mass.

ROLLING OBJECTS

Consider a rigid body rolling without slipping down the plane inclined at an angle θ with the horizontal.

Required to determine the velocity, acceleration and minimum required coefficient of static friction (µs) for rolling.

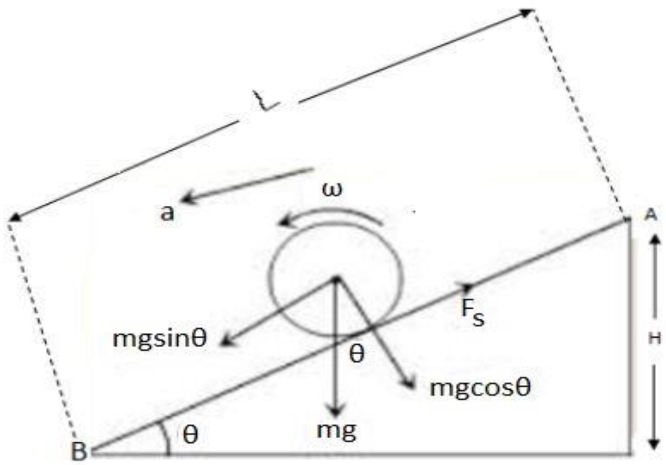


Figure. 11

By the conservation of energy we have

Total energy at A = total energy at B E .

E_A = E_B

$K.E_A$ +

$P.E_A$ =

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

1. Rolling a Hollow Sphere

$I = MR^2$

From

$$MgH = \frac{1}{2}Mv^2 \quad + \frac{1}{2}I\omega^2$$

$$MgH = \frac{1}{2}Mv^2 \quad + \frac{1}{2}MR^2\omega^2$$

But $v^2 = R^2 \omega^2$

Then

MgH = $\underline{1}$ $MR^2 \omega^2$ + $\underline{1}$ $MR^2 \omega^2$

2 2

$MgH = MR^2 \omega^2$

$gH = MR^2$

$v^2 = gH$

$$V = \sqrt{gH}$$

Is the velocity of a hollow sphere

From the figure

$$\sin \theta = \frac{H}{L}$$

$$H = L \sin \theta$$

$$v = \sqrt{gL \sin \theta}$$

For Acceleration

From the 3rd equation of motion

$v^2 = u^2 + 2aL$

Assuming the body started from rest

$u = 0$

Then

$v^2 = 2aL$

Also

$v^2 = gH$

$2aL = gH$

$$a = \frac{gH}{2L}$$

Is the acceleration of hollow sphere

$$\sin\theta = \frac{H}{L}$$

Then from

$$a = \frac{gH}{2L}$$

$$a = \frac{1}{2} gSin\theta$$

2.

ROLLING A SOLID SPHERE

From

$$MgH = \frac{1}{2} Mv^2 \quad + \frac{1}{2} I\omega^2$$

But $I = \frac{2}{5} MR^2$

$$MgH = \frac{1}{2} Mv^2 \quad + \frac{MR^2\omega^2}{5}$$

$v^2 = R^2 \omega^2$

$$MgH = \frac{1}{2} MR^2 \ \omega^2 + \frac{MR^2\omega^2}{5}$$

$$MgH = \frac{5MR^2\omega^2 + 2MR^2\omega^2}{10}$$

$$MgH = \frac{7MR^2\omega^2}{10}$$

$$gH = \frac{7R^2\omega^2}{10}$$

$$gH = \frac{7V^2}{10} \ since \ v^2 = R^2\omega^2$$

$7V^2 = 10gH$

$V^2 = \frac{10gH}{7}$

$$v = \sqrt{\frac{10}{7}gH}$$

$$\text{Sin } \theta = \frac{H}{L}$$

$$H = L \sin \theta$$

Then

$$v = \sqrt{\frac{10}{7}gH}$$

$$v = \sqrt{\frac{10}{7}Lg\text{Sin}\theta}$$

Is the velocity of solid sphere

For acceleration

From,

$v^2 = u2 + 2aL$

But u = 0

$v^2 = 2aL$

$\frac{10}{7} gH = 2aL$

$a = \frac{5}{7} \cdot \frac{gH}{L}$

$$\text{But } \sin\theta = {}^{H}/_{L}$$

$$a = \frac{5}{7} g\sin\theta$$

3. **ROLLING OF A SOLID CYLINDER**

From

$$MgH = \frac{1}{2} Mv^2 + \frac{1}{2} M.\ I\omega^2$$

$I = \frac{1}{2} MR^2$

$$MgH = \frac{1}{2} Mv^2 + \frac{1}{4} MR^2\omega^2$$

$v^2 = R^2\omega^2$

$$MgH = \frac{1}{2} MR^2W^2 + \frac{1}{4} MR^2\omega^2$$

$$MgH = \frac{2MR^2\omega^2 + MR^2\omega^2}{4}$$

$$MgH = \frac{3MR^2\omega^2}{4}$$

$$gH = \frac{3}{4}R^2\omega^2$$

$$gh = \frac{3}{4}\ v^2$$

$$v^2 = \frac{4gH}{3}$$

$$v = \sqrt{\frac{4gH}{3}}$$

Is the velocity of solid cylinder.

But $\quad Sin\,\theta = \frac{H}{L}$

$$H = LSin\theta$$

$$v = \sqrt{\frac{4}{3}gLSin\theta}$$

For acceleration

From,

$v^2 = u^2 + 2aL$

$u = 0$

$v^2 = 2aL$

$$\frac{4}{3}gH = 2aL$$

$$a = \frac{2}{3}g\frac{H}{L}$$

But,

$$Sin\theta = \frac{H}{L}$$

$$a = \frac{2}{3}gSin\theta$$

Is a acceleration of solid cylinder.

ALTERNATIVELY

Consider a solid cylinder of radius *R* and mass *M* rolling down a plane inclined at an angle θ to the horizontal.

$$MgSin\theta - F_s = Ma \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (i)$$

$$R = MgCos\theta$$

For rotational motion about the centre of mass, the only force that produce the torque is·

Torque on cylinder

$$\tau = F_s R \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (ii)$$

If I is the moment of inertia and α is the angular acceleration about the axis of rotation, then torque on the cylinder is FS

$$\tau = I\alpha \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (iii)$$

Equating equation (ii) and (iii)

$$F_s R = I\alpha$$

$$F_s = \frac{I\alpha}{R}$$

Since the cylinder rolls without slipping

$$a = R\alpha$$

$$\alpha = \frac{a}{R}$$

Then

$$F_s = \frac{Ia}{R^2} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (iv)$$

Putting the value of FS into equation (i)

$$MgSin\theta - F_s = Ma$$

$$MgSin\theta - \frac{Ia}{R^2} = Ma$$

$$MgSin\theta = Ma + \frac{Ia}{R^2}$$

$$a = \frac{MgSin\theta}{M + \frac{I}{R^2}}$$

For a solid cylinder, the moment of inertia about its symmetry axis is

$$I = \frac{1}{2}MR^2$$

$$a = \frac{MgSin\theta}{M + \frac{MR^2}{2R^2}}$$

$$a = \frac{2}{3}gSin\theta$$

All uniform rolling solid cylinders have the same linear acceleration down the incline irrespective of their masses and radii.

The linear acceleration is just 2/3 as large as it would be if the cylinder could slide without friction down the slope

$$a = gSin\theta$$

Minimum μ_s Required for Rolling

From

$$F_s = \frac{Ia}{R^2}$$

$$I = \frac{1}{2}MR^2, \qquad a = \frac{2}{3}gSin\theta$$

$$F_s = \frac{1}{2}MR^2 \bullet \frac{2}{3}gSin\theta \bullet \frac{1}{R^2}$$

$$F_s = \frac{1}{3}MgSin\theta$$

$$\mu s = \frac{F_s}{R}$$

$$\mu s = \frac{1}{3}MgSin\theta \bullet \frac{1}{MgCos\theta}$$

$$\mu_s = \frac{1}{3}\tan\theta$$

1.The frictional force is static, the cylinder rolls without slipping and there is no relative motion between the cylinder and the inclined plane at the point of contact.

2. If there were no friction between the cylinder and the incline, the cylinder would have slipped instead of rolled.

MOTION OF A MASS TIED TO THE STRING WOUND ON A CYLINDER

Consider a solid cylinder of radius *R* and mass *M* capable of rotating freely without friction about its symmetry axis.

Suppose a string of negligible mass is wrapped around the cylinder and a heavy point mass *m* is suspended from the free end.

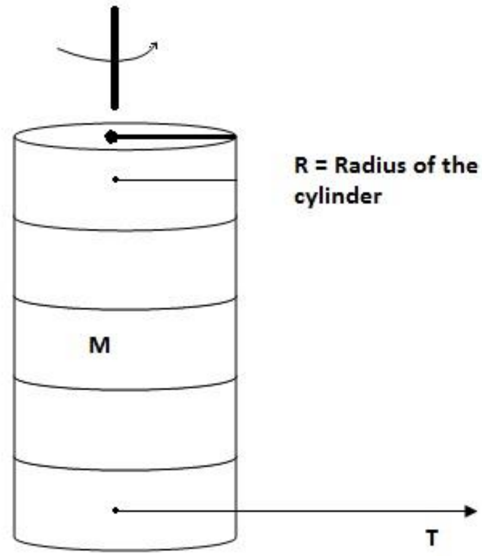


Figure 12 (a)

When the point mass m is released, the cylinder rotates about its axis and at the same time the mass falls down due to gravity and unwinds the string wound on the cylinder

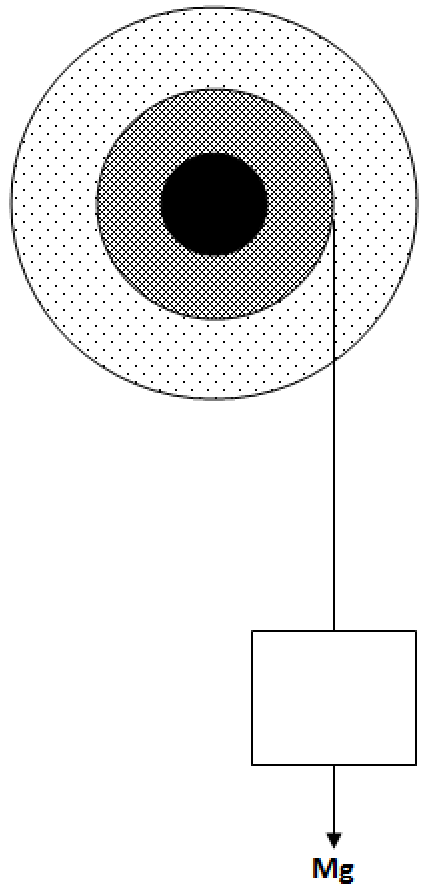
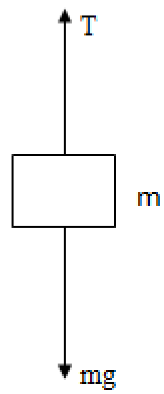


Figure 12 (b)

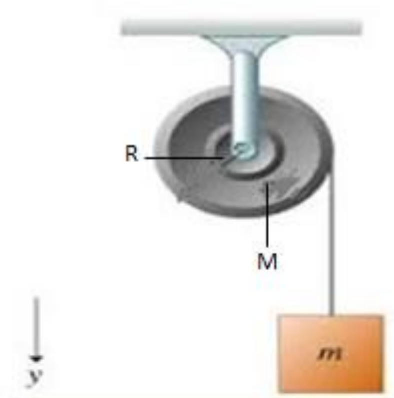
Linear Acceleration of Point Mass (m)

We first examine the forces of the point mass m as shown below



mg – T = ma……………………(i)

Looking at the cylinder end; we see that tension in the string exerts a torque on the cylinder



$\tau = TR$ ________________ (ii)

If I is the moment of inertia of the cylinder about the axis of rotation and α is the angular acceleration produced in the cylinder.

$\tau = I \propto$ ………………………(iii)

By equating equations (ii) and (iii)

$$RT = I\alpha$$

$$T = \frac{I\alpha}{R}$$

Since the string unwinds without slipping the linear acceleration *a* of the point mass *m* and the angular acceleration α of the cylinder are related as

$$a = R\alpha$$

$$T = \frac{I\alpha}{R}$$

$$T = \frac{I}{R} \bullet \frac{a}{R}$$

$$T = \frac{Ia}{R^2} \text{———————————} (iv)$$

Putting the value of T into equation (i)

$$\mathrm{mg} - \mathrm{T} = \mathrm{ma}$$

$$\mathrm{mg} - \frac{Ta}{R^2} = ma$$

$$\mathrm{mg} = \mathrm{ma} + \frac{Ia}{R^2}$$

$$\mathrm{a}\left[m + \frac{I}{R^2}\right] = \mathrm{mg}$$

$$a = \frac{mg}{m + \frac{I}{R^2}}$$

$$a = mg \times \frac{R^2}{MR^2 + I}$$

$$a = \frac{mR^2}{MR^2 + I}$$

$$T = \frac{Ia}{R^2}$$

$$T = \frac{I}{R^2}\frac{mgR^2}{mR^2 + I}$$

$$T = \frac{mgI^2}{mR^2 + I}$$

1. The tension T in the string is always less than mg.
2. As moment of inertia increases T approaches mg.

NEWTON'S LAWS OF ROTATIONAL MOTION

1. **First law**

   States that "everybody continues in its state of rest or of uniform rotational motion about an axis until it is compelled by some external torque to change that state".

2. **Second law**

   States that "the rate of change of angular momentum about an axis is directly proportional to the impressed external torque and the change angular momentum takes place in the direction of the applied torque".

$$\tau \propto \frac{I\,\omega_{2} - I\,\omega_{1}}{t}$$

$$\tau \propto \frac{I(\omega_{2} - \omega_{1})}{t}$$

$$\tau = k\,\frac{I(\omega_{2} - \omega_{1})}{t}$$

where k = 1.

$$\tau = I \propto$$

3. Third law

To every external torque applied, there is an equal and opposite restoring torque.

SIMPLE PENDULUM

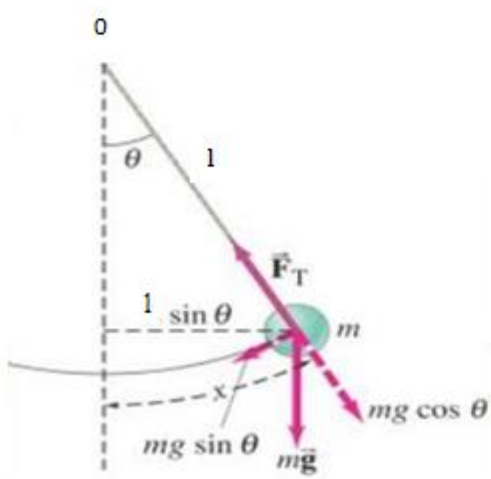


Figure. 13

It consists of a string of length l suspended from a fixed point O and carrying a bob of mass m at its free end.

When the bob is slightly displaced from the equilibrium position and then released, it executes rotational motion about an axis through the suspension point O.

Suppose at anytime t, the bob is at point *P* making an angle θ with the vertical let ω be the angular velocity and α be the angular acceleration at that instant.

The only force which exerts a torque on the bob is that due to the weight of the bob.

$$\tau = mgl\ \sin\theta \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (i)}$$

The negative sign is used since this is a restoring torque i.e. a torque which force the bob toward θ = 0°.

The angular momentum of the bob about the axis of rotation at the considered instant (at point P)

L = mvr

v = ωr

L = mωr.r

$L = m\omega r^2$

But r = lsinθ where θ = 90º

Therefore $L = m\omega l^2$

$$\frac{dL}{dt} = \frac{d(m\omega\ell^2)}{dt}$$

$$\frac{dL}{dt} = m\ell^2\ \frac{d\omega}{dt}$$

$$\therefore \ \tau = \frac{dL}{dt} = m\ell^2 \ \frac{d\omega}{dt}$$

$$\tau = \ m\ell^2 \frac{dL}{dt} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (ii)}$$

From equation (i) and (ii) we have

$$m\ell^2 \ \frac{d\omega}{dt} = mg \sin\theta$$

$$\frac{d\omega}{dt} = \ -\frac{g}{\ell} \ \sin\theta$$

$$\frac{d\omega}{dt} = \alpha$$

Now , the angular acceleration of the bob at time *t*

$$\alpha = -\frac{g}{\ell} Sin\theta$$

This governs the oscillation of simple pendulum,in the vertical plane.

(i) When θ = 0º

$$\alpha = -gSin\theta$$

$$\alpha = -\frac{g}{\ell} Sin\, 0^{0}$$

$$\alpha = 0$$

Therefore, the bob has zero angular acceleration at the equilibrium position.

(ii) The negative sign shows that the tangential acceleration is directed toward the mean position.

(iii) If θ is small, Sin $\theta = \theta$

$$a = -\frac{g}{\ell} Sin\theta$$

$$a = -\frac{g}{\ell}\theta$$

(iv) Time period

$$\omega^2 = \frac{g}{\ell}$$

$$\frac{4\pi^2}{T^2} = \frac{g}{\ell}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Also

From

$$\tau = -mg\ell\sin\theta$$

$$Sin\theta = \theta$$

$$\tau = -mg\ell\sin\theta$$

$$Ia = -mg\ell\theta$$

$$a = \frac{d^2\theta}{dt^2}$$

$$I\frac{d^2\theta}{dt^2} = mg\ell\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{mg\theta}{I}$$

$$\frac{d^2\theta}{dt^2} = \left[-\frac{Mg\ell}{I}\right]\theta$$

For S.H.M

$$a = \omega^2 y$$
$$\alpha = \omega^2\theta$$

$$\omega^2 = \frac{mg\ell}{I}$$

$$\frac{4\pi^2}{T^2} = \frac{mg\ell}{I}$$

-

$$T = 2\pi\sqrt{\frac{I}{mg\ell}}$$

From

Parallel axis theorem

$$I = I_G + m\ell^2$$
$$I_G = mK^2$$

$$I = mK^2 + m\ell^2$$

$$I = m\,(K^2 + \ell^2)$$

$$\frac{I}{m} = K^2 + \ell^2$$

Then

$$T = 2\pi\sqrt{\frac{I}{mg\ell}}$$

$$T = 2\pi\sqrt{\frac{K^2 + \ell^2}{g\ell}}$$

If $\frac{K^2 + \ell^2}{\ell} = L$

Then $T = 2\pi\sqrt{\frac{L}{g}}$

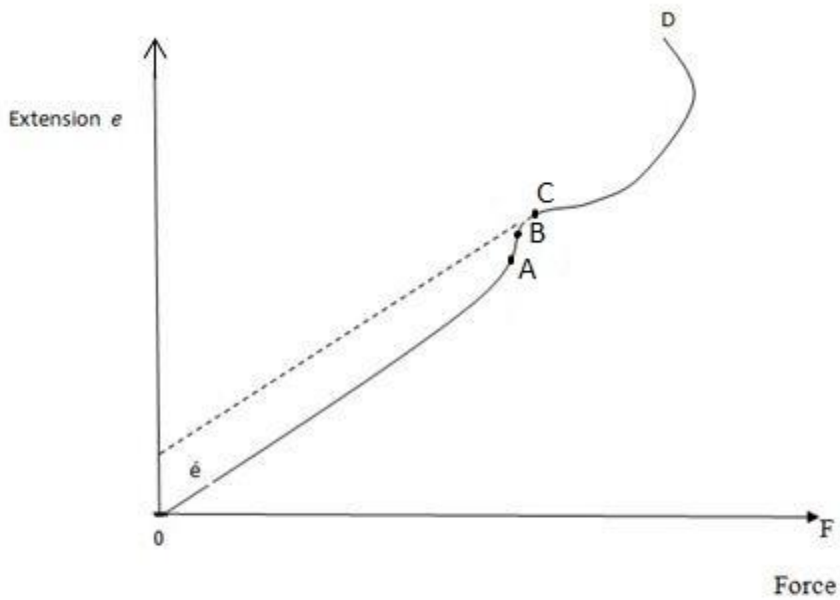
STRENGTH OF MATERIALS

Young's Modules of Elasticity Y= $\frac{Stress}{Strain}$

Y = $\frac{\sigma}{\epsilon}$ = $\frac{\frac{F}{A}}{\frac{e}{e}}$

A tensile or longitudinal force is a force which stretches or increases the length of a material

Fig 1: The graph of extension against weight or Force.



Key of the graphs

A = Proportionality limit

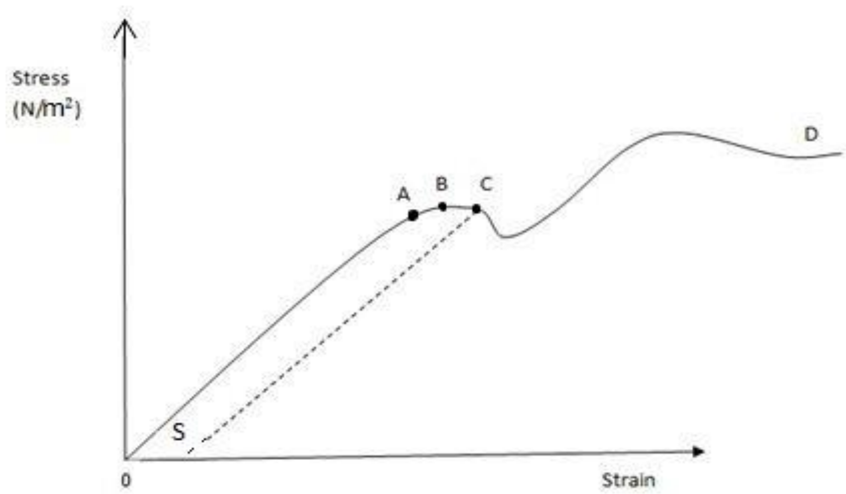
B = Elastic limit

C = Yield point

D = The material becomes thin and breaks breaking point

é = Permanent extension

Fig 2: The graph of stress against strain



S = Permanent strain.

0A = Hooke's law obeyed.

0B = Elastic deformation region.

0C = Region of plastic deformation.

Features of the graphs

- The proportionality limit is the maximum stress beyond which Hooke's law is not obeyed or the ratio of stress/strain is no longer constant

- 0A: Hooke's law is obeyed i.e. F = KX.

  Stress α Strain i.e. stress/strain = constant.

- Elastic limit is the maximum stress before which the material returns to its original shape and size.

- The yield point is the point beyond which the material changes from elastic behavior to plastic behavior.

- Plastic behavior is a situation where a force may cause a large increase in length unexpectedly and it will not return to its original shape or size.

- Energy stored in a strained wire suppose a wire of length l_0 is stretched by a force F.

Work done = Average force x distance

$= \frac{1}{2}$ Fe in Joules as SI unit

This is the elastic energy stored in the wire

$\therefore$ Energy = $\frac{1}{2}$ Fe

But, Y = $\frac{Stress}{Strain}$ = Y = $\frac{\frac{F}{A}}{\frac{e}{l_0}}$

F = $\frac{YeA}{l_0}$

Energy = $\frac{1}{2}$ Fe = $\frac{1}{2} \cdot \frac{YeA}{L_0}$ e

Energy = $\frac{1}{2}\frac{YA}{L_0}e^2$

The energy per unit volume of a wire $= \frac{1}{2}\frac{\frac{YAe^2}{l_0}}{Al_0}$

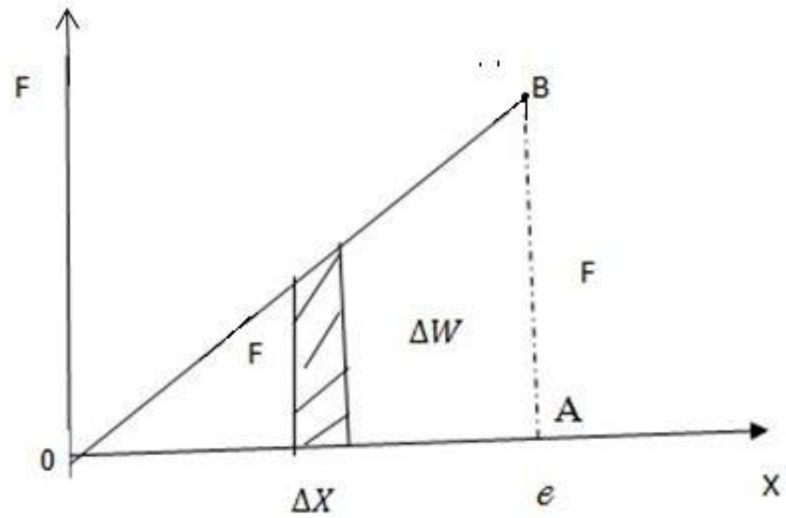
$= \frac{1}{2}\frac{YAe^2}{Al_0^2} = \frac{1}{2} Y \frac{e^2}{l_0^2}$

$= \frac{1}{2}(\frac{F}{A} \cdot \frac{l_0}{e}).\frac{e^2}{l_0^2} = \frac{1}{2} \cdot \frac{F}{A} \cdot \frac{e}{l_0}$

$= \frac{1}{2}$ x Stress x strain

$\therefore$ Energy per unit volume = $\frac{1}{2}$ x Stress x Strain

Fig 3: Graph of F V_s â„¯ and Energy if elastic limit is not exceed



Energy stored = work done = F x Δx

Total work done = Area under the curve

= $\frac{1}{2}$ x 0A x AB

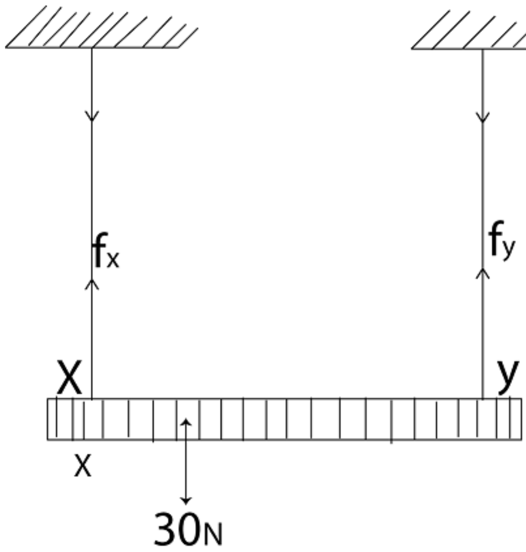
= $\frac{1}{2}$ x e x F

Energy $= \frac{1}{2} Fe$; If Hooke's law is obeyed

This agrees with the earlier derived equation.

Plastic behavior is exhibited when the material is permanently strained after the force is removed. The energy is converted to heat.

Example 1: Two vertical wires X and Y suspend at the same horizontal level are converted by a light rod XY at the lower ends as shown. The wires have the same length l and cross-sectional area A. A weight of 30N is placed at O on the rod, Where $xo: oy = 1: 2$. Both wires are stretched and the rod XY then remains horizontal



If wire X has young's modulus 1.0 x 10N/M[2] calculate young's modulus for wire Y assuming that the elastic limit is not exceed for both wires (N&P).

Solution

Since XY is horizontal e_x = e_y = e

Let F_x and F_y be the forces in X and Y

Taking moments

About point X: 30 x 1 = F_y x 3

$F_y = \frac{30}{3} = 10N$

About point Y: 30 x 2 = F_x x 3

$F_x = \frac{60}{3} = 20N$

For wire X, $Y_x = \frac{\frac{F_x}{A}}{\frac{e_x}{l_x}} = \frac{F_x l_x}{e_x A_x}$

$e_x = \frac{20l}{1.0 \times 10^{11} \times A}$

For wire Y: $e_y = \frac{F_y l_y}{Y_y \times A_y} = \frac{10l}{Y_y\ A}$

$e_x = e_y = \frac{20l}{1.0 \times 10^{11} \times A} = \frac{10l}{Y_y\ A}$

$20 Y_y = 1.0 \times 10^{11} \times 10$

$Y_y = \frac{10 \times 10^{11}}{20}$

$Y_y = 5 \times 10^{10}$ NM^{-2}

Example 2:

A rubber cord of a catapult has a cross-sectional area of 2mm^2 and an initial length of 0.20m and is stretched to 0.24m to fire a small stone of 10g.Calculate the initial velocity of the object when it just leaves the catapult(Assume Young's modulus for rubber is b x 10^8 pa and the elastic limit is not exceeded).

Solution

K.E of the fire stone =Energy stored in stretched rubber

$\frac{1}{2} m v^2 = \frac{1}{2} Fe$

$V = \sqrt{\frac{Fe}{m}}$

Where F is given by Y= F is given by Y = $\frac{F}{A} . \frac{l}{e}$

$F = \frac{YAe}{l}$

$F = \frac{6 x 10^8 x (2 x 10^{-6} x (0.24-0.20)}{0.20}$

$F = \frac{6 x 2 x 0.04 x 10^2}{0.20} = 240N$

V = $\sqrt{\frac{Fe}{m}}$ = $\sqrt{\frac{240 \, x \, 0.04}{10 \, x \, 10^{-3}}}$

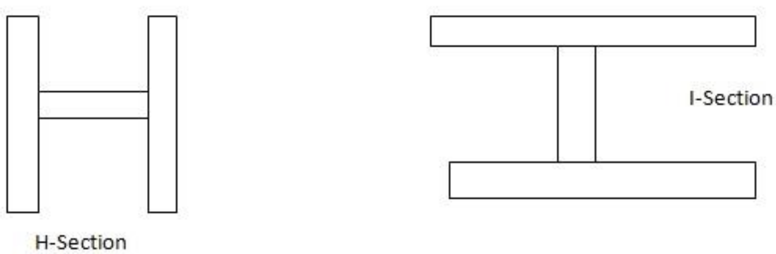
V = $\sqrt{240 \, x \, 4}$ = 31.0ms^{-1}

Applications of elasticity

1. In the design of structures the engineer has to ensure that the stresses to be applied to the structure do not exceed the elastic limit of the structure

2. Steel has very high strength (high modulus of elasticity) so it is used in construction of girders spring etc. It can withstand very high stresses

3. In construction I and H cross section beams are made using steel bars in such a way that the beams can withstand large stresses



Example 3:

A steel wire of radius 5 $\times 10^{-4}m$ is fixed at the ceiling.Find the maximum weight which can be hung from it, if it is allowed strain is 10^{-3} and young's modulus is 2.0 x 10^{11} pa

From Y= $\frac{\frac{F}{A}}{\frac{e}{l}}$

Where Y = 2.0 x 10^{11} pa, $\frac{e}{l}$ = 10^{-3} and

A = πr^2 = π $\times$ (5$\times 10^{-4}$)^2 = 2.5π $\times 10^{-7}m^2$

$F = Y \times \frac{e}{l} \times A$

$F = 2 \times 10^{11} \times 10^{-3}$

F = 15.71 x 10

F = 157.1N

This force corresponds to the maximum stress of $\sigma = \frac{F}{A} = \frac{157.1}{2.5\pi X 10^{-7}}$

$= 2 \times 10^{8}$ pa or N/m^{-2}

Example 4:

The breaking stress of steel is 8.0×10^{9} Nm^{-2}. Find the greatest length of the steel wire that can hang vertically without breaking if its density 9.9×10^{3} kg m^{-3}

The wire can break under its weight

Breaking weight of the wire
$W = A \times l_0 \rho g$

$= A l_0 \rho g$

$= A l_0 \times 9.9 \times 10^{3} \times 9.8$

$= 9.702 \times 10^{4} A l_0$

Breaking force of wire:
Stress = $\frac{F}{A}$

F = Stress x A

$F = 8 \times 10^{9} \times A$

Now, W = F

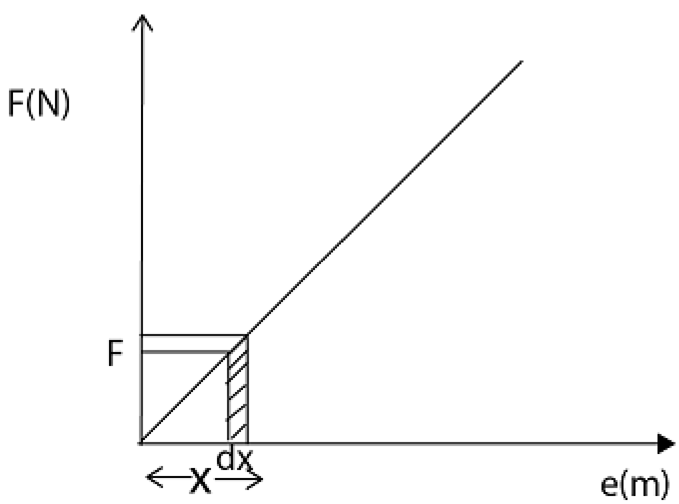
9.702 X 10^4 A l_0 = 8 x 10^9 A

$l_0 = \frac{8 \, x \, 10^9}{9.702 \, x \, 10^4}$

l_0 = 0.8245723 x 10^5

l_0 = 8.246 x 10^4m

Energy stored in a strained wire = to the area under the graph of force F against the extension X



Area of F the element

$\delta A = F\delta x$

A = $\int_0^x kxdx$

= $\int_0^x kxdx$ = $\frac{kx^2}{2}$

= $\frac{1}{2}$ kx^2 = $\frac{1}{2}$ Fx

Now from Y = $\frac{Fl}{Ax} \Longrightarrow$ Y = $\frac{kx.l}{Ax}$

$\frac{YA}{l}$ = k

TYPES OF STRESS

Stress = $\frac{Force\ Applied\ on\ an\ area}{Area}$

SI unit of stress is Nm^{-2}

Dimensions of stress

$$[\sigma] = {[F]}/{[A]}$$

$$[\sigma] = {[MLT^{-2}]}/{[L^2]}$$

$$[\sigma] = [ML^{-1}T^{-2}]$$

There are two main types of stress

i. Normal stress is the one is which force (F) is applied normally to the area (A) and is defined as the normal force applied per unit area.

i.e. Normal Stress = $\frac{Normal\ Force}{Area} = \frac{F}{A}$

Normal stress is of two kinds, namely

a) Normal tensile stress = $\frac{F}{A}$

This stress causes an increase in length

DRAWING

b) Normal compressive stress = $\frac{F}{A}$

This stress causes a decrease in length

Drawing

ii. The Tangential stress is one due to the force applied parallel to the surface on a body which is equal to tangential force/parallel to force.

TYPES OF STRAIN

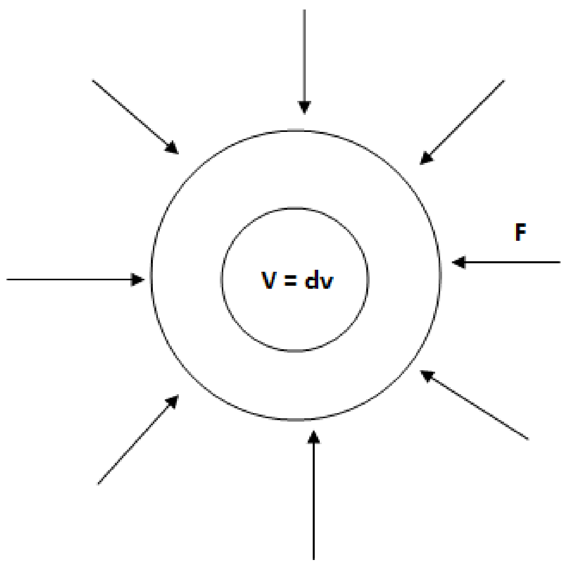
Strain = $\frac{\Delta D}{D}$ is unit less and dimensionless

There are three types of strain, namely Longitudinal, Volumetric and shearing strain

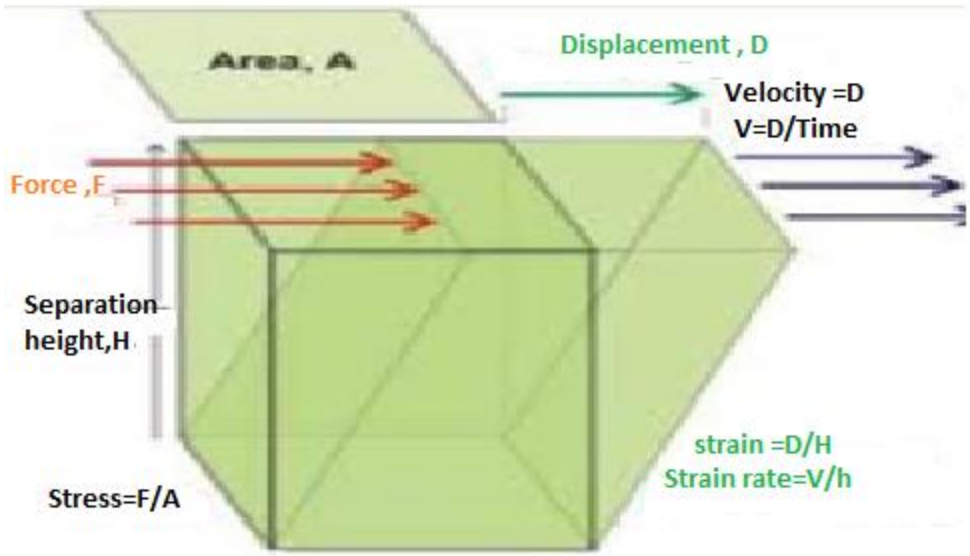
i) LONGITUDINAL STRAIN = $\frac{\Delta}{l} = \frac{e}{l}$

ii) BULK STRAIN is the ratio of the change in volume per unit original volume i.e

Bulk Strain = $\frac{\Delta V}{V}$



iii) **SHEARING STRAIN** is change in the shape of the body caused by a tangential force



Definition:

Shearing strain = $\theta = \tan\theta = \frac{AA'}{AF} = \frac{e}{l}$

$$\therefore \theta(rad) = {}^{e}/_{l}$$

θ = Angle of shear is the angle through which a vertical line is turned by a tangential force

TYPES OF MODULI OF ELASTICITY

There are three types of modulus of elasticity corresponding the three types of strain namely young's modules and shear modulus or modulus of rigidity

I. Young's Modulus of Elasticity

$$Y = \frac{Normal\ stress\ A}{Longitudinal\ strain}$$

Y = $\frac{\frac{F}{A}}{\frac{e}{l}}$ = Y = $\frac{Fl}{Ae}$

ii. The Bulk modulus of Elasticity K is defined as

K = $\frac{Normal\ stress}{Volumetric\ strain}$

= $\frac{\frac{F}{A}}{\frac{\Delta V}{V}}$

Now, $\frac{F}{A}$ = change in pressure Δp and $\frac{-\Delta V}{V}$ Volume decreases

∴ K = - $\frac{\Delta p}{\frac{\Delta V}{V}}$ in NM^{-2}

Or K = -V $\frac{\Delta P}{\Delta V}$ in NM^{-2}

Examples of moduli of elasticity

For steel K = 160 x 10^9 NM^{-2}

For water K = 2.2 x 10^9 NM^{-2}

For air at normal pressure K = 10^5 NM^{-2}

High value of K indicates the difficulty to change the volume.

COMPRESSIBILITY is a measure of how hard or how easy it is to compress the material. Compressibility is the reciprocal of the modulus of elasticity.

$$B = \frac{1}{K}$$, Its SI unit m^2N^{-1} or pa^{-1}

Example 1:

A spherical ball is subjected to a normal uniform pressure of 10^{10} N/M^2. If the bulk modulus of the material of the ball is 10^R NM^2, find the volume strain it suffers.

Solution:

$\Delta p = 10^{10}$ NM^{-2}, K = 10^{12} N M^{-2}

Using K = -v $\frac{\Delta p}{\Delta V}$ $\Rightarrow$ K = $\frac{\Delta p}{\frac{\Delta V}{V}}$

$10^{12} = \frac{10^{10}}{\frac{\Delta V}{V}}$

$\frac{\Delta V}{V} = \frac{10^{10}}{10^{12}} = 10^{-2}$

Example 2

The density of ocean water at the surface is 1030 kg/m^3. The bulk modulus water is 2.0 x 10^9 NM^{-2}. If the atmospheric pressure is 10^5 NM^{-2},what is the density of the ocean water at a depth where the pressure is 500 Atmospheres.By what % age is the water compressed?

Solution

Density $\rho' = \frac{m}{Volume}$

$\rho' = \frac{1030}{v'}$

V' = final volume

The mass of water is the same 1030kg

From K= -V$\frac{\Delta p}{\Delta V}$

V = 1m^3 at the surface

2.0 x 10^9 = - $\frac{V}{\Delta V}$. (500 – 1) x 10^5

2.0 x 10^9 = $\frac{1m^3 \ x \ 499 \ x \ 10^5}{\Delta V}$

ΔV = - $\frac{499 \times 10^5}{2.0 \times 10^9}$

ΔV = -2.495 x $10^{-2} m^3 \approx$ - 0.025 m^3

V' = V - ΔV = (1-0.02495) m^3

= 0.97505 m^3

Density at 500 Atm; $\rho' = {}^{m}/_{V'}$

= $\frac{1030}{0.97505}$

ρ= 1056 kg m^{-3}

% compression: $\frac{\Delta V}{V}$ x 100% = $\frac{0.025}{1.000}$ x 100%

= 2.5%

iii. SHEAR MODULUS OR MODULUS OF RIGIDITY

It is defined as the ratio of the tangential stress applied to the shear strain

i.e. Modulus of Rigidity $\eta = \frac{Tangential\ stress}{Shear\ strain}$

$\eta = \frac{\frac{F}{A}}{\frac{e}{l}}$

or $\eta = \frac{\frac{F}{A}}{\theta}$ where θ is in Rads

∴ The SI unit of η is Nm^{-2} **or** pa

-

Young's modulus for steel = 20 x 10^{10} Nm^{-2}

Modulus of rigidity for steel = 8 x 10^{10} Nm^{-2}

Generally the modulus of rigidity for solids is much smaller than young's modulus of elasticity.

The bulk modulus of elasticity of a gas if the change in the gas is isothermal.

PV= constant

Differentiating with respect to V gives,

P $\frac{dV}{dV}$ + V$\frac{dp}{dV}$ = 0

-V$\frac{dp}{dV}$ = + p

But -V$\frac{\Delta p}{\Delta V}$ = k, the bulk modulus

$\therefore$ k = p

$\therefore$ The isothermal Bulk modulus of a gas is equal to the pressure of the gas.

The Bulk modulus of elasticity of a gas if the change is adiabatic

pV^{γ} = Constant, k

Differentiating with respect to V

$p.\gamma V^{\gamma-1}$.$\frac{dV}{dV}$ + $V^{\gamma}\frac{dP}{dV}$ = 0

$\frac{\gamma P.V^{\gamma-1}}{V^{\gamma-1}}$ + $\frac{V^{\gamma}}{V^{\gamma-1}}$.$\frac{dP}{dV}$ = 0

$$\gamma p + v\,{}^{dp}/_{dv} = 0$$

$v \, {}^{dp}/_{dv} = \gamma p$

-

The adiabatic modulus of elasticity $k_r = \gamma p$

THE VELOCITY OF SOUND

The velocity of sound depends on the density and the modulus of elasticity. By dimensional analysis

V = k$\rho^a E^b$

$[M^0LT^{-1}]$ = $[ML^{-3}]^a[ML^{-1}T^{-2}]^b$

Equating indices gives

For T -1 = -2b $\Longrightarrow$ b = $\frac{1}{2}$

For M 0 = a + b $\Longrightarrow$ a =-b = a = -$\frac{1}{2}$

For L 1 = -3a – b $\Longrightarrow$ RHS = -3 x (-$\frac{1}{2}$) - $\frac{1}{2}$ = 1$\frac{1}{2}$ - $\frac{1}{2}$ = 1 = LHS

V = k$p^{\frac{-1}{2}} E^{\frac{1}{2}}$

Mathematical analysis shows that k=1

$\therefore$ V = $\sqrt{\frac{E}{\rho}}$

Now in gases sound propagates under adiabatic conditions where E = K_γ, the bulk modulus = γP

Velocity of sound in air is given by

V = $\sqrt{\frac{\gamma \rho}{\rho}}$

FLUID IN MOTION

HYDRODYNAMICS

Hydrodynamics is the branch of physics which deals with the study of properties of fluids in motion.

Viscosity of the fluid

Is the property of a moving fluid (liquid or gas) to oppose the relative motion between its layers.

Thus viscosity is that property of a fluid that indicates its internal friction.

The greater the viscosity of a fluid, the grater is the force required to cause one layer of fluid to slide past another.

For example:

The viscosity of honey is very large as compared to that of water. This means that for the same applied external force, the rate of flow of honey will be very small as compared to that of water.

The viscosity of a fluid not only retards its own motion but it also retards the motion of a solid through it.

The greater the viscosity of a fluid, the harder it is for a solid to move through it, imagine the difference between swimming in water and honey.

CAUSE OF VISCOSITY

Viscosity is the internal friction of a fluid which opposes the motion of one layer of fluid past another.

The forces of attraction between the molecules of a moving fluid determine the viscosity of the fluid.

Viscous force

Is the tangential force that tends to destroy the relative motion between different fluid layers.

Viscous Fluid

Is the fluid which offers a resistance to the motion through it of any solid body.

Non Viscous fluid

Is the fluid which does not offer a resistance to the motion through it of any solid.

Velocity Gradient

Is the change of velocity divided by the distance in a direction perpendicular to the velocity.

$$\text{Velocity gradient} = \frac{dv}{dx}$$

NEWTON'S LAW OF VISCOSITY

Newton's law of viscosity states that "the frictional force F between the layers is directly proportional to area A of the layers in contact and to the velocity gradient"

$$F \propto A$$
$$F \propto \frac{dv}{dx}$$
$$F = \eta A \frac{dv}{dx}$$

η is a constant of proportionality and is called coefficient of viscosity.

Note that the negative sign shows that the direction of viscous drag F is opposite to the direction of motion of the liquid.

From,

$$F = \eta A \frac{dv}{dx}$$

$$\text{If } A = 1 \text{ and } \frac{dv}{dx} = 1 \text{ then } \eta = F$$

The coefficient of viscosity is the tangential force required to maintain a unit velocity gradient between two parallel layers each of unit area.

or

Is the tangential force per unit area of a layer, required to maintain unit velocity gradient normal to the direction of flow.

Coefficient of viscosity η of the liquid is a measure of the degree to which the fluid exhibits viscous effects.

Since,

$$F = \eta A \frac{dv}{dx}$$

$$\eta = \frac{F}{A\, {dv}/{dx}}$$

$$\eta = \frac{F/A}{dv/dx}$$

Units of η

The SI units of F is 1N, The SI unit or A is 1m^2 and that of velocity gradient is $1ms^{-1}$ per m

SI unit of η is NSM^{-2}. It also called decapoise.

The coefficient of viscosity of a liquid is $1\ decapoise$ if a tangential force of 1N is required to maintain a velocity gradient of $1ms^{-1}/m$ between two parallel layers each of area $1m^2$.

η is also called Dynamic viscosity or Absolute Viscosity
The viscosity of an ideal liquid is zero.

Dimensional formula of η

$$\eta = F/_{A} \times velocity\ gradient$$

$$\eta = \frac{[MLT^{-2}]}{[L^2][LT^{-1}/_{L}]}$$

$$\eta = [ML^{-1}T^{-1}]$$

The coefficient of viscosity of a liquid decrease with the increases in temperature and vice versa. However, the coefficient of viscosity of gases increases with the increase in temperature.

Fluidity

Is a measure of re ability of a fluid of flow and is equal to the reciprocals of η

$$\text{Fluidity} = \frac{1}{\eta}$$

Dimensional formula of fluidity is $[M^{-1}L^{1}T^{1}]$

NEWTONIAN AND NON NEWTONIAN FLUID

Newtonian fluid

Is the fluid with which the velocity gradient is proportional to the tangential stress.

$$\frac{F}{A} \propto \frac{dv}{dx}$$

These fluids obeys Newton's law of viscosity

Non Newtonian fluid

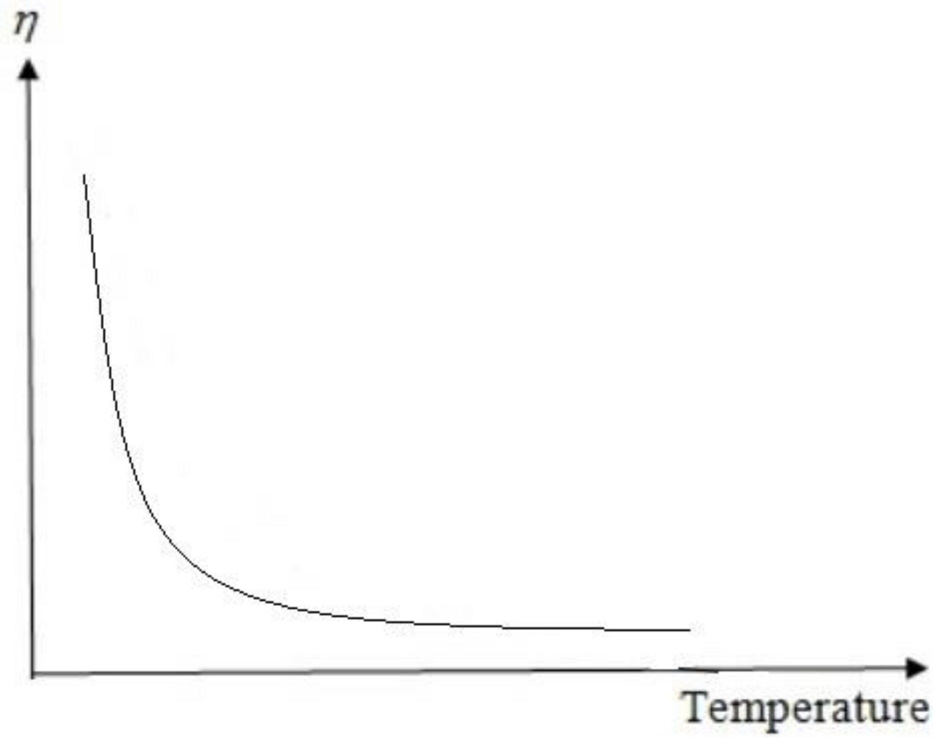
Is the fluid with which the velocity gradient is not proportional to the tangential stress. These fluids does not obey Newton's law of viscosity. They don't have constant values of η, Oil-paint is an example of a non-Newtonian liquid

VARIATION OF COEFFICIENT OF VISCOSITY WITH TEMPERATURE

1. **For liquids**

   In the case of liquids the viscosity is due to the attraction among molecules within the liquid and also between the molecules of the liquid and those of solids in contact.

With rise in temperature, the molecular attractions get weakened and hence viscosity decreases.



2. **For Gases**

The molecules are much further apart and the viscosity is due to the collisions between the fast moving (flowing) molecules and those flowing at lower velocities.

During collisions the fast molecules give up momentum to the slow molecules and are retarded in their flow.

As temperature increases molecular activity increases and this led to the increase of viscosity with rise in temperature.

Differences between friction and viscosity.

Friction	**Viscosity**
- Solid friction is independent of the temperature.	- The viscosity of a liquid decreases with the increase in temperature.
- In case of solid friction, heat is generated at the surfaces between the solids.	- Heat is generated within the fluid and not at the interface of the solid and the fluid

- Friction between two surfaces of solids is independent of the area of contact and of the relative velocity.

- Viscosity depends upon the area of contact and the velocity gradient between the layers.

Similarities between friction and viscosity

(i) Both come into play wherever there is a relative motion
(ii) Both oppose the relative motion
(iii) Both arise from intermolecular forces
(iv) Both depend on nature of surfaces.

STOKE'S LAW

Stoke's law state that "for steady motion of a small spherical body, smooth and rigid moves slowly in a fluid of infinite extent, the viscous drag force experienced on the body is given by,

$$F = 6\pi\eta v r$$

F – Viscous drag force

η - Coefficient of viscosity of the fluid

r - Radius of spherical body

v - Terminal velocity

Derivation of the Formula

Consider a sphere of radius r moving with velocity v through a fluid whose coefficient of viscosity η.

It is desired to find the expression for the viscous force F of the sphere.

For this purpose, we shall use dimensional analysis.

Stokes observed that in case of a slowly moving small sphere, the viscous force F depends on

(i) The radius r of the spherical body

(ii) The coefficient of viscosity η of the fluid.

(iii) The velocity v of the spherical body.

(iv) Shape and size of the solid body.

$$F \propto r^x \eta^y v^z$$

$$F = K r^x \eta^y v^z$$

$$[F] = [r^x][\eta^y][v^z]$$

$$[MLT^{-2}] = [L]^x[ML^{-1}T^{-1}]^y[LT^{-1}]^z$$

$$[MLT^{-2}] = [M]^y[L]^{x-y+z}[T]^{-y-z}$$

Equating the indices of M, L, T

1 = y , 1 = x – y + z, -2 = y – z

Solving gives

y = 1

z = 1

x = 1

$$F = K r^x \eta^y v^z$$

The value of k was found to be 6π

$$F = 6\pi\eta v r$$

Limitation of the Stoke's law

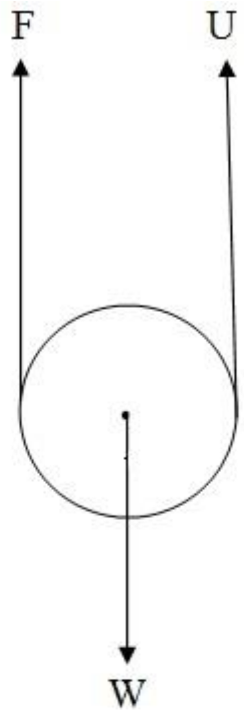
1. Strictly, the law applies to a fluid of infinite extent.
2. The law does not hold good if the spherical body is moving so fast that conditions are not streamline.
3. The spherical body must be smooth, small and rigid

Importance of Stoke's law

1. It accounts for the formation of goods
2. It is used in Millikan's experiment for the measurement of charge on an electron
3. It is used to find the size of small particles
4. It explains why large rain drops hurt much more than small ones when falling on you. It is not just that they are heavier but they are actually falling faster.

VERTICAL MOTION OF SPHERICAL BODY ON VISCOUS FLUID

Consider a small sphere falling freely from rest through a large column of a viscous fluid as shown below.



The forces acting on the sphere are

(i) Weight W of the sphere acting vertically downwards.

(ii) Upthrust U equal to the weight of the liquid displaced.

(iii) Viscous drag F (In direction opposite to motion).

When the sphere body falls with terminal velocity the body is at the equilibrium.

$$W - (U + F) = 0$$

$$W = U + F$$

Consider a small sphere of radius r falling, freely through a viscous fluid

Let,

ρ = Density of the sphere body

σ = Density of the fluid

η = Coefficient of viscosity of the fluid

Weight of sphere W

$$W = mg$$

$$W = \frac{4}{3}\pi r^3 \rho g$$

Upthrust on sphere U

$$U = \frac{4}{3}\pi r^3 \sigma g$$

Viscous drag

$$F = 6\pi\eta v r$$

From, W = U + F

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r V_T$$

$$\frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r V_T$$

$$6\pi\eta r V_T = \frac{4\pi r^3 g(\rho - \sigma)}{3}$$

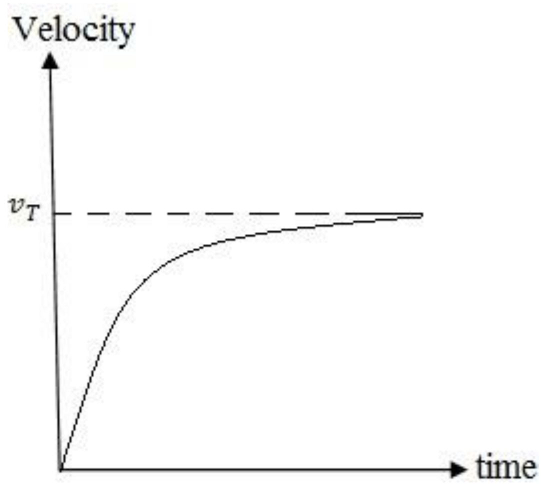
$$v_T = \frac{2}{9} X \frac{r^2(\rho - \sigma)g}{\eta}$$

Stokes made the following assumptions

1. The medium through which the sphere falls is infinite in extent.
2. The spherical body is perfectly rigid and smooth.
3. There is no slip between the spherical body and medium.
4. When the body moves through the medium no eddy current or waves should be set up in the medium.
5. The body must move through the medium slowly.
6. The diameter of spherical body must be large compared with the spaces between the molecules of the medium.
7. Terminal velocity of the body must be less the critical velocity of the medium.

Terminal Velocity V_T

Terminal velocity is the maximum constant velocity acquired by a body while falling freely through a viscous medium.



1. The terminal velocity of a spherical body falling freely through a viscous fluid is directly proportional to the square of its radius.

From,

$$V_T = \frac{2}{9} x \frac{r^2(\rho - \sigma)g}{\eta}$$

$$V_T \propto r^2$$

$$V_T = k r^2$$

$$\frac{V_T}{r^2} = k$$

$$\frac{V_{T_1}}{r_1^2} = \frac{V_{T_2}}{r_2^2}$$

$$\frac{V_{T_2}}{V_{T_1}} = \frac{r_2^2}{r_1^2}$$

This means that for a given medium, the terminal velocity of a large sphere is greater than that of a small sphere of the same material.

For this reason, bigger raindrops fall with greater velocity as compared with smaller ones.

2. The terminal velocity of a spherical body is directly proportional to the difference in the densities of the body and the fluid

$$V_T \propto (\rho - \sigma)$$

POISEUILLE'S FORMULA

When liquid flows through a horizontal pipe with wall of the pipe remains at rest while the velocity of layers goes on increasing towards the centre of the pipe, as the result the rate of flow of the liquid is slowest near the pipe walls and fastest in the centre of the pipe.

Poiseuille's studied the liquid flow through horizontal pipes and concluded that the rate of flow Q of a liquid through a pipe varies as,

(i) $Q \propto P$

(ii) $Q \propto r^4$

(iii) $Q \propto \frac{1}{\ell}$

(iv) $Q \propto \frac{1}{\eta}$

Where

Q – Volume of liquid flowing per second

P = Pressure difference across ends of the pipe

r – Radius of the pipe

ℓ - Length of the pipe

η - Viscosity coefficient of liquid

Combining these factors

$$Q \propto \frac{Pr}{\eta \ell}$$

$$Q \propto \frac{kPr}{\eta \ell}$$

Here K is a constant of proportionality and is found to be $\pi/8$

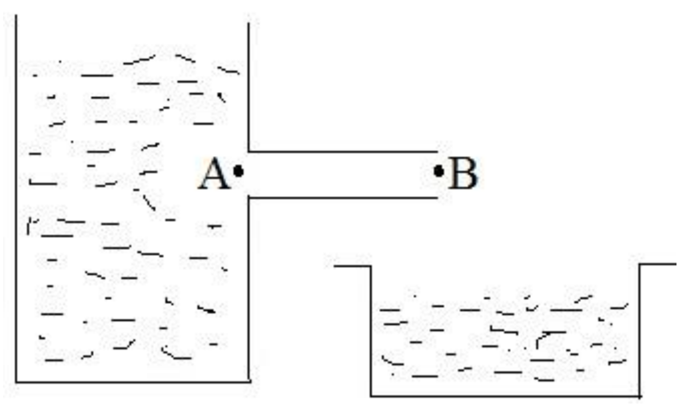
But,

$$Q = \frac{\pi P r^4}{8\eta\ell}$$

$$Q = \frac{V}{t}$$

$$\frac{V}{t} = \frac{\pi}{8}\frac{P r^4}{\eta\ell}$$

Consider the steady flow of liquid through a capillary tube AB or radius R and length ℓ.



Pressure difference at its ends

$P = p_A - P_B$

Poiseulle's formula

$$\frac{V}{t} = \frac{\pi r^4}{8}\frac{(P_A - P_B)}{\eta\ell}$$

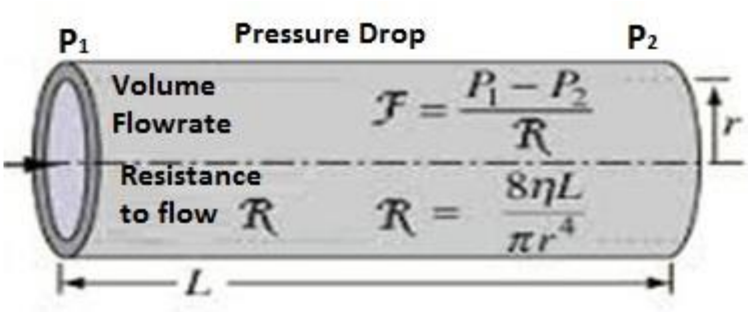
V – Volume of liquid collected in a time t

Poiseulle's made the following assumptions.

1. The flow of liquid is steady and parallel to the axis of the tube.
2. The pressure over any cross section at right angles to the tube is constant.
3. The velocity of liquid layer in contact with the sides of the tube is zero and increases in a regular manner as the axis of the tube is approached.

DERIVATION OF POISEULLE'S FORMULA BY DIMENSIONAL ANALYSIS

Consider a viscous liquid undergoing steady flow through a horizontal pipe of circular cross section as shown in the Fig. below.



Because of viscous drag, the velocity varies from a maximum at the centre of the pipe to zero at the walls.

We shall use dimensional analysis to derive an expression for the rate of flow of liquid Q through the pipe.

It is reasonable to suppose that the rate of flow of liquid through the pipe depends on

(i) The coefficient of viscosity η

(ii) The radius r of the pipe

(iii) The pressure gradient

- **Pressure gradient**

Is the pressure difference between the ends of the pipe per unit length of the pipe.

Pressure gradient = $\frac{P_A - P_B}{\ell} = \frac{P}{\ell}$

We can express the rate of flow of the liquid as

$$\frac{V}{t}\ \alpha\ \eta^{x} r^{y}\left[\frac{P}{\ell}\right]^{z}$$

$$\frac{V}{t} = k\eta^{x} r^{y}\left[\frac{P}{\ell}\right]^{z}$$

$$[L^{3}T^{-1}] = [ML^{-1}T^{-1}]^{x}[L]^{y}[ML^{-2}T^{-2}]^{z}$$

$$[L^{3}T^{-1}] =^{x} [M]^{x+z}[L]^{y-x-2z}[T]^{-x-2z}$$

Equating the induces of M, L, T

$x + z = 0$, $3 = y - (x + 2z)$, $-1 = -(x + 2z)$

On solving

$X = -1$

$y = 4$

$Z = 1$

$$\frac{V}{t} = \frac{kr^{4}P}{\eta\ell}$$

The value of k cannot be found by using dimensional analysis.

$$Q = \frac{\pi}{8}\frac{r^{4}P}{\eta\ell}$$

This formula for Q is called Poiseulle's formula

Limitation of Poiseuille's Law

The law is true only for the steady flow of a liquid through horizontal pipe.

The formula applies only to Newtonian fluids which are undergoing steady flow.

Speed of Bulk flow.

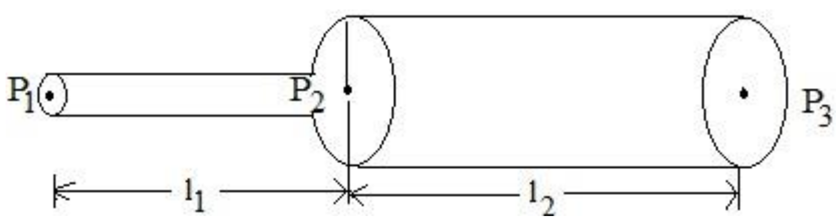
Speed of Bulk flow is the rate of volume flow divided by the cross-sectional area of the pipe.

Steady flow occurs only when the speed of bulk flow is less than a certain critical velocity. V_c.

Poiseuille's Formula does not hold good when the speed of bulk flow exceed critical velocity.

SERIES CONNECTION OF THE TUBES

When the two tubes of different diameters and lengths are connected in series the rate of fluid flow is the same in all the tubes.



Expression of the rate of volume of the fluid flow.

$$Q = \frac{v}{t} = \text{Constant}$$

For tube 1

$$\frac{V}{t} = \frac{\pi}{8}\frac{r_1^4(\triangle P_1)}{\eta L_1} \quad \delta P_1 = P_1 - P_2$$

$$\triangle P_1 = P_1 - P_2$$

$$\triangle P_1 = \frac{8(V/t)\eta L_1}{\pi r_1^4}$$

For Tube 2

$$\frac{V}{t} = \frac{\pi r_2^4(\Delta P_2)}{8\eta l_2}$$

$$\Delta P_2 = P_2 - P_3$$

$$\Delta p_2 = p_2 - p_3 = 8(V/t)\eta l_2 / \pi r_2^4 \ldots\ldots\ldots\ldots\ldots (i)$$

Adding the 2 equation

$$P_1 - P_2 + P_2 - P_3 = \frac{8(V/t)\eta l_1}{\pi r_1^4} + \frac{8(V/t)\eta l_2}{\pi r_2^4} \text{ --------------- (ii)}$$

$$P_1 - P_3 = \frac{8V\eta}{\pi t}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right]$$

$$\frac{V}{t} = \frac{\pi}{8\eta}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right]$$

Let $\triangle P$ be the pressure difference at the end of the tubes

$$\triangle P = P_1 - P_3$$

$$\triangle P = P_1 - P_2$$

$$\frac{V}{t} = \frac{\pi P}{8\eta}\left[\frac{l_1}{r_1 4} + \frac{l_2}{r_2^4}\right]^{-1}$$

Expression of the pressure at the junction

Expression of the pressure at the junction

Since,
$$\frac{V}{t} = \text{Constant}$$

For Tube 1,
$$\frac{V}{t} = \frac{\pi r_2^4 (\triangle P_1)}{8\eta \; I_1}$$

Then

$$\frac{\pi r_1^4 (P_1 - P_2)}{8\eta \; I_1} = \frac{\pi r_2^4 (P_2 - P_3)}{8\eta \; I_2}$$

$$\frac{r_1^4 (P_1 - P_2)}{I_1} = \frac{r_2^4 (P_2 - P_3)}{I_2}$$

$$r_1^4 I_2 (P_1 - P_2) = r_2^4 I_1 (P_2 - P_3)$$

$$P_1 r_1^4 I_2 - P_2 r_1^4 I_2 = P_2 r_2^4 I_1 - P_3 r_2^4 I_1$$

$$P_2 r_2^4 I_1 + P_2 r_1^4 I_2 = P_1 r_1^4 I_2 + P_3 r_2^4 I_1$$

$$P_2 (r_1^4 I_2 + r_2^4 I_1) = P_1 r_1^4 I_2 + P_3 r_2^4 I_1$$

$$P_2 = \frac{P_1 r_1^4 I_2 + P_3 r_2^4 I_1}{r_1^4 I_2 + r_2^4 I_1}$$

Applications of Viscosity

1. The quality of ink is decided by the coefficient of viscosity of ink.
2. The study of variation of viscosity with temperature helps us to pick up the best lubricant of a certain machine.
3. Applied in the study of circulation of blood. The variation in the coefficient of viscosity of blood affects the pressure and that in turn affects the efficiency of our bloody.

4. For damping the motion of certain instruments such as, shock absorber in car's suspension system.
5. Used in production and transportation of oils.
6. Liquids having higher values of coefficient of viscosity are used as buffers at railway station.

FLUID DYNAMICS

Fluid dynamics is the study of fluids in motion.

While discussing fluid flow, we generally make the following assumptions.

(i) The fluid is non-viscous
(ii) The fluid is incompressible
(iii) The fluid motion is steady

Non viscous fluid.

Non viscous fluid is the fluid which does not offer a resistance to the motion through it of any solid body.

There is no internal friction between the adjacent layers of the fluid.

Incompressible fluid.

An incompressible fluid is the fluid in which changes in pressure produce no change in the density of the fluid

This means that density of the fluid is constant.

Steady flow of a fluid

This means that the velocity, density and pressure at each point in the fluid do not change with time.

TYPES OF LIQUID FLOW

The liquid flow is of two main types

(i) Streamline flow or steady flow
(ii) Turbulent flow

Stream line flow.

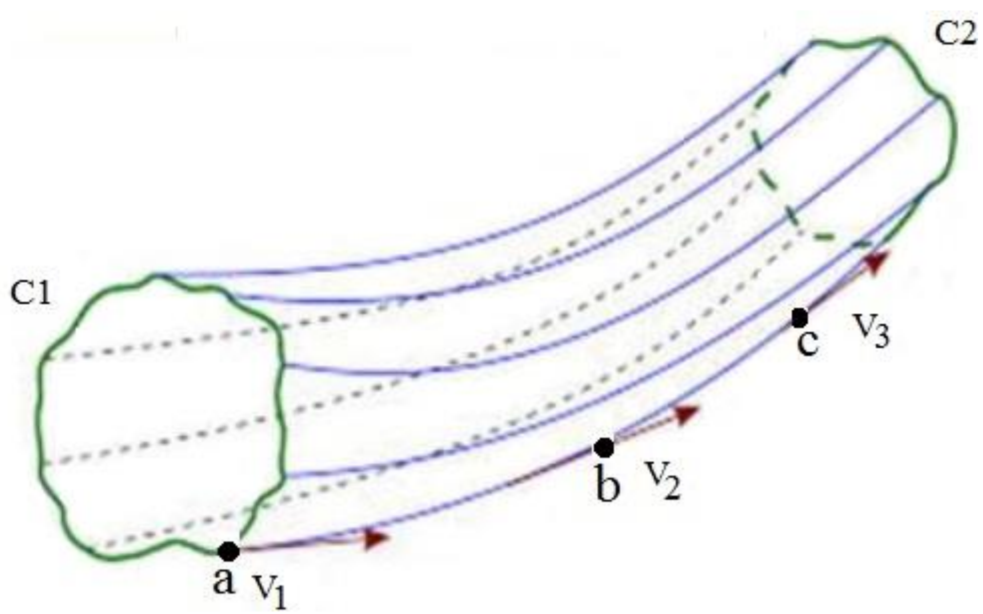
Streamline flow is the flow of a fluid when all the fluid particles that pass any given point follow the same path at the same speed.

The fluid particles have the same velocity. This flow is also called orderly flow or uniform flow.

Characteristics of streamline.

1. The velocity of a particle at any point is a constant and is independent of time.
2. The liquid layer in contact with the solid surface will be at rest
3. The motion of the fluid (liquid) follows Newton's law of viscous force

For example, consider a liquid flowing through a pipe as shown in figure below



The flowing liquid will have a certain velocity v_1 at **a**, a velocity v_2 at **b** and so on.

As time goes, the velocity of whatever liquid particle happens pas to pass be at **a** is still v_1, that at b is still v_2, then the flow is said to be steady or streamline flow.

Every particle starting at **a** will follow the same path **abc**. The line **abc** is called streamline.

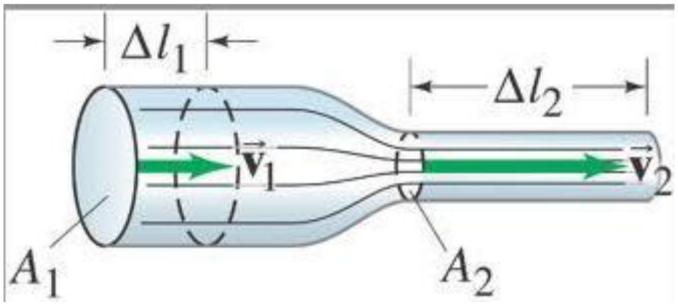
Streamline.

A streamline is a curve whose tangent at any point is along the direction of the velocity of the liquid particle at that point.

Streamlines never cross each other otherwise, particles reaching the intersection would not have a unique velocity at that point in space.

Tube of flow.

A tube of flow is a tabular region of a flowing fluid whose boundaries are defined by a set of streamlines.



Since the streamlines represent the path of particles, we see that no liquid conflation or out of the sides of a tube of flow.

In a steady flow, the velocity, density and pressure at each point in the fluid do not change with time.

Laminar flow.

Laminar flow is a special case of steady flow in which the velocities of all particles on any given streamline are the same, though the particles of different streamlines may move at different speeds.

Turbulent flow.

Turbulent flow is the flow of fluid when the speed and direction of fluid particles passing any point vary with time.

It is also known as disorderly flow.

Line of flow.

A line of flow is the path followed by a particle of the fluid.

Rotational flow

This is when the element of fluid at each point the angular velocity is equal to zero.

Irrotation flow.

Irrotation flow is the type of fluid flow by which the element of fluid at each irrotation no net angular velocity about that axis.

RATE OF FLOW

A rate of flow is the volume of a liquid that passes the cross-section of a vessel (pipe) in one second

It is denoted by the symbol Q

$$Q = \frac{V}{t}$$

The SI unit of rate of flow of liquid is m^3/s

Consider a pipe of uniform cross-sectional area A as shown below.

$$P_2 = \frac{P_1 r_1^4 L_2 + P_3 r_2^4 L_1}{r_1^4 L_2 + r_2^4 L_1}$$

If the liquid is flowing at an average velocity of v, then distance l through which the liquid moves in time t is

$$v = \frac{l}{t}$$

This may be regarded as the length of an imaginary cylinder of the liquid that has passed the section S in time t.

Then the volume of liquid that has passed section S in time t is V

Rate of flow

$$Q = \frac{V}{t}$$

$$Q = \frac{V}{t} = \frac{AVt}{t}$$

$$Q = Av$$

This is called discharge equation.

The quantity Av is called the flow rate or volume flux.

It is clear that for constant rate of flow, the velocity v is inversely proportional to the area of cross sectional of the pipe.

$$Q = Av$$
$$Q = K = \text{Constant}$$
$$v = \frac{K}{A}$$
$$v \propto \frac{1}{A}$$

If the cross sectional area of the pipe decrease the velocity of the liquid increases and vice verse.

REYNOLD'S NUMBER

Reynold's number is a dimensionless ratio which determines whether the fluid flow is streamline or turbulent.

It is denoted by Re or N_R

$$N_R = \frac{\rho v D}{\eta}$$

ρ = Density of the liquid

v = Average velocity of flow

D = Diameter of the tube or pipe

η = Coefficient of viscosity of the liquid

If $N_R < 2000$, the flow is laminar or steady

If $N_R > 3000$, the flow is turbulent

It N_R is between 2000 and 3000, the flow is unstable, it may change from laminar to turbulent and vice versa.

CRITICAL VELOCITY

Critical velocity is the velocity of liquid below up to which its flow is steady and above which its flow becomes turbulent.

When the velocity of the liquid flowing through a pipe is small, the flow is steady. As velocity is increased, a stage is reached when experiments show that for cylindrical pipes critical velocity is given by

$$V_c = \frac{1100\eta}{r\rho}$$

η = Coefficient of viscosity of the liquid

r = Radius of pipe

ρ = Density of the liquid

Derivation of expression for critical velocity

We can derive the formula for critical velocity by dimensional analysis.

The critical velocity v_c of a liquid flowing through a pipe depends upon

(i) Coefficient of viscosity of a liquid η

(ii) Density of the liquid ρ

(iii) Radius of the pipe r

$$v_c = \eta^a \rho^b r^c$$

$$v_c = k\eta^a \rho^b r^c$$

$$[LT^{-}] = [ML^{-1}L^{-1}]^a [ML^{-3}]^b [L]^c$$

$$[LT^{-1}] = [M]^{a+b} [L]^{c-a-3b} [T]^{-a}$$

Equating the powers of M, L and T

$$a + b = o, c - a - 8b = 1, -a = {}^{-}1$$

$$a = 1,\ b = -1, c = -1$$

$$v_c = k\eta\rho^{-1}r^{-1}$$

$$v_c = \frac{k\eta}{\rho_r}$$

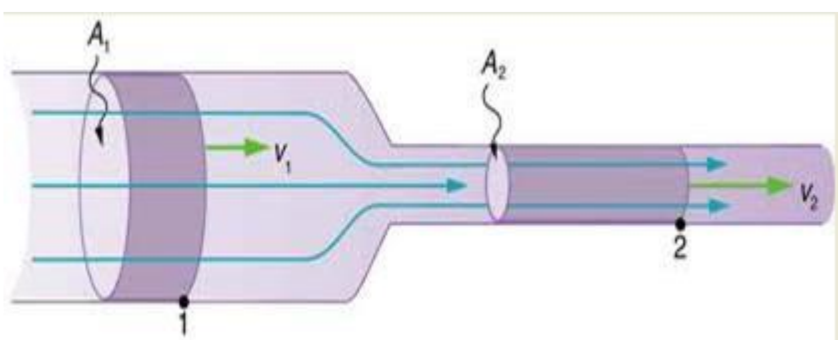
By experiment the value of k = 1100

$$v_c = \frac{1100\eta}{r\rho}$$

EQUATION OF CONTINUITY

(MASS FLOW RATE)

Consider the flowing of the fluid on the pipe PQ as shown on the figure below



Let

A_1 – Cross – sectional Area at P

A_2 – Cross – sectional Area at Q

v_1 – Velocity of fluid at P

v_2 – Velocity of fluid at Q

If ρ_1 and ρ_2 are densities of fluid at P and Q

Mass flow rate

Is the mass of liquid (fluid) flowing through a tube per second.

Then, the rate of mass of fluid leaving at P and Q at the time interval Δt be

$$\frac{\Delta M_1}{\Delta t} = A_1 V_1 \rho_1$$

$$\frac{\Delta M_2}{\Delta t} = A_2 V_2 \rho_2$$

SI unit of mass flow rate is kg/s

Applying the law of conservation of mass

$$\frac{\triangle M_1}{\triangle t} = \frac{\triangle M_2}{\triangle t}$$

$$\frac{\triangle M_2}{\triangle t} = A_2 V_2 \rho_2$$

$$A_1 V_1 \rho_1 = A_2 V_2 \rho_2$$

This is the equation of continuity for the compressible fluid flow.

If the liquid is incompressible, the density of liquid P and Q will be equal.

Flow is steady. No liquid can cross the side, so the mass of liquid passing through section P is equal to the mass of liquid passing through section Q in one second.

$$A_1V_1\rho_1 = A_2V_2\rho_2$$

$$\rho_1 = \rho_2 = \rho$$

$$A_1V_1\rho_1 = A_2V_2\rho_2$$

$$A_1V_1 = A_2V_2$$

Equation of continuity

States that "the mass of a liquid flowing through a pipe of varying cross – section is constant when density of the liquid does not change".

$$A_1V_1 = A_2V_2$$

AV = Constant.

ENERGY OF A LIQUID

A moving liquid can possess the following types of energies

(i) K.E. due to its position

(ii) P.E. due to its position

(iii) Pressure energy due to pressure of the liquid

(iv) K.E of a liquid

(i) K.E of a liquid
Is the energy possessed by the liquid due to its motion

$$K.E = \frac{1MV^2}{2}$$

M – Mass liquid

V – Velocity of a liquid flow

$$K.E \text{ per unit mass } = \frac{1MV^2}{2} \qquad m = 1$$

$$A_1V_1\rho_1 = A_2V_2\rho_2$$

$$K.E \text{ per unit volume } = \frac{1}{2} \text{ x } \frac{MV^2}{V}$$

$$K.E \text{ per unit volume } = \frac{1}{2}\rho V^2$$

(ii) P.E of a liquid

Is the energy possessed by the liquid due to its position

P.E of liquid of mass m at a height h is given by

P.E = mgh

*P.E per unit mass = mgh m=1

P.E per unit mass = G

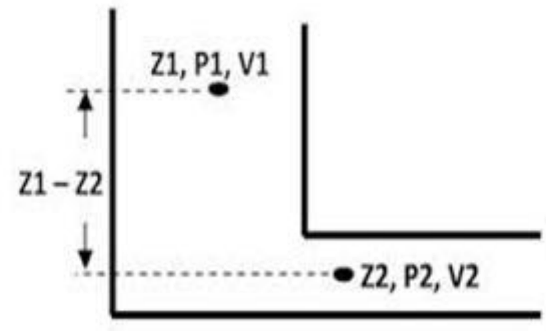
P.E. per unit volume = $\frac{mgh}{v}$

$$\rho = \frac{M}{V}$$

$$P.E \text{ per unit volume } = \rho gh$$

(iii) Pressure Energy

In figure below, is a wide tank containing a liquid



At its bottom there is a side tube fitted with a frictionless piston of area of cross section A.

Work done to move the piston through a small distance X

W = F.X

But,

$$P = F \Longrightarrow F = PA$$

P – Pressure on the piston

W = PAX

Mass of liquid pushed

$$\rho = \frac{M}{V}$$

M = ρV

But

V = AX

\ $m = \rho AX$

This work done is stored in as pressure energy of a column of liquid

Pressure energy = PAX

* Pressure energy per unit mass = $\frac{PAX}{\rho AX}$

$$\text{Pressure energy per unit mass} = \frac{\rho}{\rho}$$

*Pressure energy per unit volume = $\frac{PAX}{AX}$

Pressure energy per unit volume = P

Pressure energy of a volume V of a liquid is given by = PV

Total energy of moving liquid

$$E_T = {}^1/_2 Mv^2 + mgh + Pv$$

Total energy per unit volume

$$E_T/_v = {}^1/_2 \rho v^2 + \rho gh + P$$

Total energy per unit mass

$$E_T/_m = {}^{v^2}/_2 + gh + {}^P/_\rho$$

Pressure energy $= mgh$

But

$P = \rho hg$

$h = \frac{\rho}{\rho g}$

Pressure energy $= mg.{}^{p}/_{\rho g}$, Since $\rho = \frac{\mathrm{M}}{\mathrm{V}}$

Pressure energy $= M.\frac{P}{M}V$

Pressure energy = PV

BERNOULLI'S THEOREM

States that "for steady flow of an ideal liquid, the total energy per unit volume remains constant throughout the flow".

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{Constant}$$

P - Pressure within the fluid

ρ - Density of the fluid

h - Height of the fluid

V - Velocity of the fluid

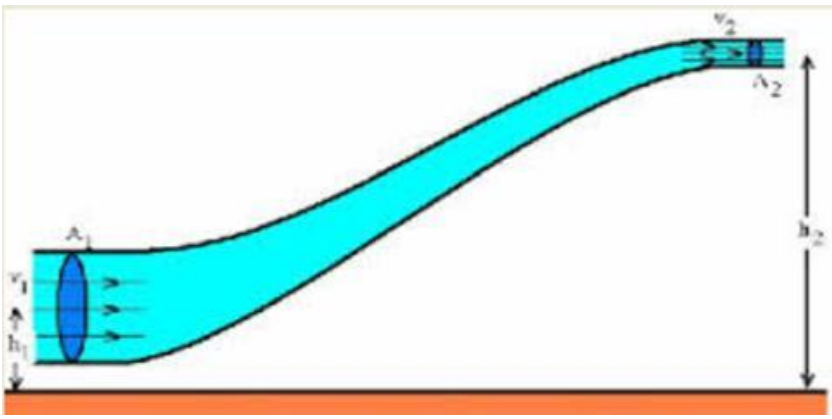
g - Acceleration due to gravity

Bernoulli's theorem is simply a statement of law of conservation of energy applied to a liquid.

The importance of this theorem lies in the fact it can be used to determine the variation of pressure in liquids as a function of velocity of the liquid and the elevation of the pipe through which the liquid is flowing.

PROOF OF BERNOULLI'S THEOREM

Consider an incompressible, non-viscous fluid flow and steadily through a pipe of variable cross – section. Consider two sections A and Q.



A_1, A_2 - The cross – sectional areas at P and Q

V_1, V_2 - Fluid velocity at P and Q

P_1, P_2 - Fluid pressure at P and Q

h_1, h_2 - Mean heights above ground level at P and Q respectively

The force acting on the liquid layer at P is

$$F = P_1 A_1$$

Under this force in time Δt the layer moves through a distance

$$= v_1 \times \Delta t$$

Work done on the fluid due to the force

$$= P_1 A_1 \times v_1 \Delta t$$

When the liquid moves from P to P'

In a time Δt at the same time the fluid at Q moves to Q', doing work against the pressure P_2 at B.

Work done by the fluid against pressure P_2

$$= P_2 A_2 \times v_1 \Delta t$$

Network done on the fluid = work done on the fluid – work done by the place

$$= P_1 A_1 \times V_1 \Delta t - P_2 V_2 A_2 \times \Delta t$$

Equation of continuity

Since, $A_1 V_1 \Delta t = A_2 V_2 \Delta t = V$

$$V = \frac{m}{\rho}$$

This equation means, whatever mass of fluid enters the pipe at P in a certain time will leave the pipe at Q at the same time.

Total work done on the fluid = $P_1 V - P_2 V$

$= (P_1 - P_2) v$

$$\Delta W = (P_1 - P_2)\, {}^{m}/_{\rho}$$

Total energy = ΔK.E + ΔP.E

$$\Delta K.E = \frac{1}{2} m(v_2^2 - v_1^2)$$

Increase in P.E. $= mgh_2 - mgh_1$

$$\Delta P.E = Mg(h_2 - h_1)$$

Work done on the fluid = $\Delta P.E + \Delta K.E$

From work energy theorem ΔW = ΔK.E + ΔP.E

$$(P_1 - P_2)\frac{M}{\rho} = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$$

$$P_1\frac{m}{\rho} - P_2\frac{M}{\rho} = mgh_2 - mgh_1 + \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\frac{P_1}{\rho} - \frac{P_2}{\rho} = gh_2 - gh_1 + \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\frac{P_1}{\rho} + gh_1 + \frac{V_1^2}{2} = \frac{P_2}{\rho} + gh_2 + \frac{V_2^2}{\rho}$$

$$P/\rho + gh + V/2 = constant$$

$$P + \rho gh + \frac{1}{2}\rho V^2 = constant$$

FLOW IN A HORIZONTAL PIPE

When the liquid flows through a horizontal pipe

$$P_1 + \rho gh_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho V_2^2$$

$$h_1 = h_2 = h$$

$$P_1 + \rho gh_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho V_2^2$$

$$P + \frac{1}{2}\rho V^2 = \text{Constant}$$

P = Static pressure

$\rho v^2/_2 = Dynamic\ pressure$

DIFFERENT FORMS OF BERNOULLI'S THEOREM

1. $P + \rho gh + {}^1/_2 \rho v^2 = constant$
2. $P/_\rho + gh + v^2/_2$ = Constant
3. $P/_{\rho g} + h + v^2/_{2g}$ = Constant

Where by

$P/_{\rho g} = Pressure\ head$

$h = Elevation\ head$

$v^2/_{2g} = velocity\ head$

LIMITATION OF BRNOLLI'S THEOREM

1. In derivation of Bernoulli's equation, it is assumed that the liquid is non-viscous i.e. the liquid has zero viscosity (no friction).
   However, a real liquid does have some viscosity so that a part of mechanical energy is lost to overcome liquid friction. This fact is not taken into account in this elevation.

2. In derivation of Bernoulli's equation, it is assume that the rate of flow of liquid is constant
   $A_1 . V_1 = A_2 V_2$

But this is not correct in actual practice. Thus in the case of liquid flowing through a pipe, the velocity of flaw is maximum at the center and goes on decreasing towards the walls of the pipe. Therefore, we should take the average velocity of the liquid.

3. In derivation of Bernoulli's equation, it is assumed that there is no loss of energy when liquid is motion. In practice, this is not true e.g. A part of K.E of flowing liquid is converted into heat and is last forever.

4. If the liquid is flowing along a curved path, the energy due to centrifugal force must be considered.

APPLICATIONS OF BERNOULL'S THEOREM

1. Flow meter - Venturimeter

Is a device that is used to measure the flow speed (flow rate) of a liquid through a pipe. It works on Bernoulli's theorem.

It consists of two tubes A and C connected by a narrow coaxial tube B with a constriction called the throat.

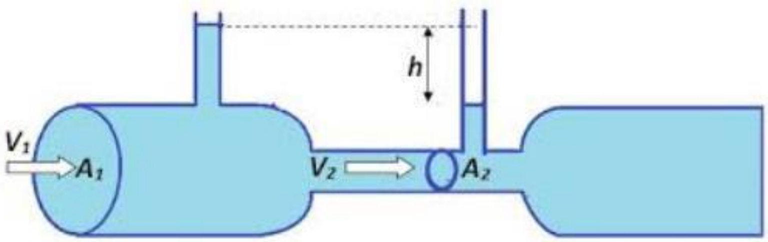
Using the two tubes D and E the difference in pressure of the liquid flowing through A and B can be round out.

As the liquids flows from A to B the velocity increases, due to decreases in cross-sectional area.

Let the velocities at A and B isV_1 and A_1 and $A_2$and V_2 and cross-sectional areas and A and B be A_1 and A_2 respectively.

By equation of continuity

$$A_1 V_1 = A_2V_2$$



Applying the equation of continuity

Q = $A_1 v_1 = A_2 v_2$

Where Q is the volume of liquid flowing in one second

Applying Bernoulli's theorem at A and B

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 = \frac{P_2}{\rho} + \frac{1}{2}\frac{V_2^2}{}$$

$$P_1/\rho - P_2/\rho = P_1{}^2/2 - v_v{}^2/2$$

$$P_1 - P_2 = \frac{\rho}{2}(V_2^2 - V_1{}^2)$$

But, $A_1 > A_2$, $V_2 > V_1$ $P_1 > P_2$ hence the level of the liquid in D is higher than that in E.

$$P_1 - P_2 = \rho h g$$

H – Difference of levels of the liquid in the tubes D and E.

From, $A_1 V_1 = A_2 V_2$

$$A_1^2 V_1^2 = A_2^2 V_2^2$$

$$V_2^2 = \frac{A_1^2}{A_2^2} V_1^2$$

$$\rho' hg = \frac{\rho}{2}\left[\frac{A_1^2 V_1^2}{A_2^2} - V_1^2\right]$$

$$\rho' hg = \frac{\rho V_1^2}{2}\left[\frac{A_1^2}{A_2^2} - 1\right]$$

$$\rho' hg = \frac{\rho V_1^2}{2}\left[\frac{A_1^2}{A_2^2} - A_2^2\right]$$

$$2\rho' hg = \rho V_1^2\left[\frac{A_1^2 - A_2^2}{A_2^2}\right]$$

$$\rho V_1^2 (A_1^2 - A_2^2) = 2\rho^1 hg - A_2^2$$

$$V_1 = A_2\sqrt{\frac{2\rho^1 hg}{\rho(A_1^2 - A_2^2)}}$$

If $\rho^1 = \rho = \rho$

Also

$$V V_1 2 A_2 \sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$

$$V_2 \; A_1 \sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$

From

Q $= A_1V_1 = A_2 V_2$

$A_1V_1 = Q \quad A_2V_2 = Q$

$V_1 = \frac{Q}{A_1} \qquad V_2 = \frac{Q}{A_2}$

$V_1^2 = \frac{Q^2}{A_1^2} \qquad V_2^2 = \frac{Q}{A_2}$

$$\rho' hg = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$\rho' = \rho$$

$$2hg = V_2^2 - V_1^2$$

$$2hg = \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}$$

$$2hg = Q^2\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]$$

$$2gh = Q^2\left[A_1^2 - A_2^2 \Big/ A_1^2 A_2^2\right]$$

$$Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$\text{Since } Q = \frac{V}{t}$$

$$\frac{V}{t} = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

TORRICELLI'S THEOREM

States that "if the difference in levels between the hole and the upper liquid surface in a drum is h, then the velocity with which the liquid emerges from the hole in $\sqrt{2gh}$ States that the velocity of efflux is equal to the velocity which a body attains. in falling freely from the surface of the liquid to the orifice

Velocity of efflux

Is the velocity of liquid at the orifice.

Or

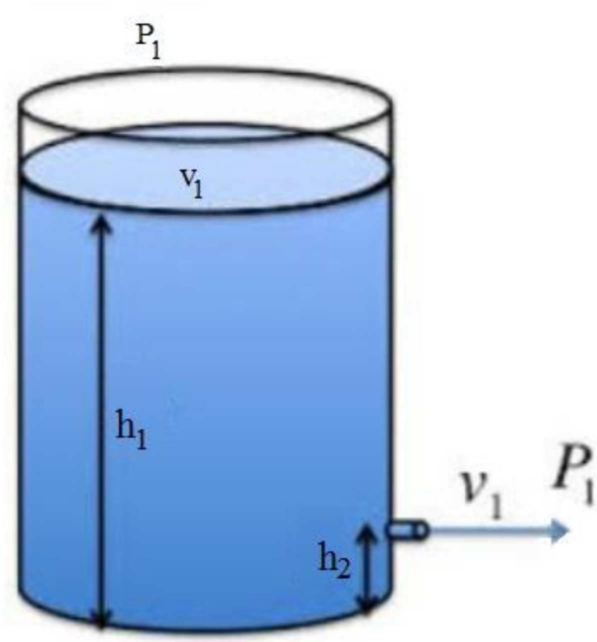
Is the velocity of emerging fluid from the orifice

This theorem applies to a liquid flowing from a drum with a horizontal opening near the base.

This is the same velocity which freely falling object will acquire in falling from rest through a vertical distance h.

PROOF OF TORRICELLI'S THEOREM

Suppose an ideal liquid flows through a hole H at the bottom of a wide drum as shown below



Let ρ be the density of the liquid.

According to Bernoulli's theorem, at any point of the liquid

$$\rho + \rho hg + \frac{1}{2}\rho v^2 = \text{constant} \text{-------------------} \quad (i)$$

At point 1

The point 1 is at the surface of the liquid in the drum

$$P = Pa\rho gh = \rho gh\frac{1}{2}\rho V^2 = 0 = \text{constant}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

$$\text{Since } h = h_1 - h_2$$

At point 2

The point 2 is at the place where liquid leaves the hole.

$$P = Pa,\ \rho gh = 0,\ \frac{1}{2}\rho v^2 = \frac{1}{2}\rho v_2^2$$

$$\rho a + \frac{1}{2}P v_2^2 ,= \text{constant}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

By equating equation (ii) and (iii)

$$Pa + \rho gh = Pa + \frac{1}{2}\rho v^2$$

$$\rho gh = \frac{\rho V_2^2}{2}$$

$$V_2^2 ,= 2gh$$

$$V_2 = \sqrt{2gh}$$

*The velocity of the liquid emerging from the hole depends only upon the depth h of the hole below the surface of the liquid.

$* P = \rho hg$

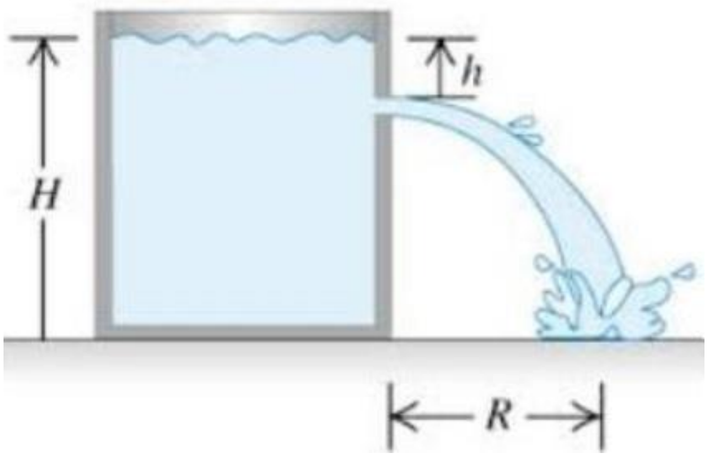
\

$$gh = \frac{P}{\rho}\ h = \frac{P}{\rho g}$$

$$V_2 \sqrt{\frac{2P}{\rho}}$$

Horizontal Range

The liquid flows out of the hole in the form of a parabolic jet and strikes the ground at a distance R from the base of the drum.



The distance R is the horizontal range of the liquid coming out of the hole.

At the hole P, the velocity V_2 of the emerging liquid is along the horizontal direction.

$$R = V_2 T$$

$$R = \sqrt{2ghxT}$$

T - Time taken by the parabolic jet to strike the ground after emerging from the hole P

h^1 - Height of hole above the bottom of the drum

Therefore, the vertical distance covered by the set in time T is h'

$$h' = u + \frac{1}{2}gt^2$$

At point H, Vertical velocity is zero, since the liquid is emerging horizontally sin $0^0 = 0$

$T = t$

$$h' = 0 + \frac{1}{2}gT^2$$

$$\frac{2h'}{g} = T^2$$

$$T = \sqrt{\frac{2h'}{g}}$$

$$\text{Horizontal range} = \sqrt{2gh}\sqrt{\frac{2h'}{g}}$$

$$R = 2\sqrt{hh'}$$

$$H = h + h'$$

$$h' = H + (-h)$$

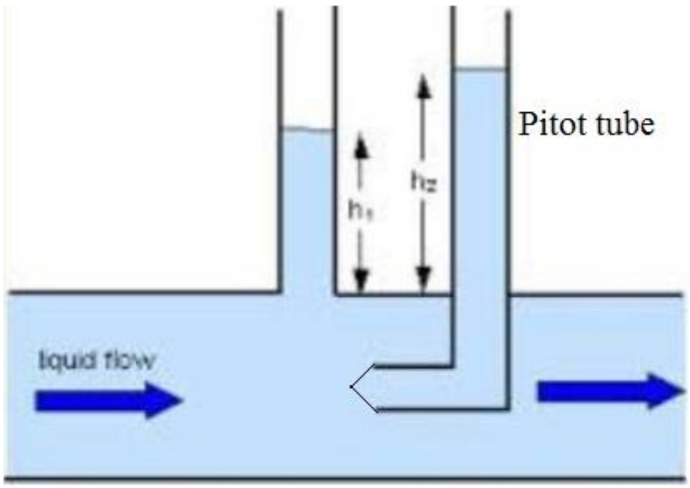
$$h' = H - h$$

$$R = 2\sqrt{h(H-h)}$$

PITOT TUBE.

Pitot tube is a device used for measuring the velocities of flowing liquids and hence the rates of flow of the liquids.

Its working is based on Bernoulli's theorem.



Pitot tube is an open ended L-shaped tube immersed in the liquid with its aperture and its nose B facing the flow of liquid, so that the plane of the aperture is normal to the direction of another tube A with a small opening at its bottom.

The plane of aperture of A is parallel to the direction of flow, so that it measures the static pressure P_A at A, which is the pressure of the undisturbed liquid.

The flow of liquid is stopped in the plane of aperture B, there by converting the Kinetic energy of the liquid into P.E.

So the liquid rises in the tube T as shown. The height of the liquid in this tube gives the total pressure or the stagnation pressure.

Applying Bernoulli's Equation

$$P_A + \frac{1}{2}\rho v^2 = P_B + 0$$

P_A - Static pressure

ρ - Density of the liquid

$\frac{1}{2}\rho v^2$ - Dynamic pressure

$$\frac{1}{2}\rho v^2 = P_B - P_A$$

Let the difference between the levels of the liquid in the two tubes be h

$$P_B - P_A = \rho g h$$

$$\frac{1}{2}\rho v^2 = \rho g h$$

The velocity of flow of the liquid

$$v = \sqrt{2gh}$$

The rate of flow of liquid

$$Q = Av$$

$$Q = A\sqrt{2gh}$$

A - Area of cross-sectional of the pipe at the place where the pitot tube is placed.

Static pressure.

Static pressure is the actual pressure of the fluid at the point due to its rest position of fluid.

Dynamic pressure.

Dynamic pressure is the pressure exerted by fluid due to its own motion.

$$D_P = \frac{1}{2}\rho v^2$$

Let h be difference in the liquid levels in the two limbs.

$$\frac{1}{2}\rho v^2 = hg(\rho m - \rho)$$

$$= \frac{2hg(\rho m - \rho)}{\rho}$$

$$v = \sqrt{2hg\left[\frac{\rho m - \rho}{\rho}\right]}$$

ρm - Density of the liquid in the manometer

ρ - Density of the liquid flow

HEAT-1

· THERMOMETRY

This is the science of temperature and its measurement.

TEMPERATURE

The temperature of a body is a number which expresses its degree of hotness or coldness on some chosen scale.

It is measure of how hot or cold the body is.

THERMOMETERS

A thermometer is an instrument designed to measure the temperature of a body.

Thermometers use some measurable property of a substance (thermometric property) which is sensitive to temperature.

Thermometric property
Is that property in which quantity value of a thermometer varies linearly and continuously with temperature
The thermometric property is also called physical property of a thermometer.

Examples of thermometric properties (physical properties) of a thermometer are:

(i) Length of liquid column in a glass tube

(ii) Volume of a gas at constant pressure

(iii) Pressure of a gas at constant volume

(iv) Electrical resistance of a platinum wire

(v) Electromotive force of a thermo-couple

QUALITIES OF A THERMOMETRIC PROPERTY

A thermometric property must have:

(i) A marked degree of expansion for a small temperature rise

(ii) A uniform expansion rate

(iii) Good thermo conductivity

(iv) High boiling point and low freezing point (if it is liquid)

TEMPERATURE SCALES

(1) CELSIUS SCALE

This is a temperature scale in which the fixed points are the temperatures at standard pressure of ice in equilibrium with water (0 $^{\circ}C$) and water in equilibrium with steam (100 $^{\circ}C$)

The scale between these two temperatures is divided into 100 equal parts and each part is called a degree.

The temperature on this scale is expressed in degree Celsius ($^{\circ}C$)

(2) FAHRENHEIT SCALE

This is a temperature Scale in which the temperature of boiling water is taken as 212 °F and the temperature of pure melting ice is taken as 32 °F

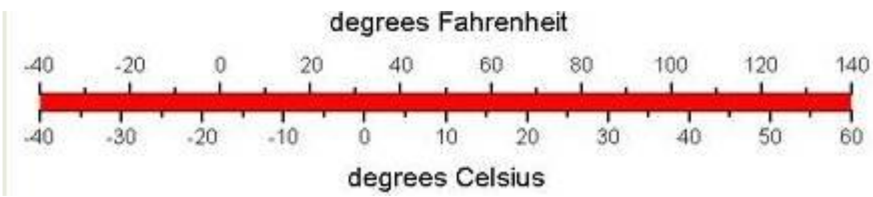
The interval between these two reference temperatures is divided into 180 equal parts and each part is called degree Fahrenheit (°F)

To convert this scale into degree centigrade (°C), the formula is:

$$°C = \frac{5}{9}(°F - 32)$$

(3) THERMODYNAMIC TEMPERATURE SCALE

This is the standard temperature scale adopted for scientific measurement.



Thermodynamic temperature is denoted by the symbol T and is expressed in Kelvin, K.

The lowest possible temperature which can be measured by this scale is assigned the value zero (OK) called absolute zero

It is the lowest temperature theoretically attainable at which the random motion of molecules and atoms in a substance is at minimum.

The temperature of a substance cannot be lowered further by decreasing the random motion.

Thermodynamic temperature is also called absolute temperature.
The degree centigrade is equal in magnitude to temperature in Kelvin i.e a change of temperature in °C is equal to a change of temperature in Kelvin.

$$\Delta(\theta°C) = \Delta T\ (K)$$

And

$$T\ (K) = 273 + \theta(°C)$$

NECESSARY CONDITIONS FOR THE TEMPERATURE SCALE TO BE ESTABLISHED

In order to establish a temperature scale one requires;

(1) SOME PHYSICAL PROPERTY OF A MATERIAL

The physical property of a material should vary linearly and continuously with temperature.

Examples

(i) The length of liquid column in a glass tube varies linearly and continuously with temperature.

$\Rightarrow$ Length $\propto$ temperature

$\Rightarrow l \propto \theta$

$\Rightarrow l = k\ \theta + 0$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

Compare $y = m\ x + c$

This is a linear relationship

(ii) The volume of a gas at constant pressure varies linearly and continuously with temperature.

$\Rightarrow$ Volume $\propto$ temperature

$\Rightarrow V \propto T$

$\Rightarrow$ V = K T + 0

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

Compare y = m x + c

This is a linear relationship

(iii) The pressure of a gas at constant volume varies linearly and continuously with temperature

$\Rightarrow$ Pressure $\propto$ temperature

$\Rightarrow$ P $\propto$ T

$\Rightarrow$ P = K T + 0

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

Compare y = m x + c

This is a linear relationship

(2) FIXED POINTS

A fixed point is a temperature that can be accurately reproduced to enable it to be used as a basis of a temperature scale.

It is a single temperature at which it can confidently be expected that a particular physical event (e.g the melting of ice under specific conditions) always takes place.

The three fixed points are:

(i) Ice point

Is the temperature at which pure ice can exist in equilibrium with water pure at standard atmospheric pressure (i.e. at 760mmHg).

(ii) Steam Point

Is the temperature at which pure water can exist in equilibrium with its vapor at standard atmospheric pressure.

(iii) Triple point

Is that unique temperature at which vapor, liquid and solid phases of a substance exist in equilibrium.

The triple point is particularly useful, since there is only one pressure at which all three phases (solid, Liquid and gas) can be in equilibrium with each other.

Example, For water the triple point is 273.16K and it occurs at a pressure of 611.2Pa

DISAGREEMENT BETWEEN SCALES

Different materials do not expand in quite the same way over a wide range of temperatures.

Thermometric properties do not keep in step as the temperature changes.

Consequently, if we calibrate different kinds of thermometers by using ice point and steam point as the reference marks they will not agree precisely (accurately).

Different results are obtained when different kinds of thermometers are used to measure the same temperature except at the calibrated fixed point.

THE TEMPERATURE SCALE EQUATION

When there is a change of temperature, the physical property of a material also changes.

Since the physical property of a material varies linearly and continuously with temperature then we have:

$$\text{Change in temperature } \alpha \text{ change in thermometric property}$$

Let X be physical property of a material

Let X_0 be physical property of a material at ice point (0 °C)

Temperature	Physical Property
0 °C	X_0
100 °C	X_{100}

θ °C	X_θ

CASE 1

When the temperature changes from 0°C to 100°C

Change in temperature = 100°C - 0°C

= 100°C

Change in property = $X_{100} - X_0$

But change in temperature $\propto$ change in thermometric property

100°C $\propto$ ($X_{100} - X_0$)

$\therefore$ 100°C = K($X_{100} - X_0$) ……………………………(1)

Where K = constant of proportionality

CASE 2

When the temperature changes from 0°C to θ °C

Change in temperature = $\theta - 0$

= θ

Change of the thermometric property = $X_\theta - X_0$

But change in temperature $\propto$ change in thermometric property

$\theta \propto (X_\theta - X_0)$

$\therefore\ \theta = k\,(X_\theta - X_0)$……………………………..(2)

Where k = constant of proportionality

Dividing $\frac{eqn\ (2)}{eqn\ (1)}$

$$\frac{\theta}{100°C} = \frac{k\,(X_\theta - X_0)}{k\,(X_0 - X_0)}$$

$$\theta = \left[\frac{X_\theta - X_0}{X_{100} - X_0}\right] \times 100°c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

RELATIONSHIP BETWEEN PHYSICAL PROPERTY OF A MATERIAL AND TRIPLE POINT

Consider a physical property P of a material which varies linearly and continuously with temperature.

Let P_{Trip} be physical property of a material at triple point temperature T_{Trip}

Since change in temperature $\propto$ change in thermometric property we have

$T_{Trip} \propto P_{Trip}$

$\therefore\ T_{Trip} = k\,P_{Trip}$ ………………………………….(1)

Where K = constant of proportionality

Let P_T be physical property of a material at a temperature T (in Kelvin)

Similarly $T \propto P_T$

$\therefore\ T = kP_T$…………………………….(2)

Dividing $\frac{eqn\ (2)}{eqn\ (1)}$

$$\frac{T}{T_{Trip}} = \frac{k\,P_T}{kP_{Trip}}$$

For water $T_{Trip} = 273.16K$

$$\frac{T}{T_{Trip}} = \frac{P_T}{P_{Trip}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3)$$

Equation (3) above can also be written as:

$$T = \left[\frac{P_T}{P_{Trip}}\right] x\ T_{Trip}$$

- For water $T_{Trip} = 273.16K$

$$T = \left[\frac{P_T}{P_{Trip}}\right] x\ 273.16K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

TYPES OF THERMOMETERS

There are so many types of thermometers but they all have common fact that they depend on some physical property of a material which changes with temperature.

Some important thermometers are: -

(i) Liquid in glass thermometers

(ii) Gas thermometers

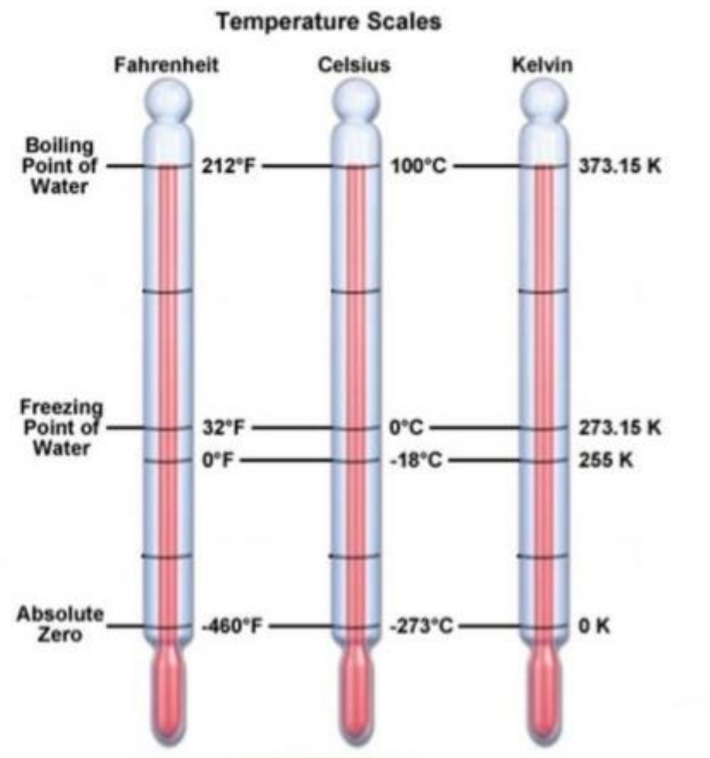
(iii) Platinum resistance thermometers

(iv) Thermocouple thermometers

(v) Radiation thermometers (Pyrometers)

(i)LIQUID IN GLASS THERMOMETERS

One of the most familiar thermometer uses mercury as a thermometric liquid



In this type of thermometer the length of mercury in a glass tube is the thermometric property of a material when the temperature changes. This means that in the temperature scale equation, x becomes length l

The temperature θ of the thermometer in terms of length l of mercury is given by:

$$\theta = \left[\frac{l_\theta - l_0}{l_{100} - l_0}\right] \text{ x } 100°C$$

When, l_0 = length of mercury at ice point

l_{100} = length of mercury at steam point

l_θ = Length of mercury at the unknown temperature θ°C

ADVANTAGES OF USING MERCURY IN THE LIQUID GLASS THERMOMETER

The use of mercury has the following advantages.

(i) Its boiling point 357 $^{\circ}C$ and its freezing point is - 36 $^{\circ}C$

Therefore it can be used over a wide range of temperature.

(ii) Its expansion is nearly uniform over the ordinary range of temperatures.

This makes the calibration of the thermometer easier.

(iii) It can be easily seen through glass because it is opaque and shining liquid.

(iv) It does not wet glass $i.e$ no mercury remains on the sides of the glass tube when the mercury level falls.

(v) It has low specific heat capacity. Therefore it does not absorb much heat from the body whose temperature is to be measured.

DISADVANTAGE OF USING MERCURY IN THE LIQUID IN GLASS THERMOMETERS

It cannot be used to measure very low as well as very high temperatures.

(ii) GAS THERMOMETERS

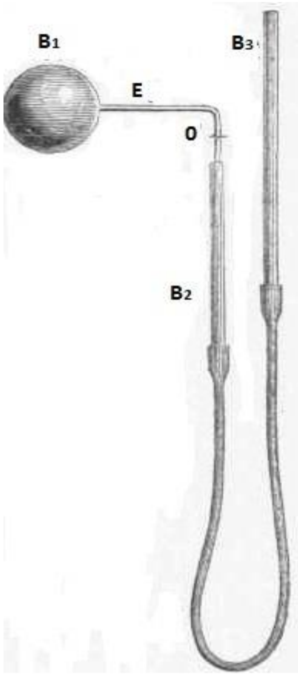
In most accurate work, temperatures are measured by gas thermometers.

There are two types of gas thermometers:

(i) Constant volume gas thermometer

(ii) Constant pressure gas thermometer

JOLLY'S CONSTANT VOLUME GAS / AIR THERMOMETER



B_1 is a glass bulb containing dry air
B_2 is a glass tube containing mercury

B_1 is connected to B_2 by means of a capillary tube E bent twice at right angles

B_3 is a glass tube containing mercury which is open the atmosphere at its upper end.

There is a fixed mark "O" on the glass tube B_2

The volume of air in B_1 is maintained constant by raising or lowering the glass tube B_3 until the mercury in B_2 is at the fixed mark "O".

OPERATION

When the thermometer is in use the bulb B_1 is placed inside the enclosure whose temperature is required.

Keeping the volume of air in B_1 constant by raising or lowering the glass tube B_3, the pressure of air in B_1 at ice point ($0^{\circ}C$), steam point ($100^{\circ}C$) and at the unknown temperature θ ($^{\circ}C$) are determined.

If P denote pressure of a gas/air at constant volume, then one can talk of pressure at $0^{\circ}C$, $100^{\circ}C$ and $\theta^{\circ}C$ as P_0, P_{100} and P_θ respectively

Now, the unknown temperature θ is given by:

$$\theta = \left[\frac{V_\theta - V_0}{V_{100} - V_0}\right] \text{ x } 100°C$$

Gas thermometer is an ideal thermometer because the increase in volume or pressure of a gas with temperature is independent of the nature of the gas.

$i.e$ All gases have the same coefficient of volume or pressure expansion.

ADVANTAGES OF USING GAS THERMOMETER

The gas thermometer has the following advantages:

(i) It is more sensitive than liquid in glass thermometer because the expansion of gases is many times greater than that of any liquid.

(ii) The expansion of the gas is uniform and regular.

(iii) Gas scale temperatures degree with absolute scale of temperature.

(iv) It can be used to measure very low as well as very high temperatures.

Example

- Hydrogen gas thermometer can measure temperature from – 200 °C to 500 °C

DISADVANTAGES OF USING GAS THERMOMETERS

The gas thermometer has the following disadvantages: -

(i) Its use is inconvenient due to its very large size.

(ii) The air in the capillary tube is not at the temperature being measured.

(iii) It cannot be used to measure the temperature of a liquid available in small quantity.

(iv) It is not a direct reading thermometer. ie require skill

SOURCES OF ERRORS WHEN USING GAS THERMOMETERS

(i) The bulb expands

(ii) Air is not an ideal gas

(iii)The air in the capillary tube is not at the temperature being measured

PLATINUM RESISTANCE THERMOMETER

The Platinum resistance thermometer is based on the principle that the electrical resistance of a pure metal increases with increasing in temperature and vice versa.

Experiments show that the resistance of a pure metal at any temperature θ is given by:

$$\theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) x \ 100°C$$

Where R_0 = Resistance at 0°C, R_θ = Resistance at a temperature θ°C It can be shown that $R_\theta = R_o(1 + a\theta + b\theta^2)$ where 'a' and 'b' are physical constant

In practice"b" is much less than "a" and hence we can ignore the term $b\theta^2$

$R_\theta = R_0(1 + a\theta)$

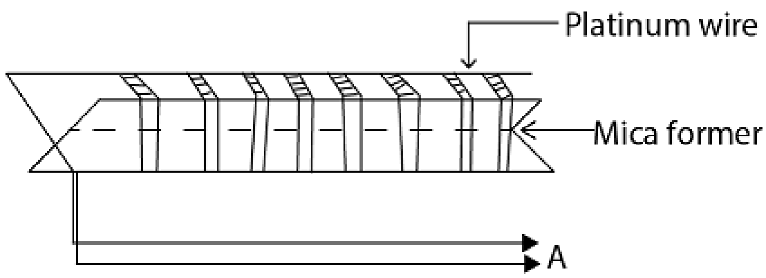
Here "a" is called temperature coefficient of resistance of the material of the wire (symbol, $\propto$)

For platinum wire $\propto = 3.8 \times 10^{-4}/°C^{-1}$

$$R_\theta = R_o(1 + \alpha\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

CONSTRUCTION

A sample from a platinum resistance thermometer consist of a platinum wire that would be around the mica former.



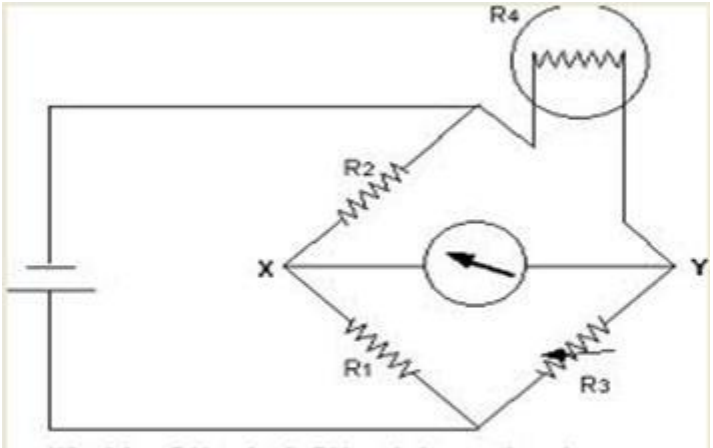
The wire is enclosed in a protective tube of quartz, glass or porcelain tube depending upon the type of application and temperature range.

The platinum wire is used because of its high temperature coefficient of resistance and high melting point (1773 ℃)

Therefore, considerable change in resistance occurs for a relatively small change in temperature.

OPERATION

The platinum resistance thermometer forms one of the four arms of the Wheatstone bridge.



R_1 and R_2 are fixed resistors while R_3 is a variable resistor.

The bridge is often kept at a considerable distance from the testing point

Under ordinary condition the bridge is balanced, that means the galvanometer shows no reading.

When the temperature changes the resistance r of the resistance thermometer also change

Consequently, the bridge no longer remains balance and some current flows through the galvanometer.

The change in resistance (and hence current through G) is a measure of the magnitude of temperature.

The accuracy of the platinum resistance thermometer depends on how accurately the bridge can be balanced.

Let R_0, R_{100} and R_θ be resistance of platinum wire at ice point (0 °C), steam point (100 °C) and at the unknown temperature θ°C respectively

The temperature θ of the scale is given by:

$$\theta = \left[\frac{R_\theta - R_0}{R_{100} - R_0}\right] \times 100°C$$

ADVANTAGES OF USING PLATINUM RESISTANCE THERMOMETER

(i) High sensitivity (0.00005 $^{\circ}C$)

(ii) Small size

(iii) Measurements can be made over a wide range (260 $^{\circ}C$ - 1200 $^{\circ}C$)

DISADVANTAGES OF USING PLATINUM RESISTANCE THERMOMETER

(i) High cost

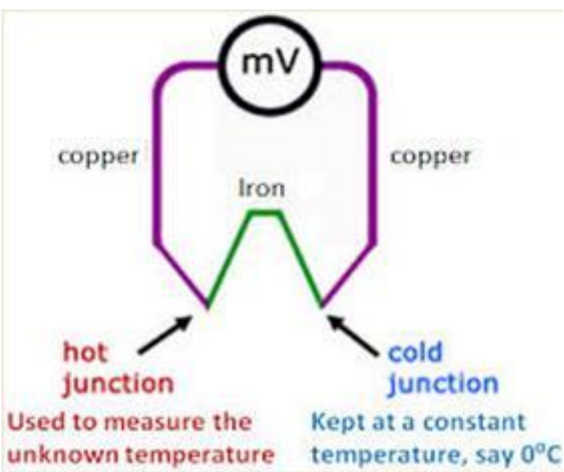
(ii) Requires additional equipments such as the bridge circuit, power supply etc.

(iii) Larger size than thermocouple

(iv)THERMOCOUPLE THERMOMETER

A thermocouple is a device consisting of two dissimilar metal wires welded together at their ends.

A thermoelectric ($e.m.f$) is generated in the device when the ends are kept at two different temperatures.



The magnitude of the $e.mf$ generated is related to the temperature difference between the two junctions.

This enables a thermocouple to be used as a thermometer over a limited temperature range

OPERATION

One of the two junctions called the hot or measuring junction is placed at the temperature to be measured.

The other junction($i.e$ the cold or reference junction) is maintained at a known reference temperature (usually 0$^{\circ}C$)

The $e.mf$ generated is measured by a suitable millvoltmeter or a potentiometer incorporated in the circuit.

The amount of the $e.mf$ generated depends upon the temperature difference between the hot and the cold junction

The greater the $e.mf$ the greater is the temperature difference between the junctions.

VARIATION OF THERMOELECTRIC $E.M.F$ WITH TEMPERATURE

When the cold junction of a given thermocouple is kept constant at 0$^{\circ}C$ and the hot junction temperature 0$^{\circ}C$ is varied. The e. m. f is found to relate with the temperature difference θ between the junction by the equation

$$E = A\theta + B\theta^2$$

Where A and B are constants.

This is a parabolic equation and hence a graph of E against θ is a parabola of the nature shown in the figure below.

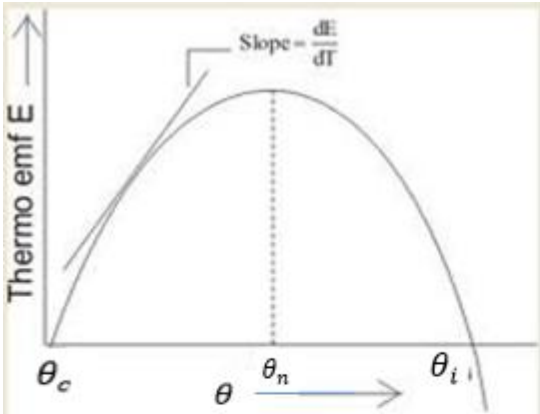


Figure 2. Variation of thermo emf with temperature of hot junction

Where T_c = Temperature of the cold junction
T_n = Neutral temperature

T_i = Inversion temperature

Neutral temperature (θ_n)

Is the temperature at which the $e.m.f$ of a thermocouple is maximum.

When the temperature is increased beyond θ_n the thermoelectric e. m. f decreases untill it becomes zero when the temperature is θ_1 called inversion temperature.

Inversion Temperature

Is the temperature to which the hot junction of a thermocouple must be raised in order that the thermoelectric e. m. f in the whole circuit becomes zero.

***EXPRESSION OF NEUTRAL TEMPERATURE OF A* THERMOCOUPLE**

From the graph above

$\theta_n - \theta_c = \theta_i - \theta_n$

$\theta_n + \theta_n = \theta_c + \theta_i$

$2\theta_n = \theta_c + \theta_i$

$$\theta_n = \frac{\theta_c + \theta_i}{2}$$

From the relationship between E and θ

E $= A\theta + B\theta^2$

Differentiating this equation with respect to θ

$\frac{dE}{d\theta} = A + 2B\theta$

When $\theta = \ \theta n$ then $\frac{dE}{d\theta}$ = 0 (slope of tangent at neutral temp)

0 = A + 2Bθ_n

2B$\theta_n = -A$

$\theta_n \ = \frac{-A}{2B}$ .... (2)

EXPRESSION OF INVERSION TEMPERATURE

From the relationship between E and θ

E = Aθ – Bθ^2

When E = 0 then θ = θ_i

$A\theta i + B\theta^2 i = 0$

$$\theta i = -\frac{A}{B}$$

$$\theta_i = \frac{-A}{B} \quad \text{.......................................} (3)$$

(1) When the inversion temperature is exceeded the thermoelectric (e. m. f) in the thermocouple is reversed.

(2) The use of a thermocouple is restricted in the temperature range between 0 and neutral temperature θ_n It is because, beyond neutral temperature θ_n the thermoelectric e. m. f decreases with increasing temperature

ADVANTAGES OF USING THERMOCOUPLES

(i) They have small heat capacities.

Therefore they have very little effect on the temperature of the body they are measuring.

(ii) They can measure rapidly fluctuating (changing) temperatures.

(iii) They are more accurate.

(iv) They are cheap and easy to use

(v) They have a wide range of temperature measurement (-200 $^\circ C$ to 1500 $^\circ C$) depending upon the metals used.

DISADVANTAGES OF USING THERMOCOUPLES

(i) Reference temperature has to be kept constant.

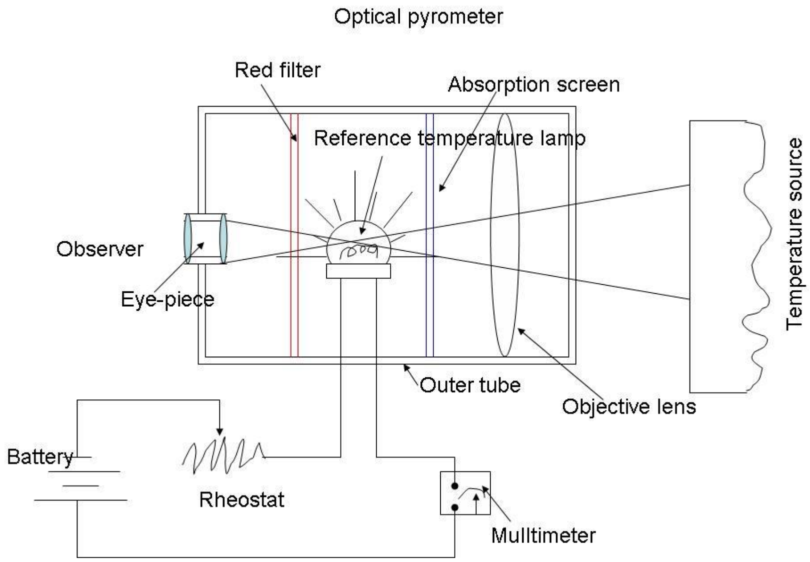
(ii) As the output voltage is less than 10mV a very sensitive meter is required.

(iii) The variation of e.m.f with temperature is non-linear

(v) RADIATION THERMOMETERS (PYROMETERS)

For measuring very high temperatures radiation thermometers (Pyrometers) are used.

In these instruments high temperatures are measured by observing the radiation from the hot body.



The thermal radiation from the hot body is compared in terms of color with thermal radiation from the lamp filament.

When a pyrometer is used a hot wire filament in the pyrometer is viewed against a glowing object.

The filament current increased from zero until it makes the filament exactly the same color as the following object.

A meter in series with the filament can then be calibrated directly in terms of source temperatures, known using the laws of radiation.

They fall into two classes:

(i) Total radiation pyrometer

Which respond to the total radiation from the hot body

(ii) Optical pyrometer

Which respond only to the visible light

Problem 01

A thermometer uses mercury as liquid in glass experiments show that the length of mercury at 0 °C and 100 °C are 5cm and 7cm respectively. At a certain temperature the length of the mercury is found to be 6.5cm, find this certain temperature. **(Answer θ = 75 °C)**

Problem 02

The pressure recorded by a constant volume gas thermometer at a Kelvin temperature T is $4.80 \times 10^4 Nm^{-2}$.

Calculate T if the pressure at triple point 273.16K is $4.20 \times 10^4 Nm^{-2}$. **(Answer. T = 312K)**

Problem 03

The resistance of a platinum wire at a temperature θ °C, measured on gas scale is given by $R_\theta = R_0 (1 + a\theta + b\theta^2)$

Where $a =$ 3.8×10^{-3} and $b = -5.6 \times 10^{-7}$

What temperature will the platinum thermometer indicate when the temperature on a gas scale is 200 °C

Problem 04

The pressure of air in a constant volume gas thermometer is 80cm and 109.3cm at 0 °C and 100 °C respectively. When the bulb is placed in hot water, the pressure is 100cm. calculate the temperature of hot water

Problem 05

The resistance of a platinum resistance thermometer is 100 Ω at room temperature of 25 °C. In an experiment for measurement of temperature, the resistance of the thermometer is found to be 115.68 Ω. find the value of temperature given that the temperature coefficient of resistance of platinum is 0.004/ °C.

Problem 06

A constant mass of a gas maintained at constant pressure has a volume of 200 cm^3 at the temperature of melting ice, 273.2 cm^3 at the temperature of water boiling under standard pressure and 525.1 cm^3 at the normal boiling point of sulphur. A platinum wire has resistances of 2.00Ω, 2.778Ω, and 5.280Ω at these temperatures. Calculate the values of boiling- point of sulphur given by the two sets of observations and comment on the results.

Problem 07

In the thermocouple, the temperature of the cold junction is 10 $^\circ C$ while the neutral temperature is 270 $^\circ C$. What is the value of temperature of inversion?.

Problem 08

In a certain thermocouple the thermo e. m. f E is given by

$$E = \propto \theta + {}^{1}/_{2}\ \beta\theta^{2}$$

Where θ is the temperature of the hot junction and the cold junction being at 0 $^\circ C$.

If $\propto$ = 10μV / $^\circ C$ and β = ${}^{-1}/_{20}\ \mu$V/$^\circ C$, find

(i) The neutral temperature

(ii) The temperature of inversion

Problem 09

(a) What does one require in order to establish a scale of temperature?

(b) A Copper – constant thermocouple with its cold junction at 0 $^\circ C$ had an EMF of 4.28mV with its other junction at 100 $^\circ C$. The EMF becomes 9.29mV when the temperature of the hot junction was 200 $^\circ C$. If the EMF E is related to the temperature different θ by the equation E = Aθ + Bθ^2, Calculate

(i) The values of A and B

(ii) The range of temperature of which E may be assumed proportional to θ without incurring an error of more that 1%?

Problem 10

The resistance R_t of a platinum varies with temperature t according to the equation $R_t =$ R$_0$ (1 + 8000bt – bt^2) where "b" is a constant.

Calculate the temperature on platinum scale corresponding to 400 °C on the gas scale

Problem 11

(a) Define the thermodynamic temperature scale

(b) The resistance of a platinum resistance thermometer is 1.20 Ω when measuring a Kelvin temperature T of a body and 1.00 Ω at the triple point of water. Find T and its centigrade equivalent.

Problem 12

(a) What do you understand by the terms

(i) Thermodynamic temperature scale

(ii) Triple point of water

(b) The resistance of a platinum wire at temperature T °C measured on a gas scale is given by

R (T) = R$_0$ ($1 + aT + bT^2$)

What temperature will the platinum thermometer indicate when the temperature on the gas scale is 200 °C?

(Take $a =$ 3.8 x 10^{-3} and $b = -$5.6 x 10^{-7})

Problem 13

(a) Define

(i) Thermodynamic temperature scale

(ii) How thermodynamic temperature donated and what is its SI unit?

(iii) Explain why a gas thermometer is seldom used for temperature measurement in the laboratory?

(b) Study the table below and answer the questions which follow:

Type of thermometer	Property	Value of property		
		Ice point	**Steam point**	**Room temp.**
Gas	Pressure in mmHg	760.0	1240.0	892.0
Thermistor	Current in mA	12.0	70.0	28.0

(i) Calculate the temperature of the room for each thermometer

(ii) Explain why thermometers disagree in their values of room temperature.

(iii) What are the advantages of gas thermometer over liquid in-glass thermometers?

Problem 14

(a) (i) Describe how mercury in glass thermometer could be made sensitive.

(ii) A sensitive thermometer can be used to investigate the difference in temperature between the top and bottom of the waterfall. Calculate the temperature difference of the water fall 50m high.

(b) (i) Platinum resistance thermometer and constant volume gas thermometer are based on different thermometric properties but they are calibrated using the same fixed points. To what extent are the thermometers likely to agree when used to measure temperature near the ice point and near the steam point.

(ii)The resistance of the element of a platinum resistance thermometer is 2.0Ω at ice point and 2.73Ω at steam point. What temperature on the platinum resistance scale would correspond to resistance value of 8.34Ω and when measured on the gas scale the same temperature will correspond to a value of 1020$^{\circ}C$? Explain the discrepancy.

Problem 15

(a). (i) What is meant by a thermometric property of a substance?

(ii) What qualities make a particular property suitable for use in practical thermometers

(b) Explain

(i) Why at least two (2) fixed points are required to define a temperature scale?

(ii) Mention the type of thermometer which is most suitable for calibration of thermometers.

This is the transfer of heat energy from one body or system to another as a result of difference in temperature. In general heat energy transfers from the region of higher temperature to the region of lower temperature.

WAYS OF HEAT TRANSFER

There are three ways by which heat can be transferred

(i) Conduction

(ii) Convection

(iii) Radiation

THERMAL CONDUCTION

This is the process in which heat flows from the hot end to the cold end of the solid body without there being any net movement of the particles of the solid.

MECHANISM OF THERMAL CONDUCTION

MECHANISM 1

The molecules of a solid vibrate about their fixed positions with an energy that increases with temperature.

When a part of the solid is heated, the molecules there start vibrating more violently.

Since neighboring molecules are bound to each other, a molecule vibrating with larger energy will transfer some of its energy to its neighbors which in turn will transfer energy to the next neighbors and so on.

MECHANISM 2

In case of metals heat energy can also be transported by the free electrons.

Since the electrons are very small, they can travel rapidly around throughout the specimen transferring energy by collision to other electrons and other molecules.

Hence, the electrons are more effective in transferring energy from the hotter part to the colder part of the material than the mechanism explained above (mechanism 1)

This explains why thermal conduction in metals is much more than that in insulators

In metals heat energy is mainly carried by the free electrons although some energy is carried by intermolecular vibration.

IMPORTANT TERMS

(1) RATE OF HEAT FLOW

Symbol, $Q/_t$ or $dQ/_{dt}$

This is the heat flow per unit time in a material

(2) TEMPERATURE DIFFERENCE

Symbol (θ_1 - θ_2) or dθ

This is the difference between higher and lower temperatures.

Heat flows from the region of higher temperature to the region of lower temperature and if $\theta_1 > \theta_2$ then,

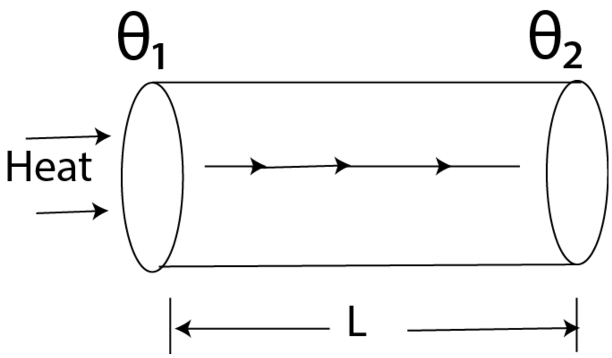
Temperature difference = $\theta_1 - \theta_2$

(3) TEMPERATURE GRADIENT

Symbol $\frac{(\theta_1 - \theta_2)}{l}$ or $\frac{d\theta}{dl}$

This is the temperature difference per unit length of the material.

It is a fall of temperature with distance between the ends of the body in the direction of heat flow.



Temperature gradient = $\frac{\theta_1 - \theta_2}{l} = \frac{d\theta}{dl}$

(4) STEADY CONDITION

This is an equilibrium point in a material when at every point the temperatures are constant.

(5) LAGGED MATERIAL

A material enclosed by an insulator (bad conductor of heat) so that the heat loss to the surrounding is negligible.

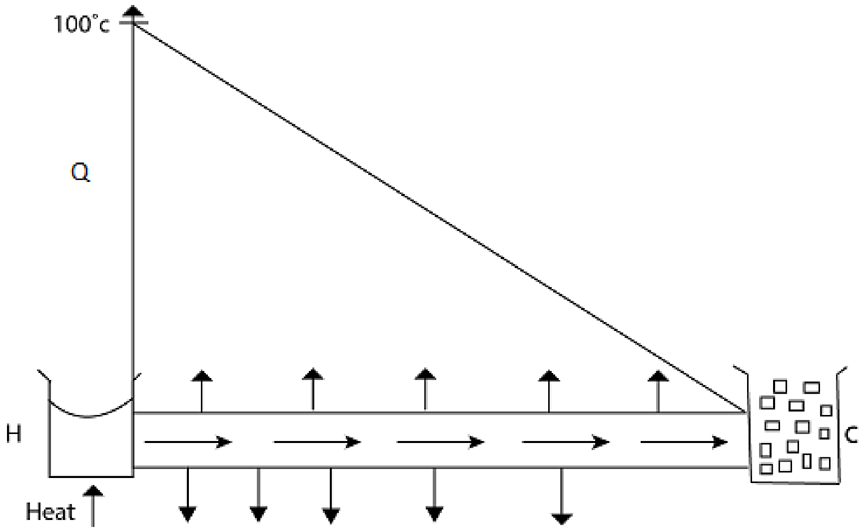
(6) UNLAGGED MATERIAL

A material which is not enclosed by an insulator so that the heat is lost to the surrounding.

TEMPERATURE DISTRIBUTION ALONG THE CONDUCTOR

1. **UNLAGGED CONDUCTOR**

Consider an unlagged metal bar AB whose ends have been soldered into the metal tanks H and C



H contains boiling water and C contains ice water.

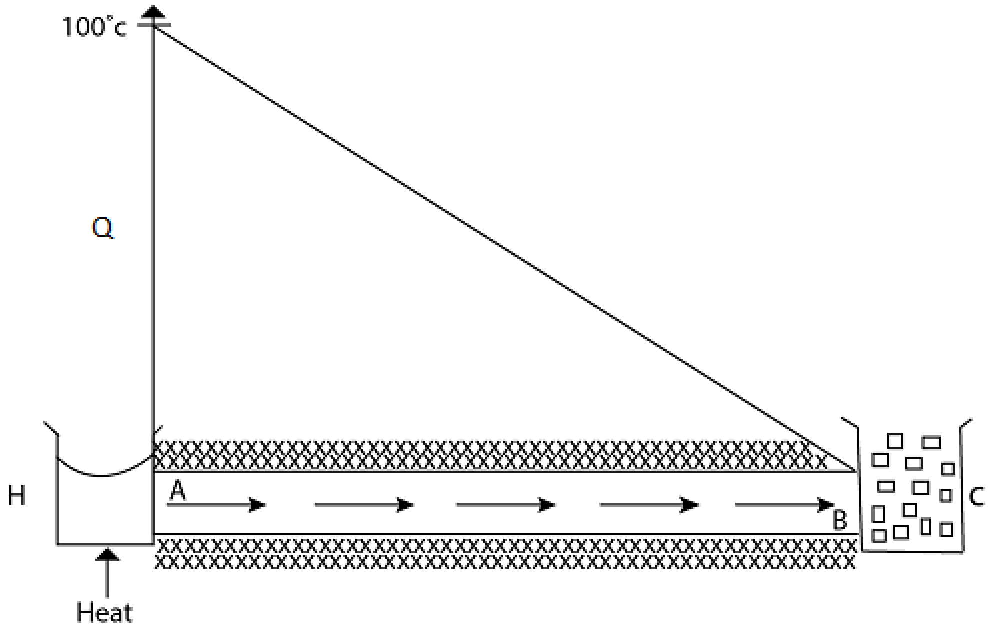
Heat flows from the hot end to the cold end of the bar and when the conditions are steady the temperature θ are measured at points along the length of the bar.

This happens simply because some amount of heat is lost to the surrounding by convection and radiation.

2. LAGGED CONDUCTOR

If the metal bar is well-lagged with a bad conductor of heat such as asbestos and wool the temperature now falls uniformly from the hot end to the cold end of the bar.

A graph of temperature against length of the bar is shown below:



Since the metal bar is well-lagged no heat is lost to the surrounding and a graph of fall of temperature against length of the bar is a straight line (see figure above)

THERMAL CONDUCTIVITY

It is measure of the ability of a material to conduct heat.

Consider a conductor of length L of cross – sectional area A

Let θ_1 and θ_2 be temperature on the opposite sides of the conductor with $\theta_1 > \theta_2$

Experiments show that the heat flow per second Q/t from the hot sides to the cold side of the conductor is:

(i) Directly proportional to the cross-sectional area A of the conductor

$$\Rightarrow \quad Q/t \propto A$$

(ii) Directly proportional to the temperature difference $(\theta_1 - \theta_2)$ between the two sides

$$Q/t \propto (\theta_1 - \theta_2)$$

(iii) Inversely proportional to the perpendicular distance between the concerned faces

$$Q/t \propto \frac{1}{l}$$

Combining the above three factors:

$$\therefore \quad \frac{Q}{t} = KA\frac{\theta_1 - \theta_2}{l} \quad \text{.............................. (1)}$$

Where K = constant of proportionality called coefficient of thermal conductivity (thermal conductivity) of the material.

Equation (1) above assumes that:

(i) The opposite sides are parallel

(ii) There is no heat loss through the sides.

From equation (1)

$$K = \frac{\frac{Q}{t}}{\frac{A(Q_1 - Q_2)}{l}} \quad \text{................................. (2)}$$

Definition

The coefficient of thermal conductivity K of a material is the rate of flow of heat per unit area per unit temperature gradient when the heat flow is perpendicular to the faces of a thin parallel – sided slab of the material under steady state conditions.

UNIT OF K

From equation (2)

$$K = \frac{\frac{Q}{t}}{\frac{A(Q_1 - Q_2)}{l}}$$

$$= \frac{Joule / sec}{Metre^2 \, x \, \frac{Kelvin}{Meter}}$$

$$= \frac{Watt}{meter \, x \, Kelvin}$$

Hence, the SI unit used $Wm^{-1}K^{-1}$

Equation (1) can be written as:

$$\frac{dQ}{dt} = - KA \frac{d\theta}{dl} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Where $\frac{d\theta}{dl}$ = Temperature gradient

The $-ve$ sign show that the heat flows in the direction of decreasing temperature i.e the temperature θ diminishes as the length l increases. The value of K for some common substances at room temperature are as shown in the table below:

Substance	K in $Wm^{-1}K^{-1}$
Silver	418

Copper	385
Aluminum	238
Iron	80
Lead	38
Mercury	8
Glass (Pyrex)	1.1
Brick	-1
Rubber	0.2
Air	0.03

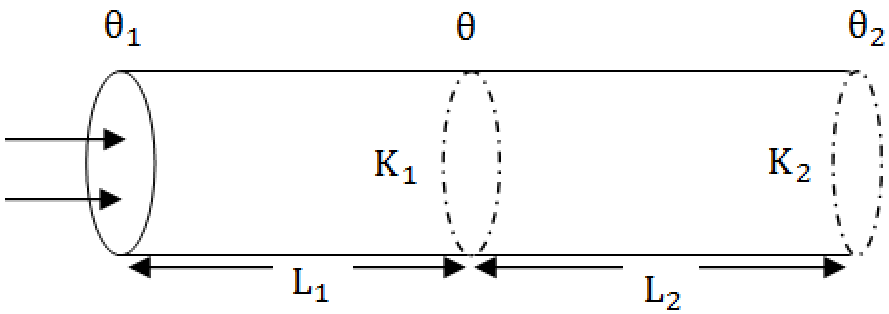
Substance for which K is small is bad conductors of heat.

Substance for which k is large conducts heat rapidly and are said to be good conductors of heat.

COMPOSITE BARS

A composite bars is that bar consisting of two or more metal bars of different materials joined end to end.

Consider a composite bar made of different materials of coefficient of thermal conductivities K_1 and K_2 respectively



Let A be cross sectional area of the bar

Assuming the bar is well – lagged so that no heat leaves from it.

Condition

At steady state condition, the heat flowing into one end of the bar is equal to that flowing out of the other end.

$i.e$ The rate at which heat flows is one material is equal to that in the other material.

$$\frac{Q}{t} = K_1A\frac{\theta_1 - \theta}{l_1} = K_2A\frac{\theta - \theta_2}{l_1}$$

Where θ_1, θ and θ_2 are the temperatures at the ends of the bars respectively.

THERMAL RESISTANCE (R)

From the heat conduction equation:

$$\frac{Q}{t} = KA\frac{\theta_1 - \theta_2}{l}$$

This equation can be linked to Ohm's law for electricity

$$I = \frac{V}{R}$$

$$I = \frac{d\theta}{dt} = {}^{V}/_{R}$$ .... (2)

From equation (1) and equation (2) above both $\frac{Q}{t}$ and I are flow quantities.

In equation (1) the heat flows per second $\frac{Q}{t}$ produced by a temperature difference $\theta_1 - \theta_2$.

In equation (2) the flow of current I (= charge flow per sec) is produces by potential difference V

Thus, the quantity $\frac{KA}{l}$ is thermal Equivalent of ${}^{1}/_{R}$

$\frac{KA}{l} = {}^{1}/_{R}$

$R = \frac{l}{KA}$ = Thermal resistance

Where l = conductor length

K = Thermal conductivity

A = across- sectional area of the conductor.

Alternative expression of R

From the heat conduction equation:

$\frac{Q}{t} = KA \frac{\theta_1 - \theta_2}{l}$

$\frac{Q}{t} = \frac{KA}{l}(\theta_1 - \theta_2)$

But $\frac{KA}{l} = \frac{1}{R}$

$$\frac{Q}{t} = \frac{1}{R}(\theta_1 - \theta_2)$$

HEAT-2

$$\therefore \frac{Q}{t} = \frac{(\theta_1 - \theta_2)}{R} = \frac{Temp.difference}{Thermal\ resistance}$$

or

$$R = \frac{(Q_1 - Q_2)}{\frac{Q}{t}}$$

UNIT OF R

From, $R = \frac{l}{KA} = \frac{m}{Wm^{-1}K^{-1} X\ m^2}$

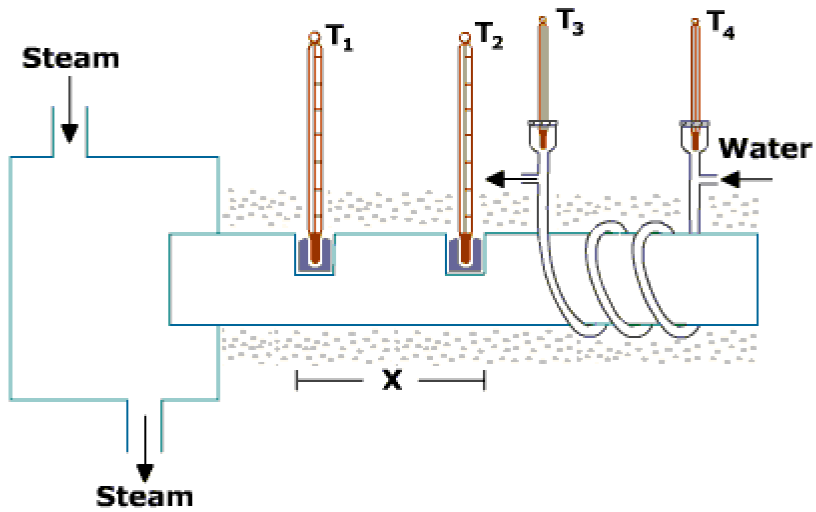
$= KW^{-1}$

From $R = \frac{(Q_1 - Q_2)}{\frac{Q}{t}} = \frac{K}{W} = KW^{-1}$

- Hence the SI unit used is Kelvin per watt (KW^{-1})

DETERMINATION OF THERMAL CONDUCTIVITY OF A GOOD CONDUCTOR OF HEAT

SEARLE'S METHOD



The heater is switched on and water is passed through the copper coil at a constant rate.

If the bar is assumed to be perfectly lagged. Then at steady state (i.e when all four thermometers give stead readings) the rate of flow of heat in distance **X** is given by:

$$\frac{Q}{t} = KA\left(\frac{\theta_1 - \theta_2}{l}\right) \qquad \text{.................. (1)}$$

Where K = Thermal conductivity of the material of the bar

Since the bar is assumed to be perfectly lagged, all of the heat which flow along the bar is being used to increase the temperature of the water.

$$\frac{Q}{t} = mc\,(\theta_4 - \theta_3) \qquad \text{............................ (2)}$$

Where m = mass of water flowing per unit time

C = specific heat capacity of water

Equation (1) = Equation (2)

…………(3)

$$KA\ \frac{(\theta_1 - \theta_2)}{l} = mc\ (\theta_4 - \theta_3)$$

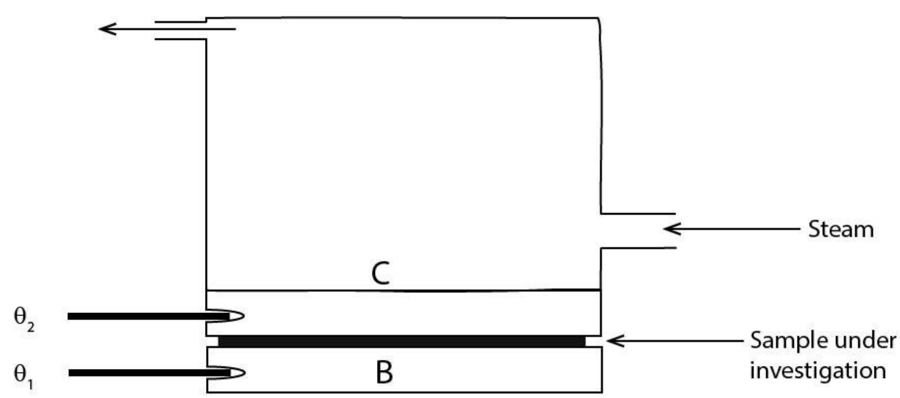
"K" can be found from this equation

The holes at **X** contain oil or mercury to ensure good thermal contact between the thermometers and the bar.

DETERMINATION OF THERMAL CONDUCTIVITY OF A BAD CONDUCTOR OF HEAT

LEES' DISC METHOD

The thermal conductivity of a bad conductor of heat such as glass, asbestos wool, paper etc can be determined by using Lees' disc method.



With this method a bad conductor of heat such as asbestos wool is sandwiched between a steam – chest C and thick brass slab B. The arrangement is suspended on three strings which are attached to B, θ_1 and θ_2 Are thermometers placed as shown, and in order to ensure good thermal contact some mercury or Vaseline is applied in the holes in which thermometers θ_1 and θ_2 are inserted.

The lower face of C and the upper face of B are made flat for uniform heat flow.

Steam is passed through the chest and the apparatus is left to reach steady state.

At steady state we have: -

$$\frac{Q}{t} = KA\frac{(\theta_1 - \theta_2)}{l} \quad ....(1)$$

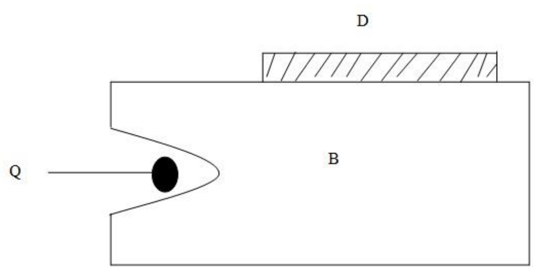
Where K = Thermal conductivity of the sample

A = Area of one face of the sample

l = Thickness of the sample

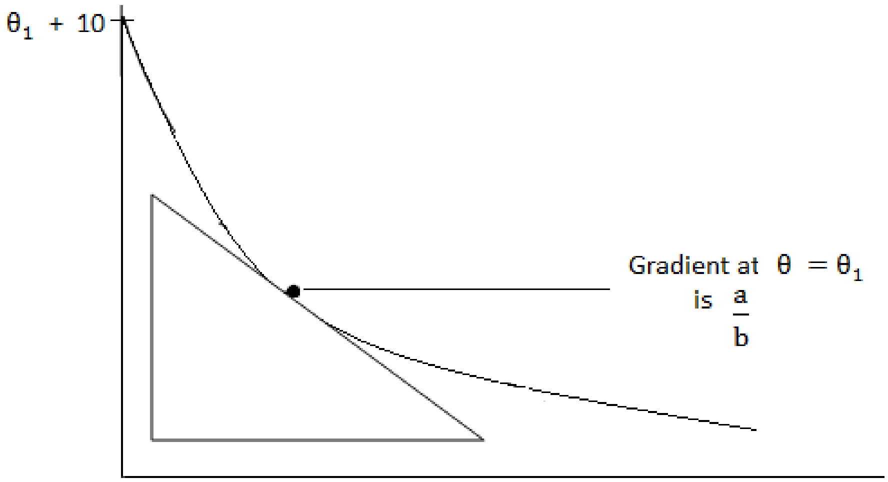
Here the rate at which heat is flowing through the sample is equal to the rate at which B is losing heat to the surroundings. Now the sample is removed so that B comes into direct contact with C and is heated by it.

When the temperature of B has risen by about 10 $^{\circ}C$, "C" is removed and the sample is replaced on top of B.



Since C is no longer present, B cools and its temperature θ is recorded at one- minute intervals until it has dropped to about 10 $^{\circ}C$ below its steady state temperature θ_1

A graph of temperature against time is plotted.



Slope of tangent at $\theta_1 = \frac{a}{b} = \frac{\Delta\theta}{\Delta t}$

heat lost per second = $M_B \quad C_B \quad \frac{\Delta\theta}{\Delta t}$

$$\text{Heat lost per second} = M_B \ C_B \ \left({}^{a}/_{b}\right) \quad \text{..........(2)}$$

Where M_B = mass of brass slab B

$C_B =$ Specific heat capacity of brass slab B

The condition under which B is losing heat are the same as those at steady state and hence

Equation (1) = Equation (2)

$$KA\frac{(\theta_1 - \theta_2)}{l} = M_B C_B \left(\frac{a}{b}\right) \quad \text{...........(3)}$$

-Thus K can be determined from this equation.

Problem 16

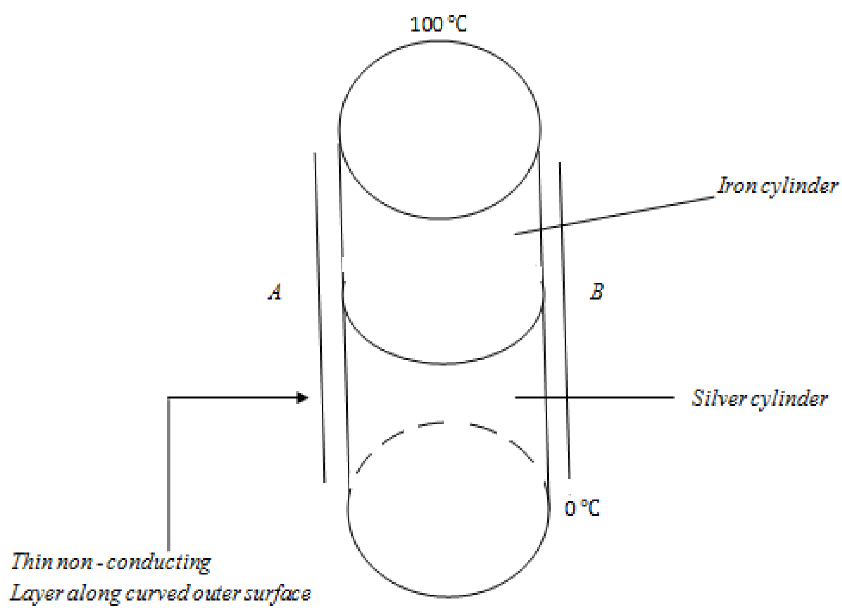
Calculate the quantity of heat conducted through $2m^2$ of brick-wall 12cm thick in 1 hour if the temperature on one side is 8 °C and the other side is 28 °C. Given that thermal conductivity of brick = $0.13Wm^{-1}K^{-1}$

Problem 17

Estimate the rate at which ice melts in a wooden box 2cm thick and inside measurements 60cm x 60cm x 60cm, assume that external temperature is 27 °C and coefficient of thermal conductivity of wood = 0.1674 $Wm^{-1}K^{-1}$. Specific latent heat of fusion of ice = 336×10^3 Jkg^{-1}

Problem 19

Two cylinders of equal physical dimensions are placed one on top of the other as illustrated below.



The lower surface of the silver cylinder is kept at 0 °C and the upper surface of the iron cylinder is kept at 100 °C. Given that the thermal conductivity of silver is eleven times that of iron, calculate the temperature of the surface AB

Problem 20

A composite bar is made of a bar of copper 10cm long, a bar of iron 8cm long and a bar of aluminium 12cm long, all having the same cross – sectional area. If the extreme ends of the bars are maintained at

100 °C and 10 °C respectively, find the temperature at the two junctions. Given that thermal conductivity of copper, iron and aluminium are 400, 40 and 20 $Wm^{-1}K^{-1}$ respectively.

Problem 21

A window of height 1.0m and 1.5m contains a double glazed unit consisting of two single glass panes, each of thickness 4.0mm separated by an air gap of 70mm. Calculate the heat energy per second conducted through the window when the temperature difference across the unit is 10K.

Problem 22

An electric heater is used in a room of total wall area $137m^2$ to maintain a temperature of 20 °C inside it when the outside temperature is 10 °C. The walls have three layers of different materials. The innermost layer is of wood of thickness 2.5cm, the middle layer is of cement of thickness 1.0cm and the outermost layer is of brick of thickness 25cm. Find the power of electric heater. Assume that there is no heat loss through the floor and ceiling. Thermal conductivity of wood, cement and brick are 1.25 $Wn^{-1}K^{-1}$,1.5 $Wm^{-1}K^{-1}$ and 1.0 $Wn^{-1}K^{-1}$ respectively.

Problem 23

(a) What does it mean by thermal conductivity of substance.

(b) Find the heat lost per square meter through a cavity wall when the temperature difference between the inside and outside is 15 °C, given that each of the two brick layers is 100mm thick and the cavity is also 100mm across.

Brick = 1.0 $Wm^{-1}K^{-1}$

Air = 0.025 $Wm^{-1}K^{-1}$

Problem 24

(a) Assuming you are managing a metal box company what requirements for thermal conductivity, specific heat capacity and coefficient of expansion would you want a material to be used as a cooking utensil to satisfy.

(b) A hot water boiler consists of iron wall of thickness 2.0cm and effective inner area of $2.5m^2$. The boiler is heated by a furnace and generates high pressure steam of temperature 170 °C at the rate of $1.2kgmin^{-1}$. The latent heat of steam at 170 °C is 2.09 x 10^6 Jkg^{-1}. Assuming the outer face of the boiler to be at a temperature of 178 °C, what is the coefficient of thermal conductivity of iron?

Problem 25

A copper kettle has a circular base of radius 10cm and thickness 3.0mm. The upper surface of the base is covered by a uniform layer of scale 1.0mm thick. The kettle contains water which is brought to the boil over an electric heater. In the steady state condition, 5.0g of steam is produced each minute. Determine the temperature of the lower surface of the base assuming the condition of heat along the surface of the cattle can be neglected. Thermal conductivities: -

Copper $=$ 3.8 x 10^2 Wm $^{-1}$k^{-1}

Scale $=$ 1.34 Wm $^{-1}$k^{-1}

Specific latent heat of steam = 226 x 10^3 JKg $^{-1}$

Problem 26

(a) Define thermal conductivity of a material

(b) Heat is supplied at the rate of 80W to one end of a well – lagged copper bar of uniform cross – sectional area 10cm^2 having a total length of 20cm. The heat is removed by water cooling at the other end of the bar. Temperature recorded by two thermometers T_1 and T_2 at distance 5cm and 15cm from the hot end are 48 $^\circ C$ and 28 $^\circ C$ respectively.

(i) Calculate the thermal conductivity of copper

(ii) Estimate the rate of flow (in g/min) of cooling water sufficient for the water temperature to rise by 5K

(iii) What is the temperature of the cold end of the bar.

Problem 27

(a) (i) The thermal conductivity β of a substance may be defined by the end equation $\frac{dQ}{dt} = -\beta A \frac{d\theta}{dx}$

(ii) Identify briefly each term in this equation and explain the minus sign.

(iii) Describe briefly one method of measuring thermal conductivity of a bad conductor in the form of disc.

(b) One end of a well lagged copper rod is placed in a steam chest and a 0.6kg mass of copper is attached to the other end of the rod with an area of 2cm^2. When steam at

100 $^{\circ}C$ is passed into the chest and a steady-state is reached the temperature of the mass of copper rises by 4 $^{\circ}C$ per minutes; if the temperature of the surrounding is 15 $^{\circ}C$. Calculate the length of the rod. Given that:

Specific heat capacity of copper

$=$ 400Jkg^{-1}K^{-1}

Thermal conductivity of copper

= 360Wm $^{-1}$K^{-1}

U – VALUE/ THERMAL TRANSMITTANCE

The U-value of a structure ($e.g$ cavity wall or a window) is the heat transferred per unit time through unit area of the structure when there is unit temperature difference across it.

U-value = $\frac{Rate\ of\ transfer\ of\ heat}{surface\ area\ x\ Temp.difference}$

This can be obtained from;

$$\frac{dQ}{dt} = -KA\frac{(T_1 - T_2)}{l} = -\frac{KA\Delta T}{l}$$

$$U = \frac{dQ/dt}{A\Delta T}$$

or

$$U = \frac{Q/t}{A\Delta T}$$

UNIT OF U – VALUE

From the definition

$$U = \frac{Q/t}{A\Delta T} = \frac{Juole/sec}{metre^2 x\ Kelvin}$$

Hence, the SI unit used in $Wm^{-1}K^{-1}$

U-values provide architects and building engineering heat losses from buildings.

They take account not only of heat lost by conduction, but of any lost by convection or radiation.

Architects base their calculations on U- values rather than on coefficients of thermal conductivity because they are concerned with the air temperature inside and outside a room, not with the temperature on the surfaces of a piece of glass, a wall etc.

THERMAL CONVECTION

This is the process by which heat is transferred from one part of fluid to another by the movement of the fluid itself.

Here the molecules of a fluid (liquid /gas) are responsible to carry heat energy from one part of a fluid to another.

TYPES OF CONVECTION

There are two types of convection:

(1) Natural / free convection

(2) Forced convection

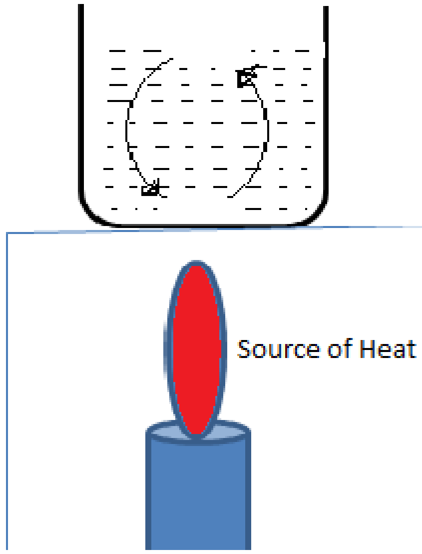
NATURAL FREE CONVECTION

This is a type of convection in which a heated fluid flows from the hot region to the cold region due to differences in density.

Example

When a fluid is heated from below, the lower part of the fluid become hot and therefore expands.

Its density decreases due to the increase in volume of fluid molecules. Its position is displaced by cold fluid from the top . This in turn gets heated and rises to the top and this process continues as shown the figure below



FORCED CONVECTION

This is a type of thermal convection in which the heated fluid is forced to move from the hot region to the cold region by means of a blower or pump. i.e by external agent

GOVERNING LAWS

Thermal convection is governed by two laws: -
(i) Newton's law of cooling
(ii)Dulong and Petit law

NEWTON'S LAW OF COOLING

Condition of the law

Newton's law of cooling is approximately true in still air only for a temperature excess of about 20k or 30k; but it is true for all excess temperature in conditions of forced convection of the air, i.e in draught.

Statement of the law

The rate of loss of heat to the surrounding air is proportional to the excess temperature over the surroundings.

Excess temperature

The excess temperature of a body over the surrounding is the difference between the temperature of the body and that of the surrounding.

Let θ be temperature of the body and θ_0 be temperature of the surrounding .

Excess temperature = $\theta - \theta_0$

Let $\frac{dQ}{dt}$ = be rate at which a body is losing heat.

From Newton's law of cooling: -

$$dQ/dt \propto (\theta - \theta_0)$$

$$\frac{dQ}{dt} \propto (\theta - \theta_0) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where K = convection coefficient

Negative sign shows that the body is losing heat

If m is the mass of a body and C is its specific heat capacity, then

$$dQ = mcd\theta$$

$$\frac{dQ}{dt} = \frac{mcd\theta}{dt}$$

For a given body mc = constant

$$\therefore \frac{dQ}{dt} \propto \frac{d\theta}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Using (2) in (1) we have: -

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

DULONG AND PETIT LAW

Dulong and Petit modified Newton's law of cooling and they stated a law which works under natural / free convection.

Condition of the law

The law works under natural /free convection where $\theta - \theta_\circ > 50k$.

Where the law works

The law works in still air $e.g$ in the laboratory

Alternative name of the law

Five – fourth power law

Statement of the law

Under natural/ free convection the rate at which a body loss heat is proportional to the five Fourth power of its excess temperature over the surroundings.

$$\frac{d\theta}{dt} = -K \quad (\theta - \theta_0)^{\frac{5}{4}}$$

→ Dulong and Petit law

Where K = Convectional coefficient

Negative sign shows that the body is losing heat.

Problem 28

A body cools from 40 °C to 30 °C in 5 minutes.The temperature of the room being 15 °C, what will be the temperature of the body after another 5 minutes?

Problem 29

In a room at 15 °C a body cools from 35 °C to 30 °C in 4 minutes. Find the further time elapse before the temperature of the body is 20 °C

Problem 30

Wind blows over a hot liquid placed in a beaker in the laboratory whose average room temperature is 27 °C. The liquid rate of cooling is 15 °C/ min when it is at a temperature of 87 °C. Calculate the liquid rate of cooling when it is at a temperature of 57 °C.

Problem 31

A body initially at 80 °C cools to 64 °C in 5 minutes and 52 °C in 10 minutes. What will be the temperature after 15 minutes and what is the temperature of the surrounding?

Problem 32

(a) State Newton's law of cooling and give one limitation of the law.

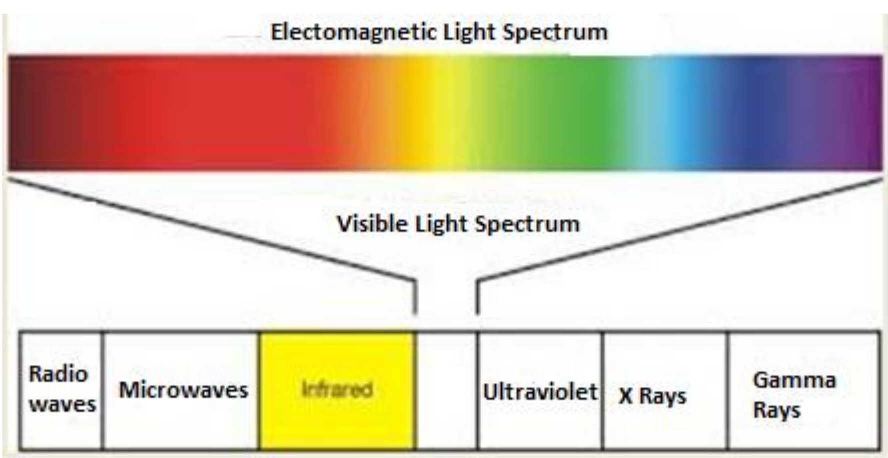
(b) A body initially at 70 °C cool to a temperature of 55 °C in 5minutes. What will be its temperature after 10minutes given that the surrounding temperature is 31 °C. (Assume Newton's law of cooling holds true)

THERMAL RADIATION

This is the transfer of heat energy from one point to another without the requirement of any material medium.

It is like a throw of radiant energy

Thermal radiation consist of electromagnetic waves with a range of wavelengths covering the infra-red and visible regions of the electromagnetic spectrum



All bodies continuously emit and absorb thermal radiation in the form of electromagnetic waves A body at higher temperature than the surrounding units it emits more radiation than it absorbs.

Thus, there is a continuous exchange of radiation between the body and the surrounding with the result that there will be a rise or fall in temperature of the body.

THE BLACK BODY

A perfectly black body is the one which absorbs completely all the radiation falling on it and reflects none

Since a perfectly black body is a perfect absorber, it will also be a perfect radiator.

When a perfectly black body is heated to a high temperature, it emits thermal radiation of all possible wavelengths.

Practical examples of perfectly black body,

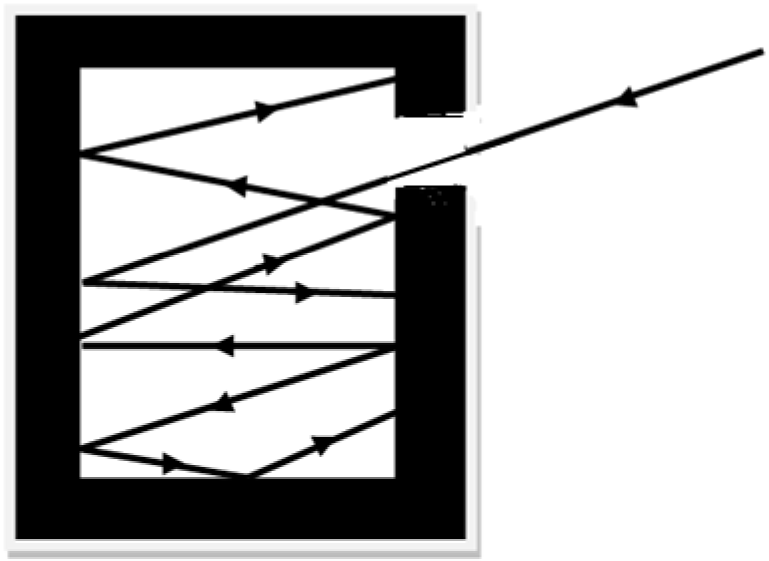
(1) The sun

(2) A surface coated with lamp- black

This surface can absorbs 96% to 98% of the incident radiation and may be considered as a perfect black body for all practical purposes

HOW TO REALIZE A BLACK BODY

A good black body can be realized simply by punching a small hole in the lid of a closed empty tin



The hole looks almost black, although the shining tin is a good reflector.

Reason

The hole looks almost black ,although the shining tin is a good reflector because the radiation that enters through it is reflected from the inside walls several times and is partially absorbed at each reflection and loses energy until no radiation is reflected back. Hence the hole absorbs all radiation falling on it.

BLACK BODY RADIATION (BBR)

Is that thermal radiation emitted by a black body at a given temperature.

Any object at a temperature greater than absolute zero emits thermal radiation of all wavelengths within a certain range.

The amount of thermal energy radiated for different wavelength intervals is different and depends on temperature and nature of the surface.

INTENSITY OF RADIATION

Symbol I

This is the rate at which radiant energy is transferred per unit area.

$$I = \frac{Energy\ emitted/time}{area}$$

$$I = \frac{Power\ emitted}{area}$$

$$I = \frac{P}{A}$$

The SI unit used is Watt/metre^2 (Wm^{-2})

LAWS OF BLACK BODY RADIATION

Law 1

WIEN'S DISPLACEMENT LAW

The wavelength λ_{max} at which the maximum amount of energy is radiated by a black body is inversely proportional to its absolute temperature

$$\lambda_{max} \propto \frac{1}{T}$$

Where K = constant of proportionality called Wien's consist (k) whose value is 2.9 x 10^{-3}mk

$$\lambda_{max} = k\frac{1}{T}$$

$\lambda_{max}T = K$

OR

$\lambda_{max}T = 2.9 \times 10^{-3}$

Law 2

STEFAN'S LAW / STEFAN'S BOLTZMAN'S LAW

The total energy emitted per unit area of a black body in unit time is directly proportional to the fourth power of its absolute temperature.

E $\propto T^4$

E $= K T^4$

Where K = consist of proportionality called Stefan's consist σ (sigma) whose value is 5.67 x 10^{-8} $Wm^{-2}K^4$

$\therefore$ $E = \sigma T^4$ .... (1)

Where E = $\frac{Energy\ radiated\ /area}{time}$

E = $\frac{Energy\ radiated}{Area\ x\ time}$

E = $\frac{Energy\ radiated\ /time}{Area}$

E = $\frac{Total\ power\ radiated}{Area}$

$\therefore$ $E = \frac{P}{A}$ .... (2)

Substitute eqn (2) in eqn (1)

$\frac{P}{A} = \sigma T^4$

$\therefore$ .... (3)

$P = A\ \sigma T^4$ → Stefan's equation

Problem 33

Assuming the total surface area of human body is $1.25m^2$ and the surface temperature is 30 °C.

Find the total rate of radiation of energy from the human body. Given that Stefan's constant, σ = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$

Problem 34

A black ball of radius 1m is maintained at a temperature of 30 °C. How much heat is radiated by the ball in 4 *second*. Given that Stefan's constant = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$

Problem 35

Two spheres made of the same material have radii 2.0cm and 3.0cm and their temperature are 627 °C and 527 °C respectively. If they are black bodies, compare:

i. (a) The rate at which they are losing heat

i. (b) The rate at which their temperature are falling.

EMISSIVITY(ε)

The emissivity (ε) of a surface is the ratio of the power radiated by a surface of a given body to that radiated by a black body at the same temperature

$$\varepsilon = \frac{\text{Power radiated by surface of a given body}}{\text{Power radiated by black body}}$$

From Stefan's law

Power / area radiated by a black body at temperature

P = ε x power of black body
$P = \varepsilon\sigma AT^4$
Where P radiated by surface of a given body.

Problems 36

A tungsten filament of total surface area of $0.45km^2$ is maintained at a steady temperature of 2227 °c. Calculate the electrical energy dissipated per second if all this energy is radiated to the surrounding. Given that emissivity of tungsten at 2227^0C = 0.3 and Stefan's constant = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$[29.9W]

Problem 37

The tungsten filament of an electric lamp has a length of 0.5m and a diameter of 6 x 10-5m.

The power of rating of the lamp is 60W. Assuming the radiation from the filament is equivalent to 80% that of a perfect black body radiator at the same temperature, estimate the steady temperature of the filament given that

Stefan's constant = 5.67 × 10^{-8} $Wm^{-2}K^{-4}$

T =935.5K

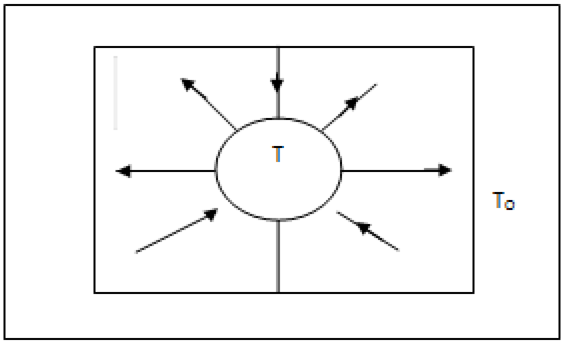
PROVOSTS THEORY OF HEAT EXCHANGE

A body radiates heat at a rate which depends only on its surface and temperature, and that it absorbs heat at a rate depending on its surface and the temperature of its surroundings

THEORY

When the temperature of a body is constant the body is losing heat by radiation and gaining it by absorption at equal rates

Consider a black body at a temperature T to be placed in an enclosure having temperature T_0



Energy radiated /sec = $A\sigma T^4$

Energy absorbed /sec = $A\sigma T_0^4$

If the body is not at the same temperature as its surrounding there is a net flow of energy between the surrounding and the body because of unequal emission and absorption

If the temperature of the body is greater than that of the surrounding, then the net energy will flow from the body to the surrounding

Net energy emitted /sec = $A\sigma T^4 - A\sigma T_0^4$

Therefore

Net energy emitted /sec = $A\sigma (T^4 - T_0^4)$

If the temperature of the body is less than that of its surrounding then the energy will flow from the surrounding to the body

Net energy absorbed /sec = $A\sigma (T_0^4 - T^4)$

Problem 38

The total external surface area of the dog's body is 0.8 m^2 and the body temperature is 37 °C at what rate is it loosing heat by radiation when it is in a room whose temperature is 17 °C? Assume that dog's body behaves as a black body and given that Stefan's constant is

5.67 x 10-8 Wm^{-2} K^{-4} [$P =$ 98.086W]

Problem 39

The cathode of a certain diode valve consists of a cylinder 2 x 10^{-2}m long and 0.1 x 10^{-2} m in diameter. It is surrounded by a co-axial anode of diameter larger than that of the cathode. The anode remains at a rate temperature of 127 °C when the power of 4 watts is dissipated in heating the cathode.

Estimating the temperature of cathode.

[1035.29]

List the assumption you have made in arriving at your estimate

Problem 40

A metal sphere with a black surface and radius 30 mm is cooled to 200k and placed inside an enclosure at a temperature of 300k. Calculate the initial rate of temperature rise of the sphere, assuming the sphere is a black body given that

Density of metal $=$ 8000 kgm-3

Specific heat capacity metal $=$ 400J$kg^{-1}K^{-1}$

Stefan's constant $=$ 5.67 $\times$ 10-8 Wm^{-2} K^{-4}

[ans dθ = 0.012 c / sec]

ENERGY DISTRIBUTION IN THE SPECTRUM OF A BLACK BODY

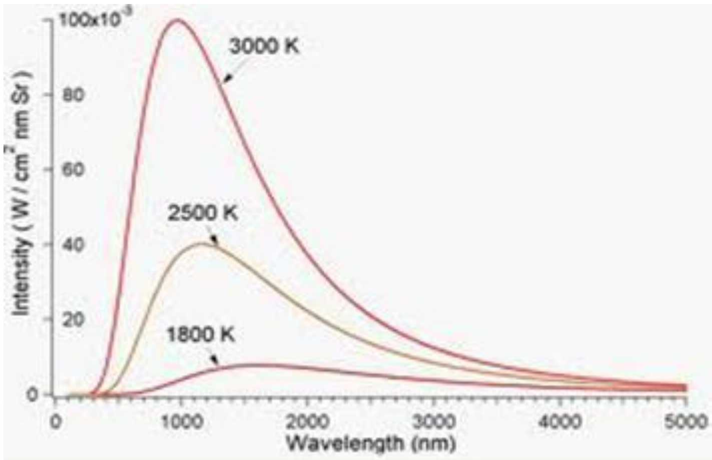
The energy radiated by a black body at constant temperature contains a continuous range of wave lights

The energy carried by the radiation is not distributed evenly across the wavelength range

However the distribution changes if the source temperature alters

The proportion of energy carried by shorter wave lengths increases as the source temperature increases

The figure below shows how the energy is distributed over the wave length range for several values of source temperature.



DEDUCTIONS

The total energy emitted by a black body increases rapidly with the increase in temperature for any wavelength.

For a given temperature, the radiant energy emitted by a black body is the maximum for a particular wave length as the temperature of the body increases the peak of the curve shifts towards shorter wave length

This is in accordance to Wien's displacement law

i.e. $\lambda_{max} \propto \frac{1}{T}$

This explains why a heated metal i.e. iron changes colours from red through yellow to white

When a metal i.e. iron is heated it first emits invisible radiation of longer wave length in infrared region

With increasing temperature the wave length of the emitted radiation becomes shorter and the metal appears red

With further increasing temperature the wave length becomes shorter and shorter and the metal emits all the colors of the visible spectrum and finally it appears white

The area enclosed by a particular curve represent the radiant energy [of all wave length] per second per unit area emitted by the black body at that temperature

When the area enclosed by a particular curve is measured, it is found to be directly proportional to the fourth power of the corresponding absolute temperature

$$E \propto T^4$$

This is in accordance to Stefan's law

EMISSIVE POWER(e_λ)

Means the emissive power of a body at a particular temperature is the total energy of all wave lengths radiated per second per unit area of the body.

ABSORPTIVE POWER(a_λ)

The absorptive power of a body at a given temperature and for a particular wavelength is the ratio of thermal energy absorbed by it in a given time to the total thermal energy incident on it for the same time , both in the unit wave length around

$$a_\lambda = \frac{thermal\ energy\ absorbed}{total\ \ incident\ energy}$$

KIRCHHOFF'S LAW OF BLACK BODY RADIATION

The law state

"The ratio of the emissive power to the absorptive power of radiation of a given wavelength is the same for all bodies at the same temperature and is equal to the emissive power of a perfectly black body at the temperature".

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = constant$$

Where

$e =$ Emissive power of a body corresponding to wave length

$a_\lambda =$ Absorptive power of a body corresponding to wave length

$E_\lambda =$ Emissive power power of a perfectly black body at the same temperature corresponding to wave length

From Kirchhoff's law

$$\frac{e_\lambda}{a_\lambda} = E_\lambda$$

$$e_\lambda = E_\lambda a_\lambda$$

Since $E_\lambda = Constant$

$$e_\lambda \; \alpha \; a_\lambda$$

If e_λ is large a_λ is also large

Therefore

If a body emits strongly the radiation of a particular wavelength, then it must also absorb the same wave length strongly.

i.e. good emitters of heat also good absorbers and vice- versa

SOLAR LUMINOSITY (Ls)

This is the amount of energy emitted by the sun per second in all directions

SOLAR CONSTANT (C)

This is the amount of energy received from the sun by the earth per unit time normally at the mean distance of the earth from the sun

Its value is 1.388 x 103 Wm $^{-2}$

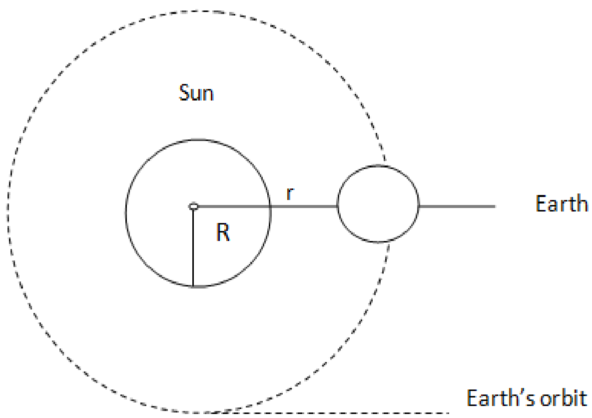
$$C = \frac{Solar\ luminosity}{Area\ of\ sphere\ of\ radius\ r}$$

$$C = \frac{L_s}{4\pi r^2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (1)$$

$L_s = 4L\pi r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$

SURFACE TEMPERATURE OF THE SUN

Consider the sun to be black body of radius R and temperature T



Let r be mean distance of the earth from the sun

If C is the solar constant then:

Solar luminosity $= 4\pi r^2 C \ldots\ldots\ldots\ldots\ldots(1)$

From Stefan's law

Energy emitted by the sun per second

$$P = A\sigma T^4$$

Where $A = 4\pi R^2 =$ surface of the sun

Therefore,

$P = 4\pi R^2 \sigma T^4$ ------------- 2

Assuming there is no loss of radiant energy

Equation 1 = equation 2

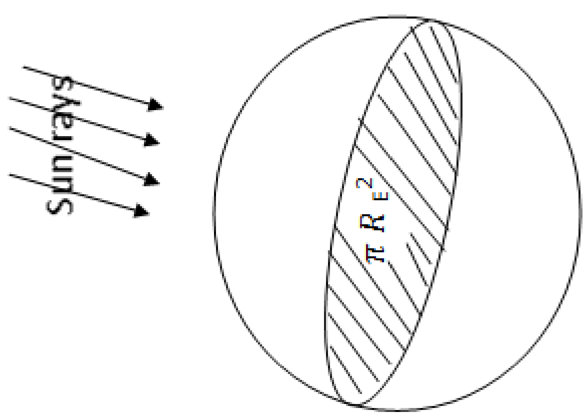
$4\pi r^2 C = 4\pi R^2 \sigma T^4$

$T^4 = {4\pi r^2 C}/{4\pi R^2 \sigma}$

$\therefore T = \left([r/R]^2 \times [C/\sigma] \right)^{1/4}$

TO CALCULATE THE TEMPERATURE OF THE EARTH

The effective area of the earth on which the sun radiation is incident normally is:



Earth.

Power which the sun radiates = $\sigma\ (4\pi R_s^2)\ Ts^4$.

Power which the Earth receive from the sun = $4\pi R\epsilon^2$.

Power which the sun radiates $4\pi R_s^2$.

Where $4\pi R_s{}^2{}_\epsilon$ is surface area on the power from the sun spreads.

Assuming the earth to be a black body then the power which the earth radiate = $\sigma(4\pi R\epsilon^2)\ T_\epsilon{}^4$

Where Tε is temperature of the earth. Assuming dynamic eradicative equilibrium .

Power which earth radiates = power which earth receives from the sun .

$$\sigma\ (4\pi R_\epsilon{}^2)\ T_\epsilon{}^4 = 4\pi R_\epsilon{}^2 \quad X\ \frac{(4\pi R_S{}^2)\ T_S{}^4}{4\pi R^2{}_{S\epsilon}}$$

$$T_\epsilon{}^4 = \frac{R_S{}^2\ T_S{}^4}{4R^2{}_{S\epsilon}}$$

$$T_\epsilon = {}^4\sqrt{Ts^4}\ \frac{1}{4}\ \left(\frac{Rs}{Rs\epsilon}\right)^2$$

$$T_\epsilon = {}^{Ts}\sqrt{\frac{1}{2}\ \left(\frac{Rs}{Rs\epsilon}\right)^2}$$

Using Ts = 6000k
Rs = 6. 96 x 10^8m
RsÑ” = 1.5 X 10^{11} m

$$T\epsilon = 6000\sqrt{\frac{6.96 \times 10^8 m}{2 \times 1.5 \times 10^{11} m}} \approx 290k$$

Example

(i) State both Stefan's law and Newton's law of cooling Stefan's law states the total power radiated per unit area is proportional to the fourth power of the absolute temp .

$$\frac{P}{A} \propto T^4$$

Newton's law of cooling ;

$$\frac{\partial Q}{\partial t} \propto A(\theta - \theta_s)$$

(ii) Mention one significant limitation of the stated laws Dulong of Petit law $(\theta - \theta_s) > 50k$ for natural convection.

Newton's law of cooling $(\theta - \theta_s) > 30K$ for natural convection.

(b)Use some of the constant on the front page. Calculate the temperature of the sun.

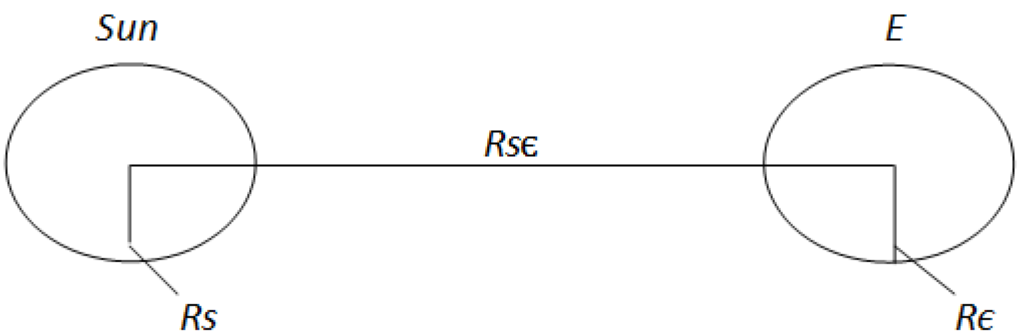
Solar constant = 1350Wm $^{-2}$

Mean earth – sun distance = 1.5 x 10^{11} m

Radius of the sun = 6.965 x 10^5 km

Stefan's constant = 5.67 x 10 – 8 Wm^{-2} K^{-4}

Solution:



Power radiated by sun

$$= \sigma A T^4$$
$$= \sigma 4\pi R^2 T^4$$

Power received or reach to the surface sphere of radius

$R\epsilon$ = Solar constant x area

$Sc \times A \rightarrow Sc\, 4\pi Rs^2\epsilon$

The power which the earth receive = power which sun radiated

$$= Sc\ 4\pi R^2 s\epsilon \ = \sigma\ 4\pi R^2 s T^4.$$

$$= Ts4 = \frac{Sc\ 4\pi R^2 s\epsilon}{\sigma\ 4\pi R^2 s}$$

$$= T\ 4 = \frac{Sc\ R^2 s\epsilon}{\sigma\ R^2 s}$$

$$T = \sqrt[4]{\frac{SC.\ R^2 s\epsilon}{\sigma Rs^2}}$$

$$= \sqrt[4]{\frac{1350 \times (1.5 \ \times 10^{11} m)^2}{5.67 \times 10^{-8} \times (6.965 \ \times 10^{8})^2}}$$

$$= T\ 1.82 \times 10^5 K\ .$$

Problem 41

The energy arriving per unit area on the earth's surface per second from the sun is 1.34 x 10^3 Wm^{-2} the average distance from the earth to the sun is 215 times as great as the sun's radius. Given that both the earth and the sun are black bodies

Stefan's constant = 5.67 x 10-8 $Wm^{-2}K^{-4}$

[T = 575k]

Problem 42

The amount of radiant heat received by the earth from the sun is $1.38 \times 10^3\, Wm^{-2}$

Suppose all these radiations on the earth are re emitted by the earth. Calculate the temperature of the earth

Given Stefan's constant

=5.67 x 10-8 Wm^{-2} K^{-1}

[T = 395K]

Problem 43

The surface temperature of the sun is 6000K. If we consider it as a perfect black body, calculate the energy radiated by the sun per second.

Given that the radius of the sun

= 6.92 $\times$ 108m and σ = 5.67 $\times$ 10-8Wm^{-2}K^{-4}

[P = 4.42 $\times$ 1026W]

Problem 44

The temperature of a furnace is 2324°C and the intensity is maximum in its radiation spectrum nearly at 12000A^0. If the intensity in the spectrum of a star is maximum is nearly at 4800A^0 then calculate the surface temperature of the star

[$T_2 =$ 6492.5K]

Problem 45

The wavelength corresponding to maximum energy for the moon is 14 x 10^{-6}m estimate the temperature of the moon if b = 2.884 x 10-3 mK

[T = 206K]

ALBEDO

Each planet absorbs a certain fraction of energy from the sun and the rest of the energy is reflected back into space

Definition

$Albedo$ Is the ratio of the sun's energy reflected by a planet to the falling on it

$$Albedo = \frac{Energy\ reflected\ by\ a\ plate}{Energy\ falling\ on\ the\ planet}$$

IMPORTANCE OF ALBEDO

It helps to know whether a certain planet has a cloud cover or not.

Clouds are a good reflector of radiant energy and hence increase the reflecting power of a planet

High value of $albedo$ means the planet has dense clouds

GAS LAWS

Gas laws try to describe the behavior of gases in relation to temperature, pressure and volume.

A gas law is obtained when two of these three quantities are variant while the third is kept constant.

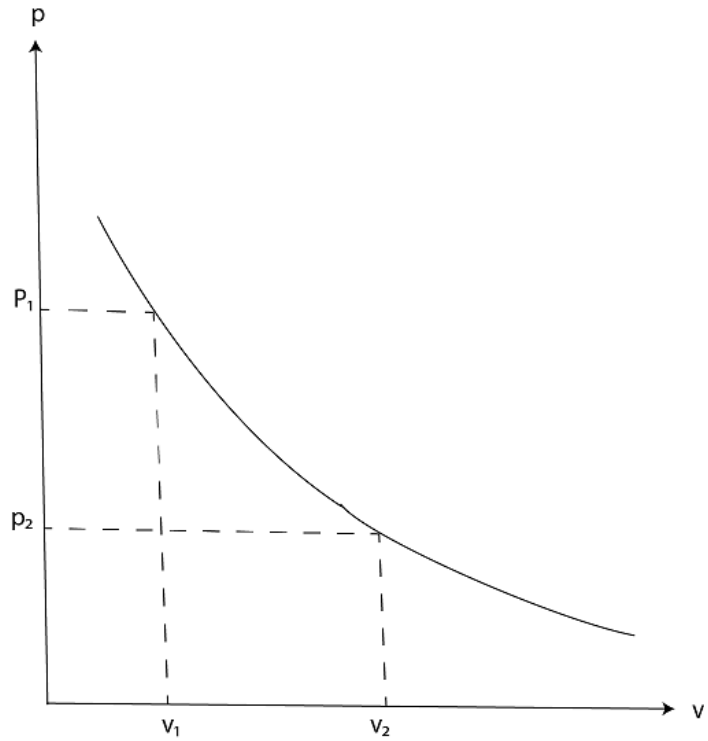
BOYLE'S LAW

The law states that

"The pressure P of a given mass of a gas is inversely proportional to its volume [V] at constant temperature".

P $\propto \frac{1}{v}$ $at\ T = constant$

The relationship can be represented graphically as shown below



The graph shows that when the volume is increased from V_1 to V_2 the pressure decreases from P_1 to P_2 and vice versa

From Boyle's law

$$P \propto \frac{1}{V}$$

$$P = K.\frac{1}{V} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Where K = constant of proportionality

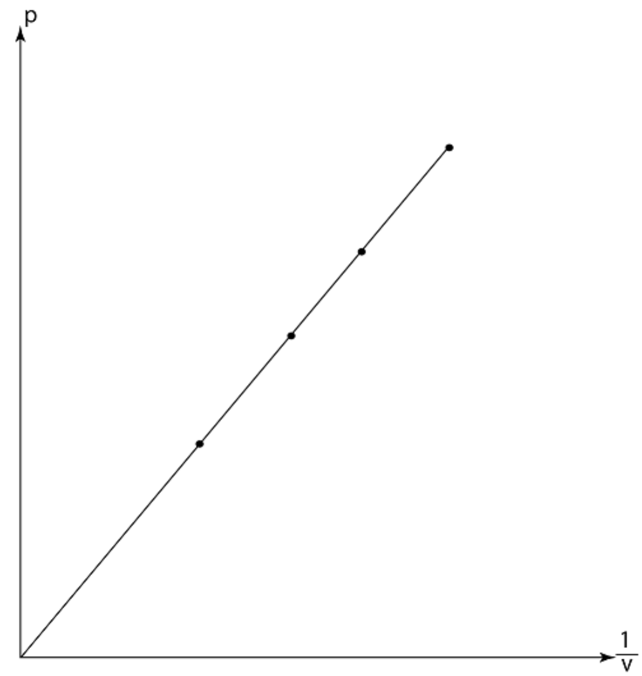
Upon comparing this equation with the linear equation we have

$$P = K.\frac{1}{V} + 0$$

↓ ↓ ↓ ↓

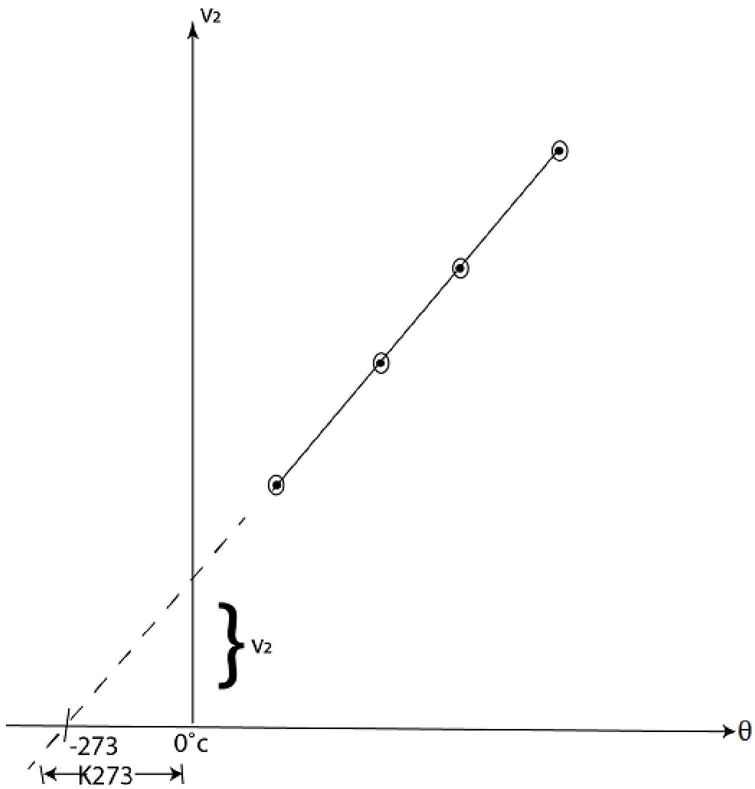
$$y = mx + C$$

A graph of P against $\frac{1}{V}$ is of nature shown below



From the equation (1) above

PV = K---------2



If a gaseous system changes from P_1 and V_1 condition to another P_2 and V_2 condition then we have

$$P_1V_1 = P_2V_2 \dots\dots\dots\dots\dots..(3)$$

CHARLE'S LAW

EXPANSIVITY OF A GAS AT CONSTANT PRESSURE

Volume coefficient (∝)

Definition

The expansively of a gas at constant pressure [volume coefficient] is the increase in volume at 0°C per degree centigrade rise in temperature when the gas is at constant pressure

$$\propto = \frac{Increase\ in\ volume}{Volume\ at\ 0^0C \times Rise\ in\ temp.}$$

$Let\ V_0\ be\ volume\ of\ gas\ at\ 0^0 C$

$Let\ V_\theta\ be\ volume\ of\ agas\ at\ \theta^0$

If 0 is rise in temperature of the gas then

$$\propto = \frac{V_\theta - V_0}{V_0 \theta}$$

Where $V_\theta - V_0 = Increase\ in\ volume\ of\ a\ gas$

-

From the definition of $\propto$

$$\propto = \frac{Increase\ in\ volume}{Volume\ at\ 0^0 C \times Rise\ in\ temp.}$$

$$= \frac{metre^3}{metre^3 \times C} = 0_{C^{-1}\ OR\ K^{-1}}$$

Hence, the SI unit used is $C^{-1}\ or\ K^{-1}$

From the equation (1) above

$$\propto = \frac{V_\theta - V_0}{V_0 \theta}$$

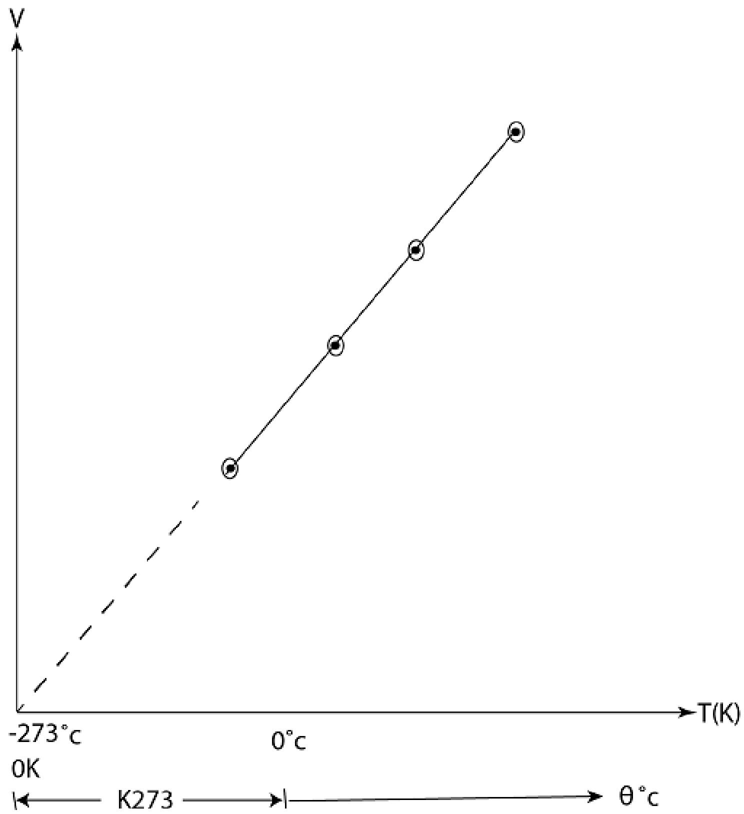
$$\propto V_0 \theta = V_\theta - V_0$$

$$\therefore V_\theta = (\alpha V_0)\theta + V_0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . (2)$$

$$\downarrow \quad \downarrow \; \downarrow \quad \downarrow$$

$$y \;=\; mx + \; c$$

A graph of V_θ against temperature θ is of the nature shown below



Deductions from the graph

Deduction 1

Slope [m] of the graph $=\propto V_0$

Therefore

$$\therefore \alpha = \frac{slope\ (m)}{V_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Knowing slope [m] and the intercept V_0 of the graph they can be found from equation(3)

Charles' found that all gases

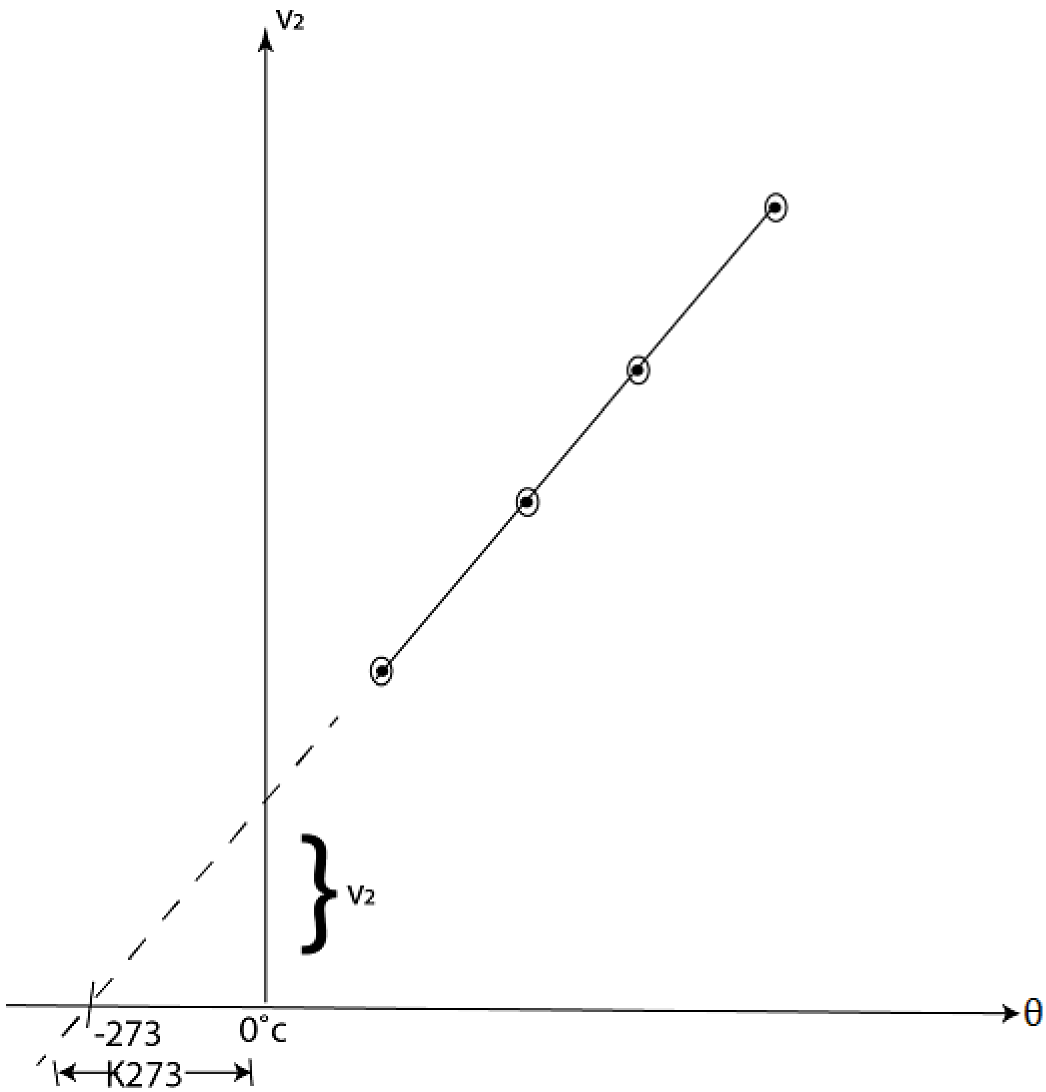
$$\propto = \frac{1}{273}{}^{0}C^{-1}$$

Deduction 2

When the graph is extra-plotted backwards it cuts the temperature axis at – 273°C

The value -273°C is called absolute ZERO [OK]

If the volume axis of the graph stands -273°C one has nature shown below



Since there is absolute zero at -273°C the axis is now called absolute temperature axis of unit Kelvin [K] which is related to the centigrade scale by the equation

$$T\ [K] = 273 + \theta\ [^0C]$$

The graph above shows that when the temperature T[K] increases the volume V of the gas also increases

$$V \propto T$$

Hence

We have Charles law which states that:

"The volume of a fixed mass of gas at constant pressure is proportional to its thermodynamic temperature".

$$V = KT$$

Where

K $=$ constant proportionality

This law is known as Gay-Lussac's law

If a gaseous system changes from one V_1 and T_1 condition to another V_2 and T_2 condition, then

$$V_1 = KT_1 \dots \dots \dots \dots \dots \dots \dots \dots \dots (1)$$

$$V_2 = KT_2 \dots \dots \dots \dots \dots \dots \dots \dots . (2)$$

$$\frac{equation\ (1)}{equation\ (2)} = \frac{V_1}{V_2} = \frac{KT_1}{KT_2}$$

$$\therefore \frac{V_1}{V_2} = \frac{T_1}{T_2} \dots \dots \dots \dots \dots \dots \dots . . (3)$$

PRESSURE LAW

EXPANSIVITY OF A GAS AT CONSTANT VOLUME

Pressure coefficient(β)

The expansivity of a gas at constant volume [pressure coefficient] is the increase in pressure of a gas per pressure at 0^0C per degree centigrade rise in temperature when the gas is at constant volume

$$\beta = \frac{Increase\ in\ pressure}{Pressure\ at\ 0^0C \times Rise\ in\ temp.}$$

$$\beta = \frac{P_\theta - P_0}{P_0 \theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where

$P_\theta =$ pressure of a gas at a temp θ^0C

$P_0 =$ pressure of a gas at 0^0C

$\theta =$ rise in temp of the gas

UNIT OF β

By definition

$$\beta = \frac{Increase\ in\ pressure}{Pressure\ at\ 0^0C\ \times Rise\ in\ temp}$$

$$= \frac{Nm^{-2}}{Nm^{-2} \times Kelvin}$$

$$= K^{-1}\ OR\quad {}^0C^{-1}$$

Hence the SI unit used is $K^{-1}\ OR\quad {}^0C^{-1}$

From equation (1) above

$$\beta = \frac{P_\theta - P_0}{P_0 \theta}$$

$$\beta P_0 \theta = P_\theta - P_0$$

$$\therefore P_\theta = (\beta P_0)\theta + P_0$$

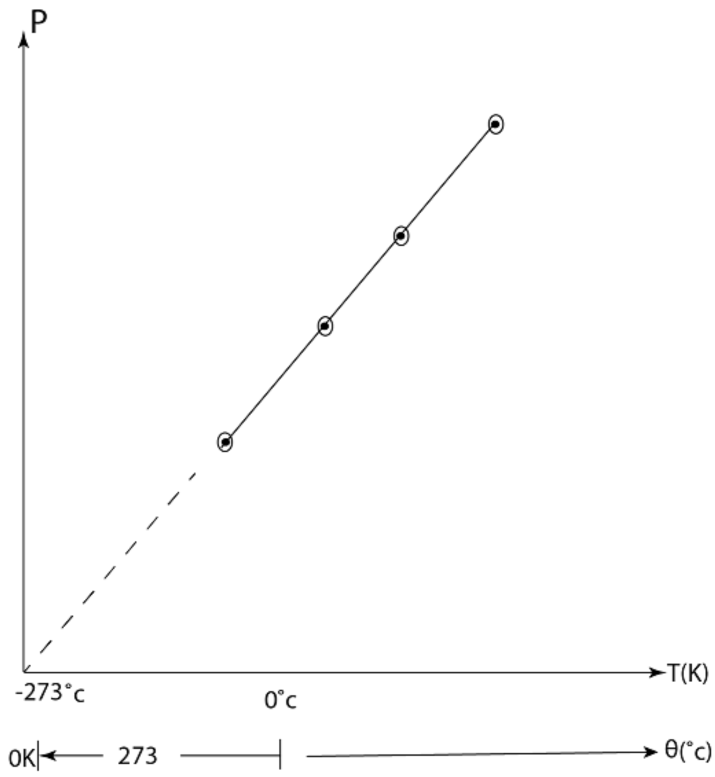
$$\downarrow \qquad\quad \downarrow \quad \downarrow \quad \downarrow$$

$$y \quad = \quad\quad mx + C$$

The SI units used is K^{-1} or $^0C^{-1}$

From the equation [1] above

A graph of P_θ against temperature θ is of the nature below



Deduction from the graph

Deduction 1

Slope [m] of the graph = βP_0

$$\therefore \beta = \frac{Slope\ (m)}{P_0}$$ ----------------2

Knowing slope [m] and the intercept P_0 of the graph then β can be found from the equation (2)

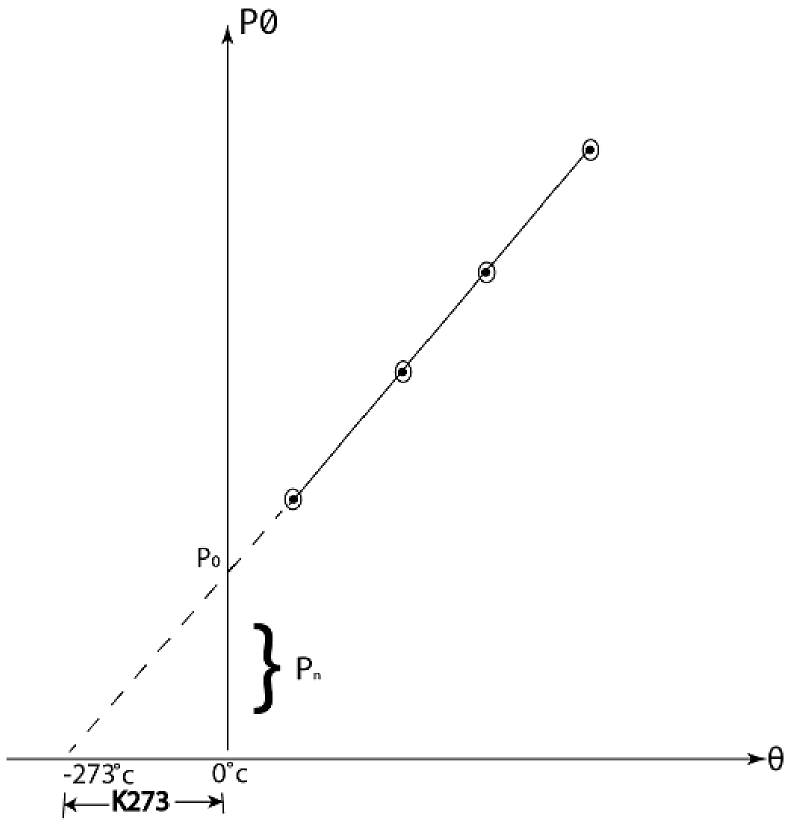
Charles's found that all gases

$$\beta = \frac{1}{273} K^{-1}$$

Deduction 2

When the graph is extra plotted backwards it cuts the temperature axis at -273°C

When P_0 – axis of the graph stands at -273 (OK) we have the nature shown below



From the graph

T(K) = 273 + θ (°C)

The graph shows that when the temperature T (K)of the gas increases, the pressure P of the gas increases

$P \propto T$

Hence we have pressure law which states that

The pressure of a fixed mass of a gas at constant volume is proportional to its thermodynamic temperature

$P = KT$

Where,

K = constant of proportionality

If a gaseous system changes from one P_1 and T_1 condition to another P_2 and T_2 condition then we have

$$P_1 = KT_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$P_2 = KT_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{Equation\ (1)}{Equation\ (2)} \rightarrow \frac{P_1}{P_2} = \frac{KT_1}{KT_2}$$

$$\therefore \frac{P_1}{P_2} = \frac{T_1}{T_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

1. There is absolute zero at – 273^0C because a graph of volume against temperature and that of pressure against temperature cut the temperature axis at -273^0C (OK)

Absolute zero occurs either at volume

V = 0 or pressure P = 0

Charles found that all gases

Pressure coefficient= volume coefficient

$$\beta = \propto = {}^{1}/_{273} K^{-1}$$

IDEAL GAS [PERFECT GAS]

Definition

An ideal gas is a hypothetical gas that obeys gas laws exactly.

It consists of molecule that occupy negligible, volume of the amount n of a gas.

Boyle's law and Charles law can be combined together to give a single equation which describe the temperature, pressure and volume behavior of the gas

Boyle's law

V $\propto \frac{1}{p}$

Charles's law

V $= \propto T$

V $\propto \frac{1}{P} and\ T$

V $\propto \frac{1}{P} x\ T$

$V = {KT}/{P}$

Where

K = constant of proportionality called universal gas constant [R]

$V = {RT}/{P}$

Therefore

PV $= RT \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (1)$

Ideal gas equation or equation of state for 1 mole of gas

For n – molecule gas the equation becomes

PV $= nRT \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$

TYPES OF GAS CONSTANT

GAS CONSTANT FOR A UNIT MASS

From the equation 1 above

PV $= RT$

But

$V = {}^{m}/_{\rho} = {}^{mass\ of\ gas}/_{density\ of\ gas}$

${}^{Pm}/_{\rho} = RT$

When m $=$ 1kg R $=$ gas constant for a unit mass $=$ r say

$P \times {}^{1}/_{\rho} = rT$

$\therefore r = {}^{P}/_{\rho T}$____________________(3)

UNIT OF r

From equation (3)

$$r = \frac{P}{\rho T} = \frac{Nm^{-2}}{Kg \times K}$$

$$= \frac{Nm}{Kg \times K}$$

$$= \frac{Joule}{Kg \times K}$$

$$= JKg^{-1}K^{-1}$$

Hence, the SI unit used is J$Kg^{-1}K^{-1}$

This unit is the same as that of specific heat capacity C

$R = {}^{p}/_{\rho t} = c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$

MOLAR GAS CONSTANT

This is the gas constant per mole of a gas.

From the ideal gas equation for n- moles

$$PV = nRT$$

$$\therefore R = \frac{PV}{nT} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (5)$$

UNIT OF R

From equation (5) :-

$$R = \frac{PV}{nT} = \frac{Nm^{-2} \times m^3}{mol \times K}$$

$$= \frac{Nm}{mol \times K}$$

$$= \frac{Joule}{mol \times K}$$

$$= Jmol^{-1}K^{-1}$$

Hence the SI unit used is $Jmol^{-1}K^{-1}$

NUMERICAL VALUE OF R

From the equation 5.

$$R = \frac{PV}{nT}$$

AT S.T.P

$$P = 1\ atm = 1.01 \times 10^5\ Nm^{-2}$$

$$n = 1\ mole$$

$$V = 22.4\ dm^3 = 22.4 \times 10^{-3} m^3$$

$$T = 273\ K$$

$$R = \frac{1.01 \times 10^5 \times 22.4 \times 10^{-3}}{1 \times 273}$$

$$\therefore R = 8.31\,Jmole^{-1}K^{-1}$$

HEAT-3

Problem 46

The gas hydrogen at $s.t.p$ has a density of 0.09g/litre. Find the gas constant for a unit mass of hydrogen

Problem 47

Calculate the density of air at 1000C and 200K pa given its density at 00C and 101 k pa is 1.29 Kgm^{-3}

Problem 48

Calculate the density of hydrogen gas at 200C and 101Kpa given the molar mass of hydrogen molecule is 2 $\times$ 10^{-3} kg assume the molar gas constant R $= 8.31\,Jmol^{-1}K^{-1}$

Problem 49

A faculty barometer tube has some air at the top above the mercury. When the length of air column is 260mm the reading of the mercury level is 740mm when the length of air column is decreased to 200mm by depressing the barometer tube further into the mercury , the reading of the mercury above the outside level becomes 747mm calculate the atmospheric pressure. (Density of mercury is 13600 kgm^{-3})

KINETIC THEORY OF AN IDEAL GAS

This explains the behaviour of gases by considering the motion of their molecules.

Assumptions

(1) Gas molecules are in random motion colliding in one another and with the walls of the container.

(2) The molecules are like perfectly elastic spheres.

(3) The attraction between the molecules is negligible.

(4) The volume of the molecules is negligible compared with the volume occupied by the gas.

(5) The duration of a collision is negligible compared with the time between the collisions.

(6) A sample of gas consist a large number of identical molecules for statistics to be applied.

PRESSURE EXTENDED BY A GAS

The pressure of a gas is due to the molecules bombarding the walls of its container.

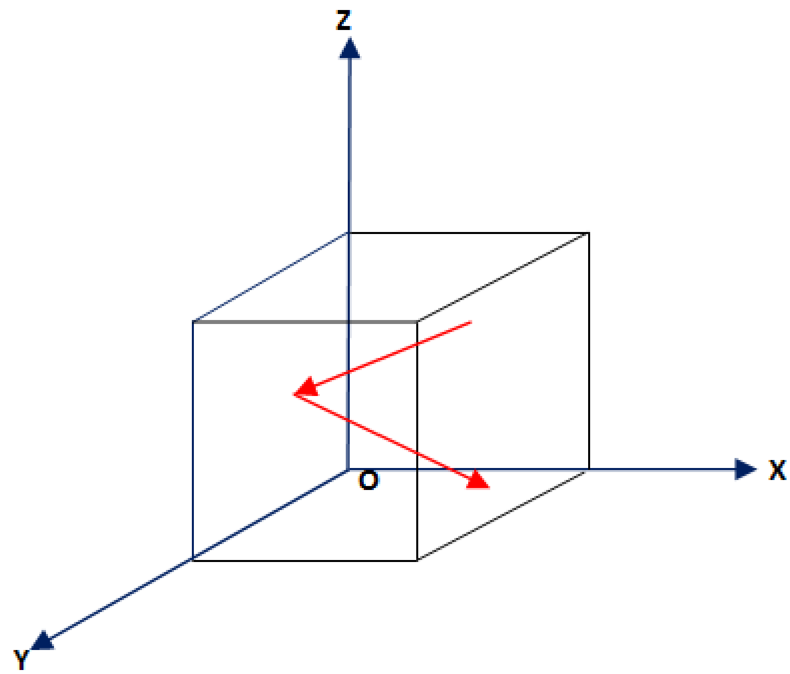
When the molecule collide with the walls they transfer momentum to it.

The total change in moment per second is the force exerted on the wall by the gas.

The pressure on the wall is the force divided by the area of the wall.

EXPRESSION OF THE PRESSURE EXERTED BY AN IDEAL GAS

Consider a cube of side L containing N molecules of gas each mass m



Let u, v and w be component velocities of a gas molecule along ox, oy and oz axes respectively

If C is the resultant velocity of a gas molecule

Then,

$$C^2 = u^2 + v^2 + w^2$$

Extension of Pythagoras theorem

Consider the force exerted on the face x of the cube due to the component U

Forward momentum $= mu$

Rebounding momentum = $-mu$

Where $-ve$ sign shows that the two momentum are in opposite directions

Momentum change on impact = mu - (-mu)

= 2mu

The time taken for the molecule to move across the cube to the opposite face and back to X = $\frac{2l}{u}$

Force F on the wall is

$$F = \frac{momentum\ chamge}{time}$$

$$F = \frac{2mu}{2l/u}$$

$$F = \frac{mu^2}{l} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..(2)$$

The pressure P' exerted on face x by one molecule is

$$P' = \frac{Force\ on\ the\ wall}{Area\ of\ the\ wall}$$

$$P' = \frac{mu^2/l}{l^2}$$

$$\therefore P' = \frac{mu^2}{l^3} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..(3)$$

This is the pressure exerted on face x by one molecule

For N- molecule let them have velocities $u_1, u_2, u_3, \ldots \ldots \ldots \ldots u_N$ towards face x

The pressure P exerted on face x by N molecule is

$$P = \frac{mu_1^2}{l^3} + \frac{mu_2^2}{l^3} + \frac{mu_3^2}{l^3} + \cdots \ldots \ldots \ldots + \frac{mu_N^2}{l^3}$$

$$P = \frac{m}{l^3}\{u_1^2 + u_2^2 + u_3^2 + \cdots \ldots . + u_N^2\} \ldots \ldots ..(4)$$

Let $\overline{u^2}$ be mean square speed of gas molecules along OX direction

$$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \cdots \dots . u_N^2}{N}$$

$$\therefore N\overline{u^2} = u_1^2 + u_2^2 + u_3^2 \dots \dots \dots + u_N^2 \dots \dots \dots \dots \dots . (5)$$

Substitute equation [5] in equation [4]

$$P = {}^{m}/_{l^3}\, N\overline{u^2} \dots\dots\dots.6$$

For random motion the mean square of the components velocities are the same

$$\overline{u^2} = \overline{v^2} = \overline{w^2} \dots \dots \dots \dots \dots \dots (7)$$

From equation 1

$$C^2 = u^2 + v^2 w^2$$

$$\overline{C^2} = \overline{u^2} + \overline{v^2} + \overline{w^2} \dots \dots \dots \dots \dots \dots \dots \dots (8)$$

- Substitute equation [7] in equation [8]

$$\overline{C^2} = 3\overline{u^2} = 3\overline{v^2} = 3\overline{w^2}$$

$$\overline{u^2} = \frac{1}{3} C^2 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (9)$$

Substitute equation [9] in equation [6]

$$P = \frac{mN}{l^3} = \frac{1}{3}\overline{C^2}$$

$$P = \frac{1}{2}\frac{mNC^2}{l^3}$$

Where $L^3 = v =$ volume of the containers

$=$ volume of the gas

$$\therefore P = \frac{1}{3}\frac{Nm\overline{C^2}}{v} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (10)$$

$$\downarrow$$

which is required equation

OR

We may write:-

$$PV = \frac{1}{3}Nm\overline{C^2}$$

NOTE

-The equation indicates that the quantities on the left hand side of the equation [P and V] are macroscopic quantities. While the quantities on the right hand side of the equation are microscopic quantities

EXPRESSION FOR PRESSURE IN ALTERNATE FORMS

- The pressure equation says

$$P = \frac{1}{3}\frac{Nm\overline{C^2}}{V}$$

Where

$$Nm/_V = {}^{total\ mass\ of\ the\ gas}/_{volume\ of\ the\ gas}$$

$$= Density\ of\ the\ gas\ (\rho)$$

$$\therefore P = \frac{1}{3}\rho C^2 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (12)$$

$$\uparrow$$

The equation (12) above Shows the expression of Pressure in terms of density (℘) of the gas.

From equation (11) above

From equation (11) above

$$PV = \frac{1}{3} Nm\overline{C^2}$$

$$P = \frac{1}{3} \frac{Nm\overline{C^2}}{V}$$

$$P = \frac{1}{2} . \frac{N}{V} \left[\frac{1}{2} m\overline{C^2} \right] \times 2$$

$$P = \frac{2}{3} . \frac{N}{V} \left[\frac{1}{2} m\overline{C^2} \right]$$

$$Where\ \frac{1}{2} mc^2 = K.E = Translational\ means\ K.E\ of\ the\ gas\ molecule$$

$$\therefore P = \frac{2N}{3V} \overline{K.E} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (13)$$

Since $N(\overline{K.E}) = Translational\ means\ K.E\ of\ the\ gas, we\ may\ write$

$$P = \frac{2}{3} \times \frac{Translational\ means\ K.E\ of\ the\ gas}{Volume}$$

From equation (11) above:-

$$PV = \frac{1}{3} Nm\overline{C^2}$$

$$P = \frac{1}{3} \frac{Nm\overline{C^2}}{V}$$

$$Where\ \frac{N}{V} = n = number\ of\ molecules\ per\ unit\ voleme$$

$$\therefore P = \frac{1}{3} nm\overline{C^2} \ldots \ldots \ldots \ldots \ldots (14)$$

OUTCOME/RESULTS OF THE KINETIC THEORY OF AN IDEAL GAS

OUTCOME 1

INTRODUCTION OF TEMPERATURE TO THE PRESSURE EQUATION

From the pressure equation

$$PV = \frac{1}{3}Nm\overline{C^2}$$

$$P = \frac{1}{3}\frac{Nm\overline{C^2}}{V}$$

$$P = \frac{1}{2}.\frac{N}{V}\left[\frac{1}{2}m\overline{C^2}\right] \times 2$$

$$\therefore PV = \frac{2}{3}N\left[\frac{1}{2}m\overline{C^2}\right] \ldots\ldots\ldots\ldots\ldots..(1)$$

From ideal gas equation for 1 mole

$$PV = RT$$

Equation (1) above can be written as

$$RT = \frac{2}{3}N\left[\frac{1}{2}m\overline{C^2}\right]$$

$$\frac{1}{2}m\overline{C^2} = \frac{3}{2}\frac{RT}{N}$$

$$\frac{1}{2}m\overline{C^2} = \frac{3}{2}\left[\frac{R}{N}\right]T \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Where $^1/_2\, m\overline{C^2} = Translational\ means\ K.E\ of\ a\ gas\ molecule$

If $N = N_A = Avogadros\ number\ of\ molecules, then$

$$\frac{R}{N_A} = K = Boltzmann'sconstant$$

$$\therefore \frac{1}{2}m\overline{C^2} = \frac{3}{2}KT \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

This equation shows that the translational means K.E of a gas molecule is proportional to its absolute temperature.

It is the gas constant per molecule

From equation (3) above

$$\frac{1}{2}m\overline{C^2} = \frac{3}{2}KT$$

$$K = \frac{{}^1/_2 m\overline{C^2}}{\frac{3}{2}T} = \frac{Joule}{Kelvin} = JK^{-1}$$

The SI unit of K is J/K

Numerical value of K

From the definition of K

$$K = \frac{R}{N_A} = \frac{8.31\,Jmol^{-1}K^{-1}}{6.02 \times 10^{23}}$$

$$\therefore K = 1.38 \times 10^{-23}\,JK^{-1}$$

The value of boltzman's constant is $1.38 \times 10^{-23}\,JK^{-1}$

DEDUCTIONS FROM THE EQUATION (3)

Equation 3 above says

$$\frac{1}{2}m\overline{C^2} = \frac{3}{2}KT$$

Deduction 1

Since $\frac{3}{2}$ and K are constant, we have

$$\frac{1}{2}m\overline{C^2} \propto T$$

$i.e$ The translational mean K.E of a gas molecule is proportional to absolute temperature [T].

Deduction 2

Since ½ . $\frac{3}{2}$ and K are constant we have

Therefore

Since $1/2, 3/2\ and\ K$ are constant we have

$$\overline{C^2} \propto T$$

$i.e$ The mean square speed of a gas molecule is proportional to absolute temperature.

$$\overline{C^2} = KT$$

Where K $=$ constant proportionality

If $\overline{C_1^2}$ and $\overline{C_2^2}$ are the mean square speeds of a gas molecule at temperature T_1 and T_2 respectively then we have

$$\overline{C_1^2} = KT_1 \ldots\ldots\ldots\ldots\ldots\ldots. (i)$$

$$\overline{C_2^2} = KT_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Dividing equation (i) by equation (ii)

$$\frac{\overline{C_1^2}}{\overline{C_2^2}} = \frac{KT_1}{KT_2}$$

$$\therefore \frac{\overline{C_1^2}}{\overline{C_2^2}} = \frac{T_1}{T_2}$$

ROOT MEAN SQUARE [R.M.S] SPEED OF A GAS MOLECULE

The root mean square [$r.m.$] speed of a gas molecule is the square root of the mean square speed of a gas molecule

R.M.S speed = $\sqrt{\overline{C^2}}$

From deduction 3,

$$\overline{C^2} \propto T$$

$$\sqrt{\overline{C^2}} \propto \sqrt{T}$$

$i.e$ The root mean square speed of a gas molecule is proportional to the square root of absolute temperature

$$\sqrt{\overline{C^2}} = K\sqrt{T}$$

Where

K $=$ constant of proportionality

At two different temperatures T_1 and T_2 we have

$$\frac{\sqrt{\overline{C_1^2}}}{\sqrt{\overline{C_2^2}}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

Where

$\sqrt{\overline{C_1^2}}$ = R.M.S speed of a gas molecule at a temperature T_1

$\sqrt{\overline{C_2^2}} = R.M.S$ Speed of the gas molecules at a temperature T_2

DIFFERENT EXPRESSIONS OF R.M.S SPEED OF GAS MOLECULES

The R.M.S speed of gas molecule can be given in several alternate forms

From the pressure equation terms of density

$$P = \frac{1}{3}\rho\overline{C^2}$$

$$\overline{C^2} = \frac{3P}{\rho}$$

$$\therefore R.M.S, speed = \sqrt{\overline{C^2}} = \sqrt{\frac{3P}{\rho}}$$

(2) From the pressure equation:

$$P = \frac{1}{3}\frac{Nm\overline{C^2}}{V}$$

$$PV = \frac{1}{3}Nm\overline{C^2}$$

$$\overline{C^2} = \frac{3PV}{Nm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(*)$$

If $N = N_A = \text{ Avogadro's number of the molecule then}$

$$\overline{C^2} = \frac{3PV}{N_A m}$$

Where $N_A m = molar\ mass\ of\ a\ gas\ (M)$

$V = volume\ of\ one\ mole\ of\ a\ gas$

$$\overline{C^2} = \frac{3PV}{N_A m}$$

$$\therefore R.M.S\ speed = \sqrt{\overline{C^2}} = \sqrt{\frac{3PV}{M}} \ldots\ldots\ldots\ldots\ldots..(2)$$

(3) From equation (1)

$$\overline{C^2} = \frac{3PV}{Nm}$$

If N is the total number of molecules making up the gas, then

Nm = total mass of gas (M_T)

V = total volume of the gas

$$R.M.S\ speed = \sqrt{\overline{C^2}} = \sqrt{3PV/_{M_T}} \ldots\ldots\ldots\ldots\ldots..(3)$$

(4) From the equation (2) above

$$R.M.S\ speed = \sqrt{\overline{C^2}} = \sqrt{\frac{3PV}{M}}$$

Ideal gas equation for 1 mole requires:

PV = RT

$$\therefore R.M.S\ speed = \sqrt{\overline{C^2}} = \sqrt{\frac{3RT}{M}} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

(5) From $^1/_2 m\overline{C^2} = {^3/_2} KT$

$$\overline{C^2} = \frac{3KT}{m}$$

$$\therefore R.M.S\ speed = \sqrt{\overline{C^2}} = \sqrt{\frac{3KT}{m}} \ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

$Where \quad m = mass\ of\ a\ molecule$

$K = Boltzmann's constant$

OUTCOME 2

DERIVATION OF GAS LAWS

(1) BOYLE'S LAW

Boyle's law requires

V $\propto \frac{1}{p}$ or PV $=$ constant

Condition of the law

Temperature must be kept constant

Derivation

According to kinetic theory of gases, the pressure exerted by a gas is given by

$$P = {}^{1}/_{3}\, {}^{Nm\overline{C^2}}/_{V}$$

$$PV = \frac{1}{3} Nm\overline{C^2}$$

But Nm$= Total\ mass\ of\ the\ gas\ M$

$$PV = \frac{1}{3} M\overline{C^2}$$

From kinetic theory of an ideal gas

$$\overline{C^2} \propto T$$

$\therefore \overline{C^2} = KT$

Where $K = constant\ of\ proportionality$

$PV = \frac{1}{3}MKT$

For a given mass of a gas at constant temperature $^1/_3\, MKT = constant$

$\therefore PV = constant \qquad (Boyle's\ law)$

(2) CHARLE'S LAW

Charles' law requires

$V \propto T$

Condition of the law

Pressure must be kept constant

Derivation

According to kinetic theory of gases

$$P = \frac{1}{3}\frac{Nm\overline{C^2}}{V}$$

$$V = \frac{1}{3}\frac{Nm\overline{C^2}}{P}$$

But, Nm$= Total\ mass\ of\ the\ gas\ M$

$$V = \frac{1}{3}\frac{M\overline{C^2}}{P}$$

According to kinetic theory of an ideal gas

$\overline{C^2} \propto T$

$\therefore \overline{C^2} = KT$

$\Longrightarrow V = \frac{1}{3}\frac{MKT}{P}$

For a given mass of a gas at constant pressure $^1/_3\,{}^{MKT}/_P = constant$

V ∝ T (Charles law)

(3) PRESSURE LAW

Pressure law requires

$P \propto T$

Condition of the law

Volume must be kept constant

Derivation

According to kinetic theory of gases:

$P = \frac{1}{3}\frac{Nm\overline{C^2}}{V}$

But Nm$= Total\ mass\ of\ the\ gas\ M$

$P = \frac{1}{3}\frac{M\overline{C^2}}{V}$

According to kinetic theory of gases

$\overline{C^2} \propto T$

$\therefore \overline{C^2} = KT$

$P = \frac{1}{3}\frac{M}{K}KT$

For a given mass of a gas at constant volume $^1/_3\,{}^{MK}/_V = constant$

$\therefore P \propto T$

IDEAL GAS EQUATION

Ideal gas equation requires

PV $=$ RT (for 1 mole of a gas)

Derivation

According to kinetic theory of gases:

P = $\frac{1}{3}\ \frac{Nm\overline{C^2}}{V}$

PV = $\frac{1}{3}Nm\overline{C^2}$

But Nm $= Total\,mass\ of\ gas\ M$

$$PV = \frac{1}{3}M\overline{C^2}$$

According to kinetic theory of gases: $\overline{C^2} = KT$

$$PV = \frac{1}{3}MKT$$

For a given mass of a gas $^1/_3\,MK = constant = R$

PV = RT (Ideal Gas equation)

OUTCOME 3

PROOF OF AVOGADRO'S HYPOTHESIS

Avogadro's hypothesis

Under the same conditions of temperature and pressure equal volumes of all gases contain the same number of molecules

PROOF

Consider two gases (gas 1 and gas 2 say)

Let T be same temperature of gases

Let P be same pressure of gases

Let V be same volume of gases

The pressure equations for the gases are

GAS 1

P = $\frac{1}{3}\,\frac{N_1 m_1 \overline{C_1^2}}{V}$

Therefore

PV = $\frac{1}{3} N_1 m_1 \overline{C_1^2}$ ……………..1

GAS 2

Similarly PV = $\frac{1}{3} N_2 m_2 \overline{C_2^2}$…………………..2

Where,

$N_1 m_1 \overline{C_1^2}$ and $N_2 m_2 \overline{C_2^2}$ are number of molecules mass of each molecule and mean square speed of molecule of gas 1 and gas 2 respectively

Equation (1) $=$ equation (2)

$$\frac{1}{3} N_1 m_1 \overline{C_1^2} = \frac{1}{3} N_2 m_2 \overline{C_2^2}$$

$$N_1 m_1 \overline{C_1^2} = N_2 m_2 \overline{C_2^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .3$$

According to kinetic theory of ideal gas

$$m_1\overline{C_1^2} \propto T \quad and \quad m_2\overline{C_2^2} \propto T$$

Since the absolute temperature T is the same for both gases we have

$$m_1\overline{C_1^2} = m_2\overline{C_2^2} = m_3\overline{C_3^2} \ldots\ldots\ldots\ldots\ldots\ldots \ m_n\overline{C_n^2}$$

$$N_1 m\overline{C^2} = N_2 m\overline{C^2}$$

$\therefore N_1 = N_2$ (Avogadro's hypothesis)

OUT COME 4

PROOF OF DALTON'S LAW OF PARTIAL PRESSURE

Definition

Partial pressure that could be exerted by a gas if it were present alone and occupies the same volume as the mixture of gases

DALTON'S LAW OF PARTIAL PRESSURE

The law states that

"The total pressure exerted by a mixture of gases is always equal to the sum of the partial pressure of the constituent of gases"

$P = P_1 + P_2$

Where;

$P =$ total pressure of the mixture

P_1 and P_2 are partial pressures of the constituent gases

PROOF

Consider two gases nitrogen (N_2) and hydrogen (H_2) in a container of volume V at the surrounding temperature T

Let

N_1, m_1, $\overline{C_1^2}$, and N_2, m_2 $\overline{C_1^2}$ be number of molecules mass of each molecule and the mean square speed of the molecules of N_2 and H_2 gases respectively

If P_1 and P_2 are partial pressure of N_2 and H_2 gases respectively then the pressure equation for N_2 and H_2 gases are

For N_2

$P_1 = \frac{1}{3} N_1 \frac{N_1 m_1 \overline{C_1^2}}{V}$

$$\therefore P_1V = \frac{1}{3}N_1m_1\overline{C_1^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

For H_2

$P_2 = \frac{1}{3}N_2m_2\overline{C_2^2}/V$

$$\therefore P_2V = \frac{1}{3}N_2m_2\overline{C_2^2} \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Adding equation (1) and equation (2) above

$$P_1V + P_2V = \frac{1}{3}N_1m_1\overline{C_1^2} + \frac{1}{3}N_2m_2\overline{C_2^2}$$

$$(P_1 + P_2)V = \frac{1}{3}(N_1m_1\overline{C_1^2} + N_2m_2\overline{C_2^2}) \ldots\ldots\ldots\ldots\ldots\ldots.(3)$$

According to kinetic theory of an ideal gas

$$m_1\overline{C_1^2} \propto T \quad and \quad m_2\overline{C_2^2} \propto T$$

Since the absolute temperature T is the same, we have

$$m_1\overline{C_1^2} = m_2\overline{C_2^2} = m\overline{C^2}\ say$$

Equation (3) above becomes

$$(P_1 + P_2)V = \frac{1}{3}\left[N_1 m\overline{C^2} + N_2 m\overline{C^2}\right]$$

$$\therefore (P_1 + P_2)V = \frac{1}{3}(N_1 + N_2)m\overline{C^2} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Compare this equation with the pressure equation of an ideal gas

$$(P_1 + P_2)V = \frac{1}{3}(N_1 + N_2)m\overline{C^2}$$

↓ ↓ ↓ ↓ ↓

$$P \quad V = \frac{1}{3}\ N \quad m\overline{C^2}$$

Where N = total number of molecules

Therefore;

N = $N_1 + N_2$

P = total pressure of the mixture

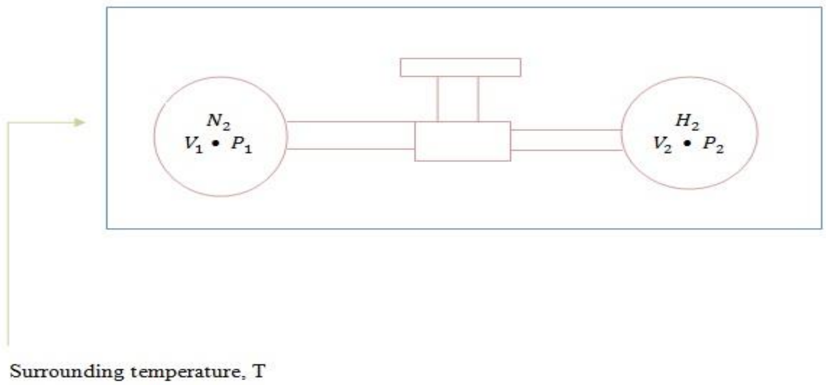
Therefore

P = $P_1 + P_2$

Dalton's law of partial pressure

CALCULATION FORMULA OF DALTON'S LAW

Consider two gas containers of volumes V_1 and V_2 containing nitrogen (N2) and hydrogen (H_2) gases respectively at pressure P_1 and P_2



Let the two containers to be connected by a narrow tube of negligible volume with a stopper's

Before the stopper S is opened

Ideal gas equation gives

For N_2

$P_1 V_1 = n_1 RT$

$n_1 = \frac{P_1 V_1}{RT}$ ....(1)

Where $n_1 =$ number of moles of N_2 in the container of volume V_1

For H_2

$P_2 V_2 = n_2 RT$

$n_1 = \frac{P_2 V_2}{RT}$ ....(2)

Where n_2 = number of moles of H_2 in the container of volume

When the stopper "S" is opened

The two gases are mixed up and finally the system will have a common pressure

P = total pressure of the mixture

Volume of mixture = $V_1 + V_2$

Total number of moles of the mixture = $n_1 + n_2$

Ideal gas equation for the mixture of the two gases gives:

$$P(V_1 + V_2) = (n_1 + n_2)RT \text{ ------------ } (3)$$

Substitute equation (1) and equation (2) in equation (3)

$$P = P_1\left[\frac{V_1}{V_1+V_2}\right] + P_2\left[\frac{V_2}{V_1+V_2}\right]$$

Partial pressure of N_2 ↑ Partial pressure of H_2 ↑

NOTE

$$\frac{V_1}{V_1+V_2} = \text{Volume fraction of } N_2$$

$$\frac{V_2}{V_1+V_2} = \text{Volume fraction of } H_2$$

OUTCOME 5

MEAN FREE PATH

Symbol, λ

Definition

The mean free path a gas molecule is the average distance covered by a gas molecule during successive collisions.

It is given by:

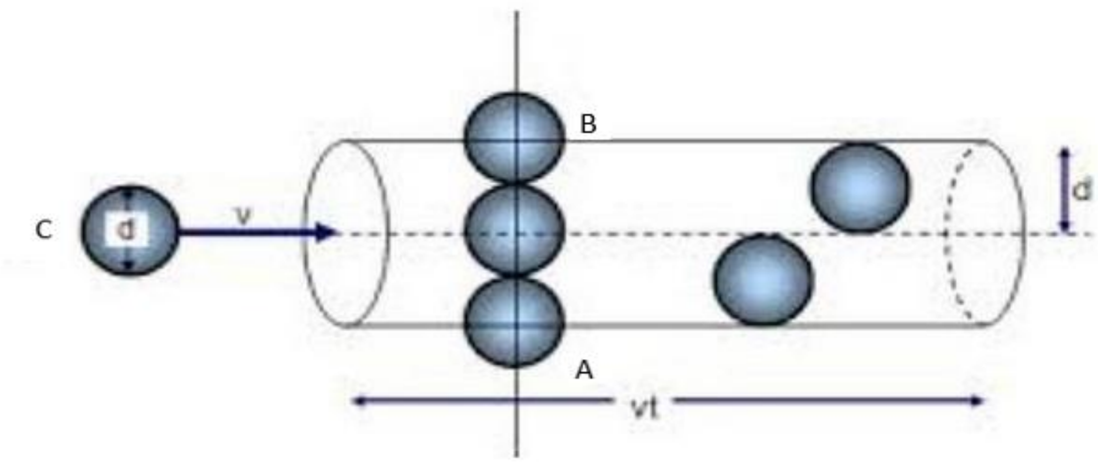
$$= \frac{Total\ distance\ covered}{Total\ number\ of\ molecules\ in\ collision}$$

Condition

For the collision to happen the distance between centres of the molecule must be equal to the diameter of a molecule.

Assumption

We assume that during collision, one molecule move and others are stationary target



Assumption molecule C moves while A and B are stationary.

The molecule with which C just make collision are those whose centers lie inside the volume of the cylinder of radius.

d = 2r = diameter of a molecule.

Where r = radius of a molecule

Volume of the cylinder = $\pi d^2 l$

Let n be the number of the molecules per unit volume in the cylinder.

Total number of molecules in collision =$n\pi d^2 l$

Where = total distance covered by a gas molecule

From the definition of mean free path

$$= \frac{Total\ distance\ covered}{Total\ number\ of\ molecules\ in\ collision}$$

$$= \frac{l}{n\pi d^2 l}$$

$$\lambda = \frac{1}{n\pi d^2} \text{ --------------- (1)}$$

This equation is true for a stationary target

From kinetic theory of an ideal gas all molecules move colliding one another and with the walls of the container, hence equation (1) above is statistically modified to:

$$\lambda = \frac{1}{n\pi d^2\sqrt{2}} \text{ ---------------- (2)}$$

This equation is true for moving target

ALTERNATIVE EXPRESSION OF MEAN FREE PATH

From equation (2) above :

$$\lambda = \frac{1}{n\pi d^2\sqrt{2}} \text{ ---------------- (2)}$$

From ideal gas equation for 1 mole

PV = RT

But, R = KN

PV = KTN

Where K = Boltzmann's constant

N = Avogadro's number of molecule

P = KTN / V

$$\frac{N}{V} = \frac{P}{KT} = n = number\ of\ molecules\ per\ unit\ volume$$

Thus equation (2) above can be written as:

$$\lambda = \frac{1}{\frac{P}{KT}\pi d^2\sqrt{2}}$$ ....(3)

This is an alternative expression of mean free path.

FACTORS DETERMINING THE MEAN FREE PATH OF A GAS MOLECULE

(1) Absolute temperature (T)

From equation (3) above:

$$\lambda = \frac{1}{\frac{P}{KT}\pi d^2\sqrt{2}}$$

$$\lambda = \frac{KT}{P\pi d^2\sqrt{2}} . T$$

First dark ring, n = 1

Second dark ring, n =2

Since a phase change of radians occurs when light is reflected at the glass plate H, the central spot (for which t = 0) is dark.

At any position where there is a bright ring:

difference = 2t = (n + 1/2) $^{\lambda}$

Where n = 0, 1, 2, 3, ………………..

First bright ring, n = 0

Second bright ring, n = 1

Third bright ring, n = 2

COLLISION FREQUENCY (f)

This is the number of collisions made by a gas molecule in unit time.

It is given by:

$$f = \frac{mean\ speed}{mean\ free\ path}$$

$$f = \frac{\bar{C}}{\frac{1}{n\pi d^2\sqrt{2}}}$$

$$\therefore f = n\pi d^2\bar{C}\sqrt{2}$$

Since n $= \frac{N}{V} = \frac{P}{KT} = number\ of\ molecules\ per\ unit\ volume$

Where K $=$ Boltzmann's constant

P $=$ pressure of the gas

$\bar{C} =$ means speed of a molecule

d $=$ diameter of a molecule

T $=$ absolute temperature

Equation 7 shows that

i) At constant temperature

$$f \propto P$$

ii) At constant pressure

$$f \propto \frac{1}{T}$$

REAL GASES

Definition

A real gas is that gas which does not have the properties assigned to an ideal gas

A real gas satisfies the $\text{Van} - \text{der} - \text{waal's}$ equation

THE VAN DER WAAL'S EQUATION

$\text{Van} - \text{der} - \text{waals}$ modified the ideal gas equation to take account that two of the Kinetic theory of an ideal gas may not be valid

1. The volume of the molecules may not be negligible in relation to the volume V occupied by the gas.

2. The attractive forces between the molecules may not be negligible.

Problem 50

0.09gm/litre. Find the gas constant for unit mass of hydrogen

Problem 51

The gas hydrogen has a density of 0.09g/liter at s.t.p. Find the mean square speed and hence root mean square speed of hydrogen at 42°C

Problem 52

a) List down any four assumptions of the kinetic theory of an ideal gas.

b) Determine the absolute temperature of a gas in which the average molecules of mass 8 x 10-26kg are moving with r.m.s speed of 500ms-1. Given that universal gas constant R $=$ 8.31 $jmol^{-1}K^{-1}$ and Avogardo's number $=$ 6.023 x 10^{23}

Problem 53

Helium gas occupies a volume of $0.04m^3$ at a pressure of 2 $\times$ 10^5pa and temperature of 300K

Calculate

a) The mass of helium

b) The $r.m.s$ speed of its molecules

c) The $r.m.s$ speed at 432K when the gas is heated at constant pressure to this temperature.

Problem 54

a) Derive the gas equation obeyed by a system consisting of N molecules each of mass m and mean square speed $\overline{C^2}$. Hence obtain the kinetic energy per molecule in terms of absolute temperature.

b) A vessel of volume 6 x 10-3m^3 contains nitrogen at a pressure of 2 $\times$ 10^2 pa and a temperature of 27^{o}C. What is

i) The number of nitrogen molecules in the vessel, and

ii) Their $r.m.s$ speed

Given that

R $=$ 8.3 $Jmol^{-1}K^{-1}$

M_N $=$ 28 $gmol^{-1}$

NA $=$ 6.0 x 10^{23}

Problem 55

a) Write down the equation of a state of an ideal gas, defining all symbols used.

b) How does the average translation kinetic energy of a molecule of an ideal gas change if

i) The pressure is doubled while the volume is kept constant?

ii) The volume is double while the pressure is kept constant?

c) Calculate the value of the root mean square of the molecules of helium at 0^0C

Problem 60

a) i) What is meant by the mean free path of a molecule

ii) Show that the mean free path of a molecule of an ideal gas at pressure p and temperature T is given by

$$\lambda = \frac{K_B T}{P\pi d^2\sqrt{2}}$$

Where K_B is the Boltzmann's constant and d is the molecule diameter

b) i) Derive an expression for the average kinetic energy of one molecule of a gas assuming the formula for the pressure of an ideal gas.

c) A cylinder of volume 2×10^{-3} m^3 contains a gas at pressure of $1.5 \times 106Nm^{-2}$ and temperature 300K. Calculate

i) The number of moles of the gas

ii) The number of molecules the gas contains

iii) The mass of the gas if its molar mass is 320×10^{-3} Kg.

iv)The mass of one molecule of the gas

Problem 61

a) (i) Write down the van der waal's equation and define each term in its usual meaning

(ii) State the assumptions upon which the equation you have written in (a) (iii) above is derived from the ideal gas equation

b) i) On the basis of the kinetic theory of gases, shows that two different gases at the same temperature, will have the same average value of the kinetic energy of the molecules.

c) Define means free path, λ of the molecules of a gas and state hoe it is affected by temperature.

d) If the mean free path of molecules of air at 0^oC and 1.0 atmospheric pressure is 2×10^{-7}m, what will be mean free path be at 1.0 atmospheric pressure and 27^oC

INTERNAL ENERGY OF GAS(Symbol, U)

The internal energy (U) of a gas is the total mean kinetic energy of all molecules making up the gas.

U = total mean K.E of all molecules

But mean K.E of a molecule $= \frac{1m\,\overline{C^2}}{2} = \frac{3KT}{2}$

Where all symbols carry their usual meaning

U = means K.E of a molecule $\times$ N

$$U = \frac{3NKT}{2}$$

Where N = total number of molecules making up the gas

For 1 mole of a gas N = N_A = Avogadro's number of molecules

Equation I above becomes

$$U = \frac{3N_A KT}{2}$$

Since K $= \frac{R}{N_A} =$ Boltzmann's constant, we have

$$U = \frac{3N_A}{2} \times \frac{R}{N_A} \times T$$

$$U = \frac{3\,RT}{2}$$

This equation is true for 1 mole of a gas

For n moles of a gas, the equation becomes

$$U = \frac{3n\,RT}{2}$$

$$\Delta U = \frac{3nR\,\Delta T}{2}$$

The internal energy U of a gas depends on the temperature T

At constant temperature the change in internal energy ΔU of the gas is zero.

THERMODYNAMICS

This deal with the study of the laws that govern the conversion of energy from one form to another, the direction in which heat will flow and the availability of energy o do work.

It is based on the facts in isolated system everywhere in the universe there is a measurable quantity of the energy known as internal energy of the system.

THE FIRST LAW OF THERMODYNAMICS

The law states
"The heat energy Q supplied to a system is equal to the sum of the increase in internal energy (Δ u) of the system and external work done (w)".

$$Q = \Delta U + W$$

1) Q: comes from outside at constant pressure

ΔU: depends on the temperature of the gas since $\Delta U = \frac{3nR\Delta T}{2}$

W: is given by:

$$W = P\Delta V = P(V_2 - V_1)$$

$\Delta V =$ change in volume

$V_1 =$ initial volume

$V_2 =$ final volume

Thus, the first law of the thermodynamic can be written as:

$$Q = \Delta U + P\Delta V$$

The first of thermodynamics represents the principle of conversation of energy

MOLAR HEAT CAPACITY OF A GAS (C)

Definition

The molar heat capacity of a gas is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin.

$$C = \frac{Heat\ supplied}{number\ of\ moles\ \times Rise\ in\ temp}$$

$$C = \frac{Q}{n\Delta T}$$

$$OR$$

$$Q = nC\Delta T$$

UNIT OF C

By definition

$$C = \frac{Q}{n\Delta T} = \frac{Joule}{mol \times kelvin}$$

Hence, the SI unit used is $jmol^{-1}\ K^{-1}$

PRINCIPLE MOLAR HEAT CAPACITIES OF A GAS

There are two principle molar heat capacities of a gas

i) Molar heat capacity of a gas at constant pressure

ii) Molar heat capacity of a gas at constant volume

i) MOLAR HEAT CAPACITY OF A GAS AT CONSTANT PRESSURE

Symbol, Cp

This is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin when the gas is at constant pressure

$$C_P = \frac{Q}{n\Delta T}$$

$Q = nC_P\Delta T$ Where Q $=$ heat supplied to the gas at constant pressure.

ii) MOLAR HEAT CAPACITY OF A GAAS AT CONSTANT VOLUME

Symbol, C_V

This is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin when the gas is at constant volume.

$$C_V = \frac{Q}{n\Delta T}$$

$$Q = nC_V\Delta T$$

Where Q $=$ heat supplied to the gas at constant volume

APPLICATIONS OF FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics can be applied to some simple processes such as:

i) Relation between C_p and C_v

ii) Boiling process

iii) Isobaric process (no change in pressure)

iv) Isochoric process (no change in volume)

v) Isothermal process (no temp change)

vi)Adiabatic process (no heat learning or enters)

RELATION BETWEEN C_p AND C_v (Mayer's equation)

Consider n – moles of an ideal gas
Suppose the gas is heated at constant volume so that its temperature increases by ΔT

The amount of heat Q supplied is given by

$Q = nC_v\Delta T$

From the 1st law of thermodynamics

Q $= \Delta u$ + PΔV

The gas is heated at constant volume and hence $\Delta V = 0$

$Q = \Delta u + P \times 0$

$Q = \Delta u \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$

Equation (1) = Equation (2)

$nC_v\Delta T = \Delta U$

$\therefore \Delta u = nC_v\Delta T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$

Suppose now that n – moles of the same gas are heated at constant pressure so that its temperature increases by the same amount ΔT

The heat Q' – supplied is given by

$Q' = nC_p\Delta T \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$

From

$Q' = \Delta u + P\Delta V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$

From ideal gas equation

$PV = nRT$

$\therefore P\Delta V = nR\Delta T \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$

Substitute equation (3), equation (4) and equation (6) in equation (5)

$$nC_p\Delta T = nC_v\Delta T + nR\Delta T$$

$$C_p = C_v + R$$

$$\therefore C_p - C_v = R \ldots\ldots\ldots\ldots\ldots..(7)$$

This equation shows that C_p is greater than C_v

Moreover;

i) When the gas is heated at constant volume, no work is done (ΔV $=$ 0) and all the heat goes into raising the internal energy and thus the temperature of the gas.

ii) When the gas is heated at constant pressure, it expands and does work so that only a part of heat is used up in increasing the internal energy and hence temperature of the gas.

Therefore, in a constant- pressure process, more heat is needed to achieve a given temperature change than that of constant volume process.

NOTE:

If C_p and C_v are specific heat capacities of the gas at constant pressure volume respectively then:

$$C_p \quad - \quad C_v = \qquad r - - - - - - - - - - - -(8)$$

↓ ↓ ↓

$Jkg^{-1}K^{-1}$ $Jkg^{-1}K^{-1}$ $Jkg^{-1}K^{-1}$

Where r = gas constant for a unit mass

It is given by

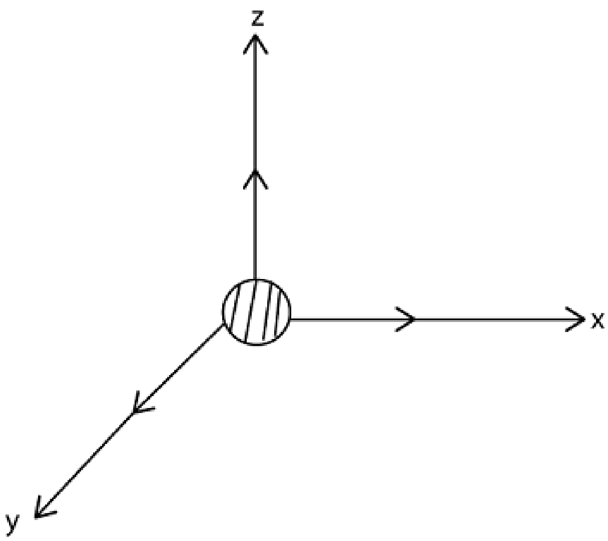
$$r = {}^{P}/_{\rho T}$$

$$\therefore C_p - C_v = {}^{P}/_{\rho T}$$

TYPES OF GASES

1) MONO-ATOMIC GAS

This is a type of a gas whose molecule consist of a single atom



Example

Helium

Neon

2) DIATOMIC GAS

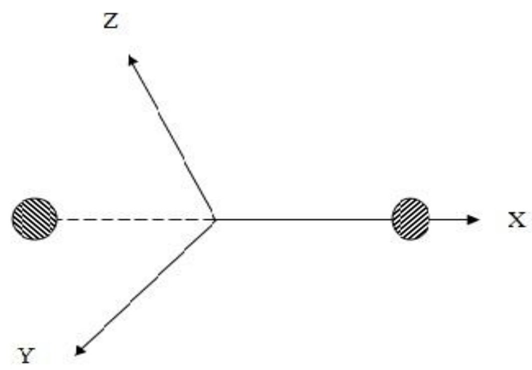
This is a type of gas whose, molecule consist of two atoms

Examples

Hydrogen (H_2)

Oxygen (O_2)

Chloride (Cl_2)



POLYATOMIC GAS

This is a type of gas whose molecules consists of more than two atoms.

Examples

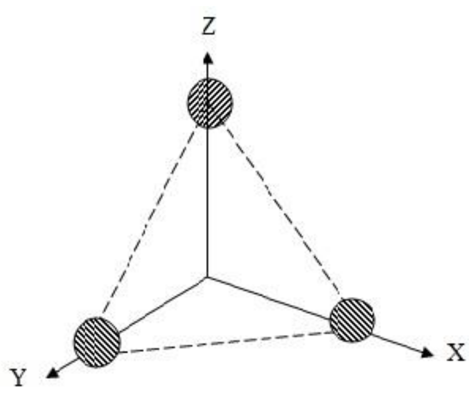
Carbon dioxide (CO_2)

Ammonia (NH_3)

Hydrogen peroxide (H_2O_2)

Steam (water gas) (H_2O)

Etc.



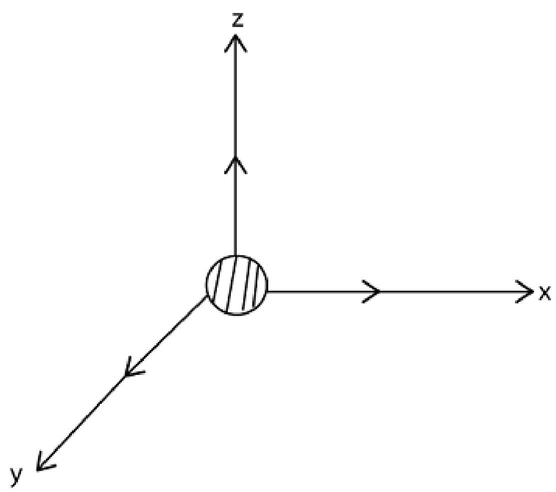
DEGREES OF FREEDOM

Degrees of freedom of a gas molecule are the number of independent ways the molecule can possess energy.

It refers to the axes x, y and z to which the gas molecule can move freely.

DEGREES OF FREEDOM FOR DIFFERENT GASES

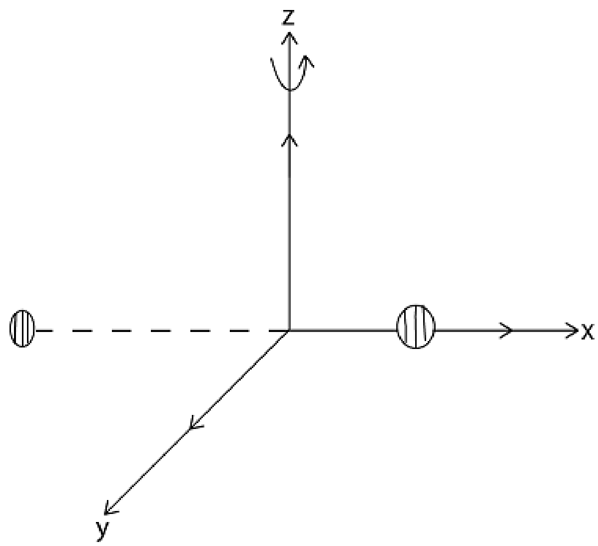
MONOATOMIC GAS



The atom can move freely along x, y and Z- axes.

Hence a monatomic gas has three degrees of freedom

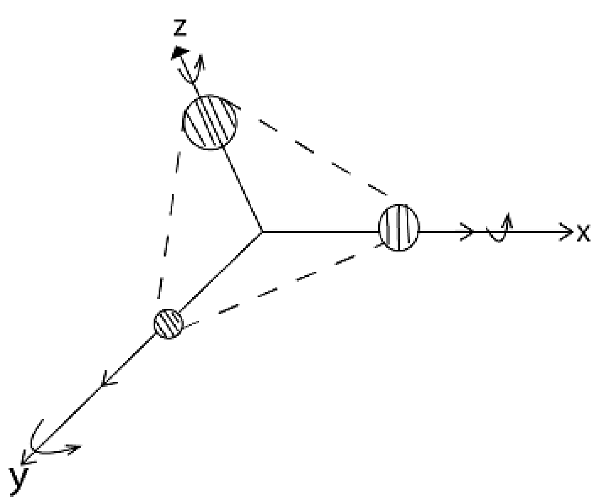
DIATOMCI GAS



The molecule has three degree of freedom in translation and two degree of freedom in rotation.

Thus, a diatomic gas has a total 5 – degrees of freedom

POLYATOMIC GAS



The molecule has 3-degree of freedom in translation and 3-degrees of freedom in rotation. Hence, a polyatomic gas has a total of 6-degrees of freedom.

EQUIPARTITION PRINCIPLE

The mean kinetic energy of a molecule is given by

$\frac{1}{2}m\overline{C^2} = \frac{3}{2}$KT ……………………………………… (1)

Where all symbols carry their usual meaning.

Since gas molecules are in random motion

$\bar{u}^2 = \bar{v}^2 = \bar{w}^2$ ………………………………….. (2)

If $\bar{c}^2$ is the resultant mean square speed of a gas molecule then: -

$\bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$ ………………………….. (3)

Substitute equation (2) in equation (3)

$\bar{c}^2 = 3\bar{u}^2 = 3\bar{v}^2 = 3\bar{w}^2$ ………………………….. (4)

Substitute equation (4) in equation (1)

$\frac{1}{2}m(3\bar{u}^2) = \frac{1}{2}m(3\bar{v}^2) = \frac{1}{2}m(3\bar{w}^2) = \frac{3}{2}$KT

$\therefore \frac{1}{2}m\bar{u}^2 = \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m\bar{w}^2 = \frac{1}{2}$KT ………………….(5)

When $\bar{u}^2$, $\bar{v}^2$, and $\bar{w}^2$ are the main square speed of gas molecules along x, y and z-axes respectively, and the axes of the degrees of freedom.

Equation (5) above express the principal of equitation of energy which says: The mean energy of the molecules of a gas is equally divided among their available degrees of freedom, the average for each degree of freedom being $\frac{1}{2}$KT

Where K = Boltzmann Constant

T = Thermodynamic temperature

DEDUCTIONS FROM EQUATION (5)

(1) MONOATOMIC GAS

Has three degrees of freedom $\Rightarrow$ Mean K.E of a molecule $= \frac{1}{2}$ KT $\times$ 3

$$\text{Mean K.E of a molecule} = \frac{3}{2}\text{KT}$$

(2) DIATOMIC GAS

Has 5 – degrees of freedom

Mean K.E of a molecule = $\frac{1}{2}$ KT X 5

$$\text{Mean K.E of a molecule} = \frac{5}{2}\text{KT}$$

(3) <u>POLYATOMIC GAS</u>

Has 6 – degrees of freedom

Mean K.E of a molecule = $\frac{1}{2}$ KT $\times$ 6

$$\text{Mean K.E of a molecule} = 3\text{ KT}$$

RATIO OF MOLAR HEAT CAPACITIES OF A GAS

The ratio $\frac{C_p}{C_v}$ is called gamma (γ)

$$\gamma = \frac{C_p}{C_v}$$

This ratio has different values for different gases

FOR MONOATOMIC GAS

This means the internal energy U is given by:

U = $\frac{3}{2}$ KT $\times N_A$

Where K $= \frac{R}{N_A}$ = Boltzmann constant

U $= \frac{3}{2} \times \frac{R}{N_A} \times$ T $\times N_A$

$$U = \frac{3}{2} RT$$

If Cv is the molar heat capacity of a gas at constant volume then

U = nC_vT

For 1 mole of a gas n = 1

∴ U = C_vT …………………………………… (2)

Equation (1) $=$ equation (2)

$\frac{3}{2}$ RT $= C_vT$

$C_v = \frac{3}{2} R$ ....(3)

According to Mayer's equation

$C_p - C_v = R$

$C_p = R + C_v$

Substitute equation (3) in this equation:

$C_p = \frac{5}{3} R$

Now $\gamma = \frac{C_p}{C_v} = \frac{5R/2}{3R/2} = 1.67$

$\therefore$ $\gamma = \frac{C_p}{C_v}$ = for mono atomic gas = 1.67

FOR DIATOMIC GAS

The internal energy U is given by:

$U = \frac{5}{2} KT \times N_A$

Where $K = \frac{R}{N_A}$

$U = \frac{5}{2} \times \frac{R}{N_A} \times T \times N_A$

$\therefore$ $U = \frac{5}{2}RT$ .... (1)

└→ For 1 mole of a gas

The internal energy of the gas can also be given by

$U = C_v T$ .... (2)

└→ For 1 mole of a gas

Equation (1) = equation (2)

$\frac{5}{2}RT = C_v T$

$\therefore$ $C_v = \frac{5}{2}R$ .... (3)

According to Mayer's equation

$C_P - C_V = R$

$C_P = R + C_V$

Substitute equation (3) in this equation

$C_P = R + \frac{5}{2} R$

$$\therefore \quad C_P = \frac{7}{2} R \quad \text{.........................} (4)$$

Now, $\gamma = \frac{C_p}{C_v} = \frac{7^{R}/_{2}}{5^{R}/_{2}} = 1.40$

$$\gamma = \frac{C_p}{C_v} = \text{for diatomic gas} = 1.40$$

FOR POLYATOMIC GAS

The internal energy (U) of a gas is given

$U = 3KT \times N_A$

Where $K = \frac{R}{N_A}$

$U = 3 \times \frac{R}{N_A} T \times N_A$

$\therefore$ ……………. (1)

$$U = \frac{5}{2}RT$$

→ For 1 mole of a gas

The internal energy U of the gas can also be given by:

$U = C_v T$ ……………………………….. (2)

→ For 1 mole of a gas

Equation (1) = Equation (2)

$3RT = C_v T$

$C_v = 3R$ ………………………………(3)

According to Mayer's equation

$C_P - C_V = R$

$C_P = R + C_V$

Substitute equation (3) in this equation

C_P = R + 3R

∴ C_P = 4R .... (4)

Now, $\gamma = \frac{C_p}{C_v} = \frac{4R}{3R} = 1.33$

∴ $$\gamma = \frac{C_p}{C_v} = \text{for Polyatomic gas} = 1.33$$

APPLICATION OF $\gamma = C_P/C_V$

It is used to solve adiabatic problems

BOILING PROCESS

Suppose a liquid of mass m vaporizes at constant pressure P

Let V_1 be volume in the liquid state.

Let V_2 be volume in the vapor state

Here expansion takes place at constant pressure and hence the work done by the system is

W = $P\Delta V$

W = P $(V_2 - V_1)$ .... (3)

$\therefore \Delta u = mL - P(V_2 - V_1)$

Where Δu = Increase in internal energy of the system

MELTING PROCESS

When a solid changes into liquid state (melting), its internal energy increases.

This can be calculated from, the 1st law of thermodynamics.

Let m = mass of the solid

L = specific latent heat of fusion

Heat (Q) absorbed during the melting process is

$Q = mL$ …………………………….. (1)

Since during melting process, the change in volume (ΔA) = 0

From the 1st law of thermodynamics

$Q = \Delta u + W$

$Q = \Delta u + P\Delta V$

$mL = \Delta u + P \times 0$

$\therefore \quad mL = \Delta u$ ....(2)

During melting process, internal energy increase by mL.

Since temperature remains constant during melting the kinetic energy remains the same.

Therefore, the increase in potential energy.

ISOBARIC PROCESS

This is the process which occurs at constant pressure.

According to 1st law of thermodynamics

Q = Δu + $P\Delta V$

In this case, the heat Q added increases the does internal energy of the gas as well as the gas does external work.

ISOCHORIC PROCESS

This is the process which occurs at constant volume (i.e ΔV = 0)

In such a process, external work done W is zero

W $= P\Delta V =$ P $\times$ 0 $=$ 0

According to 1st law of thermodynamics:

Q = Δu + W

$$Q = \Delta u + 0$$

$$\therefore$$

$$Q = \Delta u$$

We conclude that if heat is added to a system at constant volume, all the heat goes into increasing the internal energy of the system.

ISOTHERMAL CHANGE /PROCESS

Definition

An isothermal change is that change which takes place at constant temperature.

In such a process heat is, if necessary, supplied or removed from the system at just the right rate to maintain constant Temperature.

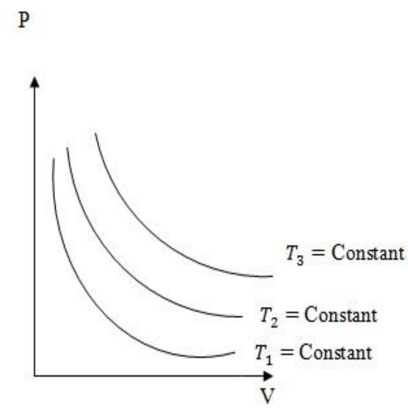
Conditions for isothermal change

(1) The gas must be held in a thin-walled, highly conducting vessel, surrounded by a constant temperature bath.

(2) The expansion or contraction must take place slowly. So that the heat can pass in or out to maintain the temperature of the gas at every instant during expansion or contraction.

Isothermal change represented graphically

When the temperature is constant the pressure of a gas varies with volume and a graph which shows this variation is a curve known as isothermal curve



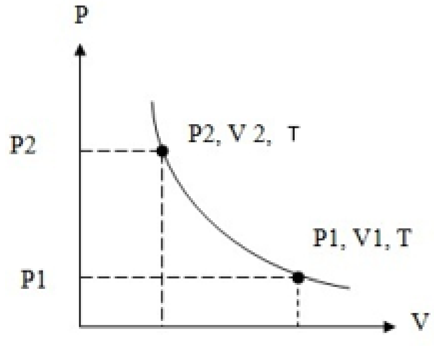
Where $T_3 > T_3 > T_1$

This graph is also called PV - curve or PV – Indicator diagram

When a gas expands, or is compressed, at constant temperature, its pressure and volume vary along the appropriate isothermal, and the gas is said to undergo an isothermal compression expansion

Isothermal reversible change

When the gas is compressed isothermally from P_1, V_1, T to P_2, V_2, T then a graph which show this variation is:



If the gas is allowed to expand isothermally so that the state of the gas is brought back from (P_2, V_2, T) through exactly the same intermediate stage then the gas is said to undergo isothermal reversible change:

Definition

An isothermal reversible change is that change which goes to and from through exactly the same intermediate stages at constant temperature.

Isothermal reversible change equation

Since the temperature is constant, and is isothermal change obeys Boyle's law.

$$PV = \text{constant.}$$

$\therefore$ $P_1V_1 = P_2V_2$ Isothermal reversible change equation

Work done during isothermal change

Consider a gas pressure (P) expanded isothermally from volume V_1, to volume V_2. The gas does some work during the expansion given by:

$dW = Pdv$ …………………….. (1)

Where all symbols carry their usual meaning.

According to ideal gas equation

$PV = nRT$

P = $\frac{nRT}{V}$ …………………………. (2)

Substitute equation (2) in equation (1)

dW = $\frac{nRT}{V}$ dV ……………………………….(3)

When n, R and T are constants

The total work done W is obtained by integrating equation (3) above from volume V_1, to volume V_2.

$\int dw = \int_{V_1}^{V_2} \frac{nRTdV}{V}$

W = $NRt \int_{V_1}^{V_2} \frac{dV}{V}$

W = NRt$[\lfloor nV]_{V_1}^{V_2}$

W = NRt $[\lfloor nV_2 - \lfloor nV_1]$

∴ $W = Nrt \lfloor n \left[\frac{V_2}{V_1}\right]$ ……………………… (4)

The work done W can either be positive or negative depending on whether the gas undergoes isothermal expansion or compression.

If it is isothermal expansion then the work done is positive and the work is said to be done by the gas.

If it is isothermal compression then the work done is negative and the work is said to be done on the gas by the compressing agent.

The first law of thermodynamics applies to an isothermal change.

According to first law of thermodynamics

Q = Δu + $P\Delta V$

Where,

Q $=$ Heat input

$\Delta u =$ Increase in internal energy of the gas

$P\Delta V =$ External work done

For an isothermal change the temperature of the gas is constant and hence for an ideal gas there is no change in internal energy of the gas, since

$\Delta u = nC_V\Delta T$ or $\Delta u = \frac{3}{2}$ nRΔT

Thus, for an isothermal change $\Delta T =$ 0

Hence $\Delta u = nC_V \times$ 0 OR $\Delta u = \frac{3}{2}$ Nr $\times$ 0

$\therefore \Delta u =$ 0

Applying the first law of thermodynamics

Q = Δu + $P\Delta V$

Q = 0 + $P\Delta V$

Q = $P\Delta V$ External work done

Which means the whole amount of heat energy supplied (Q) is used to perform external work $P\Delta V$

ADIABATIC CHANGE /PROCESS

An adiabatic change is the change which takes place without exchange of heat between inner and outer of the system.

It is the one which takes place at constant heat.

In general, an adiabatic change involves a fall or rise in temperature of the system.

Condition for adiabatic change

No heat is allowed to enter or leave the gas. **therefore.**

(1) The gas must be held in a thick – walled, badly, conducting vessel.

(2) The change in volume must take place rapidly to give as little time as possible for heat to escape.

Examples of adiabatic process/change

i. i. The rapid escape of air from a burst Tyre.

i. ii. The rapid expansions and contractions of air through which a sound wave is passing.

Adiabatic change represented graphically
A curve which relates the pressure and volume when the heat content of the gas is kept constant is called an adiabatic.
Adiabatic curves and isothermal curves are similar except that adiabatic are steeper than isothermals.
If the gas is compressed adiabatically from volume V_0 to volume V_1 its temperature rises to T2 so that its new position is (P_1V_1) on the new isothermal.

Similarly, if the gas is left to expand adiabatically from volume V_0 to volume V_2 its temperature is lowered to T_1 so that its new position is (P_2, V_2) on the new isothermal

Adiabatic reversible change

Definition

An adiabatic reversible change is the change which goes to and fro through exactly the same intermediate stages without exchange of heat between inner and out of the system.

Adiabatic reversible change equation

Consider an adiabatic change and the first law of thermodynamics

Q $= \Delta u$ + $P\Delta V$

But $\Delta u = C_V \Delta T$ - for 1 mole of a gas

Since that is an adiabatic change Q = 0 i.e. no heat is allowed to enter or leave the system.

0 = $C_V \Delta T$ + $P\Delta V$ ………………………….. (1)

From ideal gas equation for 1 mole

PV = RT

Differentiating this equation

PdV + VdP = RdT

We may write:

$P\Delta V + V\Delta P = R\Delta T$

$\therefore \ \Delta T = \frac{P\Delta V + V\Delta P}{R}$ ………………………….. (2)

Substitute equation (2) in equation (1)

$0 = C_V \left[\frac{P\Delta V + V\Delta P}{R}\right] + P\Delta V$

$0 = C_V \ P\Delta V$ + V$C_V \ \Delta P + RP\Delta$ ………………….. (3)

According to Mayer's equation

R = C_P - C_V .... (4)

substitute equation (4) in equation (3)

$0 = C_V \ P\Delta V$ + V$C_V \ \Delta P + (C_P - C_V) \ P\Delta V$

$0 = C_V \ P\Delta V$ + V$C_V \ \Delta P + PC_P - \ C_V P\Delta V$

0 = V$C_V \ \Delta P + PC_P\Delta$

Dividing by C_V throughout

$0 = P\Delta V + \frac{PC_P}{C_V}\Delta V$

But $\frac{C_P}{C_V} = \gamma$ = ratio of molar heat

Capacities of a gas

$0 = V\Delta P + P\gamma\Delta V$

We may write

$0 = VdP + P\gamma dV$

$VdP = -P\gamma dV$

$\frac{dP}{P} = \frac{-\gamma dV}{V}$

Integrating this equation

$\int \frac{dp}{P} = \int \frac{-\gamma dv}{v}$

$\ln P = \gamma \ln V + K$

Where K = constant of integration

$\ln P + V^{\gamma} \ln = K$

$\ln P + \ln V^{\gamma} = K$

$\text{Ln}(PV^{\gamma}) = K$

$\log_e(PV^{\gamma}) = K$

$PV^{\gamma} = e^{k}$

Where e^{k} = constant

PV^{γ} = Constant .... (5)

In general

$P_1 V_1{}^{\gamma} = P_2 V_2{}^{\gamma}$ ....(6)

→ Adiabatic reversible change equation

Adiabatic reversible change equation in terms of temperature.

From equation (5)

PV^{γ} = constant

PV^{γ} = K ……………………………………….. (7)

From ideal as equation

PV = RT

P = $\frac{RT}{V}$ ………………………………………….. (8)

Substitute equation (8) in equation (7)

$\frac{RT}{V} . V^{\gamma}$ = K

$TV^{\gamma-1} = \frac{K}{R}$

But $\frac{K}{R}$ = constant

$TV^{\gamma-1}$ = Constant ....(9)

In general

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ ....(10)

→ Adiabatic equation in terms of temperature

Work done during adiabatic process

Consider a gas which expands adiabatically such that its volume changes from V_1 to V_2

The work done during the expansion is given by:

W = $TV^{\gamma-1}$ .... (11)

For an adiabatic change PV^{γ} = K

∴ P = $KV^{\gamma-1}$

W = $\int_{V_1}^{V_2} KV^{-\gamma} dV$

W = K $\int_{V_1}^{V_2} V^{-\gamma} dV$

W = K $\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$

W = $\frac{K}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$

W = $\frac{1}{1-\gamma}\left[KV_2^{1-\gamma} - KV_1^{1-\gamma}\right]$

But, K = $P_1 V_1^{\gamma}$ = $P_2 V_2^{\gamma}$

W = $\frac{1}{1-\gamma}\left[\ P_2 V_2^{\gamma}.V_2^{1-\gamma} - P_1 V_1^{\gamma}.V_1^{1-\gamma}\right]$

W = $\frac{1}{1-\gamma}\left[\ P_2 V_2^{\gamma}.V_2^{1-\gamma} - P_1 V_1^{\gamma}.V_1^{1-\gamma}\ \right]$

W = $\frac{1}{1-\gamma}\left[\ P_2 V_2 - P_1 V_1\ \right]$

If the temperature of the initial state is T_1 and that of the final state is T_2 then ideal gas equation gives.

$P_1 V_1$ = nRT_1 and $P_2 V_2$ = nRT_2

W = $\frac{1}{1-\gamma}\left[nRT_2 - nRT_1\right]$

$$W = \frac{nR\,(T_2 - T_1)}{1-\gamma} \quad \text{..........................(12)}$$

The first law of thermodynamics applied to an adiabatic change / process

According to first law of thermodynamics

Q = Δu + $P\Delta V$

Where all symbols carry their usual meaning.

For an adiabatic process there is no change in heat content of the gas i.e Q = 0

0 = Δu + $P\Delta V$

$\therefore\ \Delta u$ = - $P\Delta V$

Since $\Delta u = nC_V\Delta T$ we have:

$$\Delta u = nC_V\Delta T = -P\Delta V \quad \text{.................................. (13)}$$

Here the work is done at the expense of the internal energy of the gas itself. Thus, if the gas expands adiabatically, it does work and W = $P\Delta V$ is positive, hence Δu is negative, since the temperature of the gas decreases.

If the gas is compressed adiabatically, the work is done on it and W = $P\Delta V$ is negative, hence Δu is positive, since the temperature of the gas increases.

From equation (13)

$$\Delta u = nC_V\Delta T = -P\Delta V$$

$$P\Delta V = -nC_V\Delta T$$

But $P\Delta V$ = W = work done during adiabatic process.

W = $-nC_V\Delta T$

W = $-nC_V(T_2 - T_1)$

W = $-nC_V(T_1 - T_2)$ .... (14)

From ideal gas equation: PV = nRT

$\therefore T_1 = \frac{P_1V_1}{nR}$ and $T_2 = \frac{P_2V_2}{nR}$

Equation (14) becomes:

W = $nC_V\left[\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right]$

W = $\frac{nC_V}{nR}[P_1V_1 - P_2V_2]$

W = $\frac{C_V}{R}[P_1V_1 - P_2V_2]$ …………………………….. (15)

But $\frac{C_V}{R} = \frac{C_V}{CP - C_V} = \frac{1}{\gamma - 1}$

Equation (15) becomes:

W = $\frac{1}{\gamma-1}[P_1V_1 - P_2V_2]$

But $P_1V_1 = nRT_1$ *and* $P_2V_2 = nRT_2$

$$W = \frac{1}{\gamma - 1}[nRT_1 - nRT_2]$$

$\therefore$ W = $\frac{nR\,(T_1 - T_2)}{\gamma - 1}$ ………………(16)

OR

$\therefore$ W = $\frac{nR\,(T_2 - T_1)}{1 - \gamma}$ ………………(17)

Problem 63

5 moles of hydrogen initially at STP are compressed adiabatically so that the temperature becomes 400 ℃. Find:

(i) The work done on the gas

(ii) The increase in internal energy of the gas

Given that γ = 1.4 for diatomic gas.

Problem 64

At 27$^{\circ}C$ two moles of an ideal monatomic gas occupy a volume V. the gas expands adiabatically to a volume 2V. Calculate:

(i) The final temperature of the gas

(ii) The change in its internal energy

(iii) The work done by the gas during this process

Given that γ = 1.67

Problem 65

A metallic cylinder contains 10 litres of air at 3 atmospheres of pressure and temperature of 300K.

(a) If the pressure is suddenly doubled, what are the new values of volume and temperature.

(b) If the pressure is slowly doubled, what are the new values of volume and temperature.

Problem 66

(a) Define the principle molar heat capacities of a gas.

(b) Why the energy needed to raise the temperature of a fixed mass of a gas by a specific amount is greater if the pressure is kept constant than when the volume is kept constant.

(c) Find the two principal heat capacities for oxygen (diatomic molecule) whose ratio of $\frac{C_P}{C_V}$ is 1.4 at STP.

Problem 67

A quantity of oxygen is compressed isothermally until its pressure is doubled. It is then allowed to expand adiabatically until its volume is restored. Find the final pressure in terms of the initial pressure. Given that γ = 1.4

Problem 68

(a) With the help of sketch diagram distinguish between an "isothermal change" and an "adiabatic change". Illustrate your answer with an example of a gas changing from state A to state B.

(b) Argon gas (specific heat capacity ratio 1.67) is contained in a 250 cm^3 versel at a pressure of 750mmHg and a temperature of 0 $^{\circ}C$. The gas is expanded isothermally to a final volume of 400 cm^3

(i) Calculate the final pressure of the gas

(ii) By how much will the pressure will be lowered if the change is made adiabatically instead?

Problem 69

(a)

(i) Define mean "free path" for a molecule of a gas

(ii) How is the means free path of the molecule of a gas affected by temperature.

(b) The heat capacity C_v at constant volume for 8 moles of oxygen gas is 166.2KJ^{-1} .

Find the heat capacity at constant pressure for 8 moles of oxygen.

Problem 70

(a) What is the difference between an "isothermal process" and "adiabatic process"?

(b) How, much work is required to compress 5 moles of air at 20 $^{\circ}C$ and 1 atmosphere pressure at $\frac{1}{10}$ of the original volume by:

(i) An isothermal process

(ii) An adiabatic process

(c) What are the final pressure for case (b) (i) and (b) (ii) above?

(d) In a diesel engine, the cylinder compresses air from approximately standard temperature and pressure to about one sixteenth of the original volume and a pressure of about 5 atmospheres. What is the temperature of the compressed air?

γ = 1.403

R = 8.31 $mol^{-1}K^{-1}$

C_V = 20.68 $Jmol^{-1}K^{-1}$

Problem 71

100g of a gas are enclosed in a cylinder which is fitted with a movable frictionless piston.

When a quantity of heat is supplied to the gas it expands at constant pressure doing 8400J of work and heating up by 20 $^{\circ}C$. Calculate:

(i) The change in internal energy of the gas

(ii) The specific heat capacity of the gas at constant volume C_V

Given that C_P = 1.26 x $10^3 JKg^{-1}K^{-1}$

Problem 72

Given that the molar heat capacities of hydrogen at constant value and constant pressure are respectively 20.5 $Jmol^{-1}K^{-1}$ and 28.8 $Jmol^{-1}K^{-1}$. Calculate

(i) The molar gas constant

(ii) The heat needed to raise the temperature of 8g of hydrogen from 10 $^{\circ}C$ to 15 $^{\circ}C$ at constant pressure.

(iii) The increase in internal energy of the gas

(iv) The external work done

Problem 73

The density of a gas is 1.775Kgm^{-3} at 27°C and 1$\times 10^5\ NM^{-2}$ pressure and its specific heat capacity at constant pressure is 846$JKg^{-1}K^{-1}$. Find the ratio of its specific heat capacity at constant pressure to that at constant volume.

Problem 74

A gas of volume 500cm^3 and pressure 1.0 x 10^5 Nm expands adiabatically to 600cm^3. Calculate

(i) The find pressure

(ii) The work done by the gas

(iii) The final temperature if the initial temperature of the gas before expansion was 23°C

Given that γ = 1.4

Problem 75

One gram of water becomes 1671 of steam at a pressure of 1 atmosphere$(= 1.013\ x\ 10^5 Pa)$. The latent heat of vaporization at this pressure is 2256Jg^{-1}. Calculate the external work done and the increase in internal energy.

Problem 76

1.0m^3 of water is converted into 1671cm^3 of steam at atmospheric pressure and 100°C temperature. The latent heat of vaporization of water is 2.3 $\times 10^6 JKg^{-1}$. If 2.0Kg of water is converted into steam at atmospheric pressure and 100°C temperature, then how much will be the increase in its internal energy?

Given that

Density of water = 1.0 $\times 10^3 Kgm^{-3}$

Atmospheric pressure = 1.01 $\times 10^5 Nm^{-2}$

Problem 77

(a) What is an isothermal change?

(b) A cylinder fitted with a frictionless piston holds a volume of 1000cm^3 of air at a pressure of 1.10 $\times 10^5$ Pa and temperature of 300 K. The air is then heated to 375K at constant pressure. Determine the new volume of the gas. The gas is then compressed isothermally to a volume of 1000cm^3. Calculate the new pressure.

Problem 78

(a) (i) What is the difference between an isothermal and an adiabatic process?

(ii) Show that an adiabatic change follows an adiabatic equation.

PV^{γ} = constant

(b) (i) Distinguish between the specific heat capacity and the molar heat capacity. Given the unit of each.

(ii) Calculate the two principal molar heat capacities of oxygen and explain why the specific heat capacity of the gas at constant volume is less than that at constant pressure.

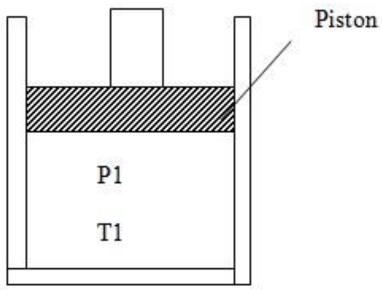
Problem 79

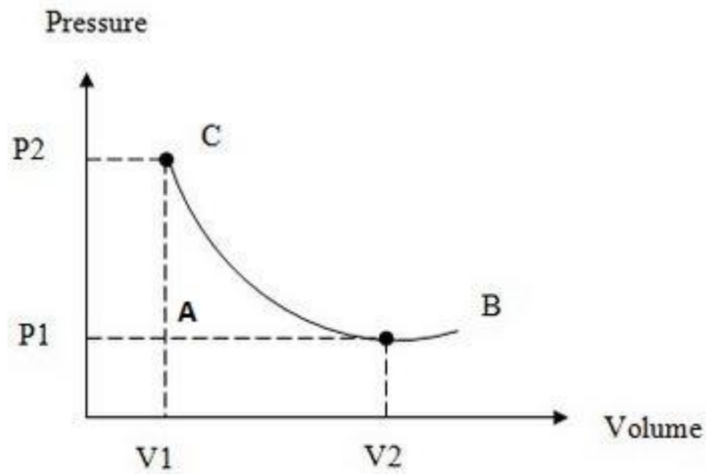
i. ( ) What is reversible change?
ii. ( ) State the condition for a reversible change to occur.
iii. ( )A litre of air at 10^5 Pa pressure expands adiabatically and reversibly to twice its volume.

Calculate the work done by the gas.

Problem 80

A cylinder in the figure below holds a volume V_1 = 1000cm^3 of air at an initial pressure of P_1 = 1.1 $\times 10^5$ Pa and temperature T_1 = 300K. Assume the air behaves like an ideal gas.





i. (i) AB – the air heated to 373 K at constant pressure. Calculate the new volume.

i. (ii) BC – the air is compressed isothermally to volume V_1. Calculate the new pressure P_2

i. (iii) Calculate the root mean square speed of nitrogen molecules at a temperature of 27 $^{\circ}C$

Problem 81

a. ( ) State the 1st law of thermodynamics and write its equation.
b. ( ) A liter of air initially at 25 $^{\circ}C$ and 760mmHg is heated at constant pressure until the volume is doubled. Determine:

(i) The final temperature

(ii) The external work done by the air in expanding it.

(iii) The quantity of heat supplied

Problem 82

0.15 mol of an ideal mono atomic gas is enclosed in a cylinder at a pressure of 250 KPa and a temperature of 320K. The gas is allowed to expand adiabatically and reversibly until its pressures is 100KPa

(a) Sketch a P – V curve for the process.

(b) Calculate the final temperature and the amount of work done by the gas.

Problem 83

i. ( ) Define the bulk modulus of a gas

i. (i) Find the ratio of the adiabatic bulk modulus of a gas to that of its isothermal bulk modulus in terms of the specific heat capacities of the gas.

Problem 84

(a) A gas expands adiabatically and its temperature falls while the same gas when compressed adiabatically its temperature rises. Explain giving reasons why this happens.

(b) A mole of oxygen at 280K is insulated in an infinitely flexible container is 5 $\times 10^5 Nm^{-2}$. When 580J of heat is supplied to the oxygen the temperature increases to 300K and the volume of the container increases by 3.32 $\times 10^{-4} m^3$. Calculate the values of the principal molar heat capacities and the specific universal gas constant.

Given that molar mass of oxygen = 32 $\times 10^{-3} kg$

Problem85

(a) (i) Why is heat needed to change liquid water into vapour?

What amount of energy is needed

(ii) The molar heat capacity of hydrogen at constant volume

is 20.2 $Jmol^{-1}K^{-1}$

What is the molar heat capacity at constant pressure?

(b) In an industrial refrigerator ammonia is vaporized in the cooling unit to produce a low temperature. Why should the evaporation of ammonia reduce the temperature in the refrigerator?

How much energy is needed to convert 150g of water at 20 ℃ into steam at 100 ℃

Problem 86

An ideal gas is kept in thermal contact with a very large body of constant temperature T and undergoes an isothermal expansion in which its volume changes from V_1 to V_2.

Derive an equation for the work done by the gas.

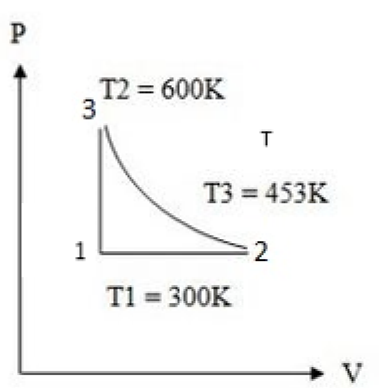
Problem 87

A heat engine carries I mole of an ideal gas around a cycle as shown in the figure below. Process 1 – 2 is at constant volume, process 2 – 3 is adiabatic and process 3 – 1 is at a constant pressure of 1 a.t.m. The value of γ for this gas is $5/3$.

Find:

i.) The pressure and volume at points 1, 2 and 3

ii) The net work done by the gas in the cycle.



Problem 88

The door of a working refrigerator is left open.

What effect will this have on the temperature of the room in which the refrigerator is kept?

Explain

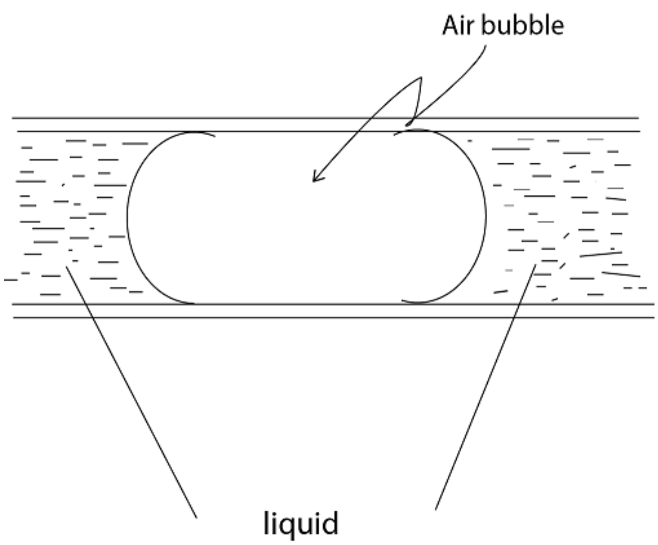
Problem 89

(a a) What do you understand by the terms:

(i) Critical temperature?

(ii) Adiabatic change?

(b b) An air bubble is observed in a pipe of the braking system of a car. The pipe is filled with an incompressible liquid (see figure below). When the brakes are applied, the increased pressure in the pipe causes the bubble to become smaller.



Before the brakes are applied the pressure is 110 $\times 10^3 Nm^{-2}$, the temperature is 290K and the length of the bubble is 15mm. When the brakes are applied quickly, the air bubble is compressed adiabatically and if the change in its length exceed 12mm the brakes fail. If the internal cross-sectional area of the pipe is 2 $\times 10^{-5} m^2$

(i) Explain briefly why the compression of the bubble is considered to be adiabatic.

(ii) What is the maximum safe pressure in the system during rapid braking if the bubbles change in length does not exceed 12mm? Take γ_{air} = 1.4

(iii) Determine the temperature of the air in the bubble at the end of the adiabatic compression.

Problem 90

(a) Find the number of molecules and their mean kinetic energy for a cylinder of volume 4 $\times 10^{-4} m^3$ containing oxygen at a pressure of 2 $\times 10^5$ Pa and a temperature of 300K

(b) When the gas is compressed adiabatically to a volume of 2$\times 10^{-4} m^3$, the temperature rises to 434K. Determine the γ, the ratio of the principal heat capacities.

Given that:

Molar gas constant = 8.31$J mol^{-1} K^{-1}$

And N_A = 6 $\times 10^{23} mol^{-1}$

Problem 91

(a) The first law of thermodynamics is a consequence of the law of conservation of energy. Explain briefly.

(b) What is the difference between isochoric process and isobaric process?

(c) Why is the energy needed to raise the temperature of a fixed mass of a gas by a specific amount is greater if the pressure is kept constant than if the volume is kept constant?

A certain volume of a dry air at S.T.P is allowed to expand four times its original volumes under:

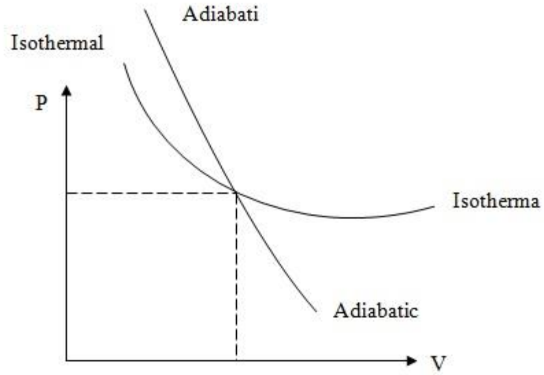
(i) Isothermal conditions

(ii) Adiabatic conditions

Calculate the final pressure and temperature in each case

Given that γ = 1.4

Problem 92



In a P – V diagram shown above, an adiabatic and an isothermal curve for an ideal gas intersect. Show that the absolute value of the slope of the adiabatic $\left|\left[\frac{dp}{dv}\right]\right|$ is γ times that of the isothermal

Hence the adiabatic curve is steeper because the specific heat ratio γ is greater than 1

Problem 93

A Tyre has air pumped at a pressure of 4 atmospheres at room temperature of 27 $^{\circ}C$. If the Tyre bust suddenly, calculate the final temperature (take γ = 1.4)

Problem 94

Two moles of oxygen are initially at a temperature of 27 $^{\circ}C$ and volume of 20 litres. The gas expanded first at constant pressure until the volume has doubled, and then adiabatically until the temperature returns to the original value.

(i) What is the total increase in internal energy?

(ii) What is the final volume?

Given that $C_P/C_V = \gamma = 2$

Problem 95

The Specific heat capacity of hydrogen at constant volume is 1.01$\times 10^4 Jkg^{-1}K^{-1}$. If the density of hydrogen at S.T.P is 0.09kgm^{-3}, calculate the specific heat capacity of hydrogen at constant pressure.

Problem 96

(i) Does a gas do work when is expands adiabatically? If so what is the course of energy needed to do this work.

(ii)Derive a relation between the bulk modulus K and density ρ of a perfect gas under isothermal conditions and adiabatic conditions.

A mass of air at 27$^{\circ}C$ and 750mmHg pressure occupies a volume of 8litres. If the air expands first isothermally until its volume increases by 50% and then adiabatically until its volume again increases by 50% each time reversibly. Calculate

(1) The final pressure

(2) The final temperature

An ideal gas expands adiabatically from initial temperatures T_1 to a final temperature T_2, prove that the work done by the gas is $C_V\ (T_1 - T_2)$

Problem 97

An ideal gas at 760mmHg is compressed isothermally until its volume is reduced to 75% of its original volume. The gas is then allowed to expand adiabatically to a volume 120% of its original volume. If the temperature of the gas is 20$^{\circ}C$

(a) Construct the P – V indicator diagram.

(b) Calculate the final pressure and temperature

Given that:

C_P = 3600 J/kg K

C_V = 2400 J/kg K

HEAT-4

Problem 98

A mono atomic gas initially at the temperature T = 25 $^{\circ}C$ and pressure of 2 atmospheres is expanded to a final pressure of 1.0 atmosphere.

a) Isothermally and reversibly

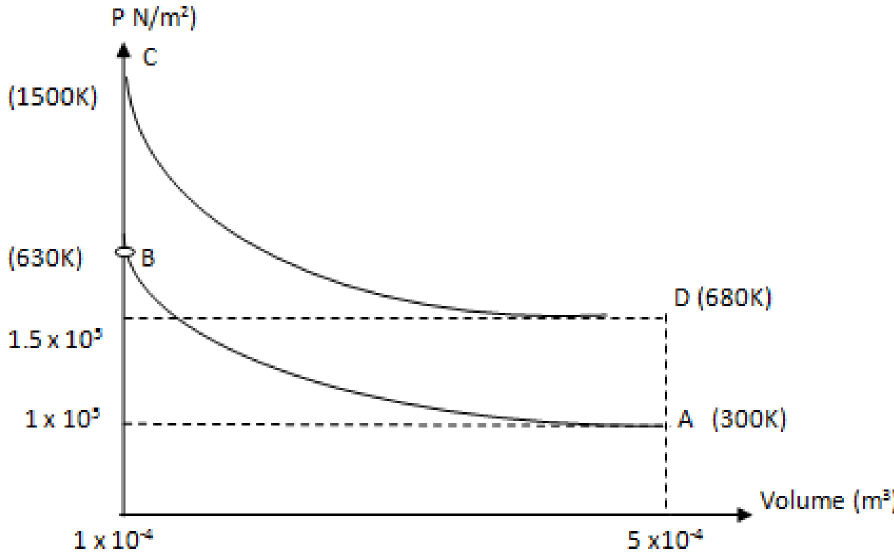
b) Isothermally against a constant pressure of 1.0 atmosphere. Calculate for each case:

(i) The final temperature of the gas

(ii) The increase of internal energy

Problem 99

The figure below shows some details concerning the behavior of a fixed mass of a gas assumed to be an ideal one in a petrol engine. The gas starts at A with a volume 5 $\times 10^{-4} m^3$, temperature 300 K and a pressure of 1 $\times 10^5 N/m^2$. In the change from A to B it is compressed to volume of 1 $\times 10^{-4} m^3$, the pressure rises to 1.5 $\times 10^5 N/m^2$. And temperature 630K



a) Using the equation of state for an ideal gas, find the number of molecules in the fixed mass of a gas.

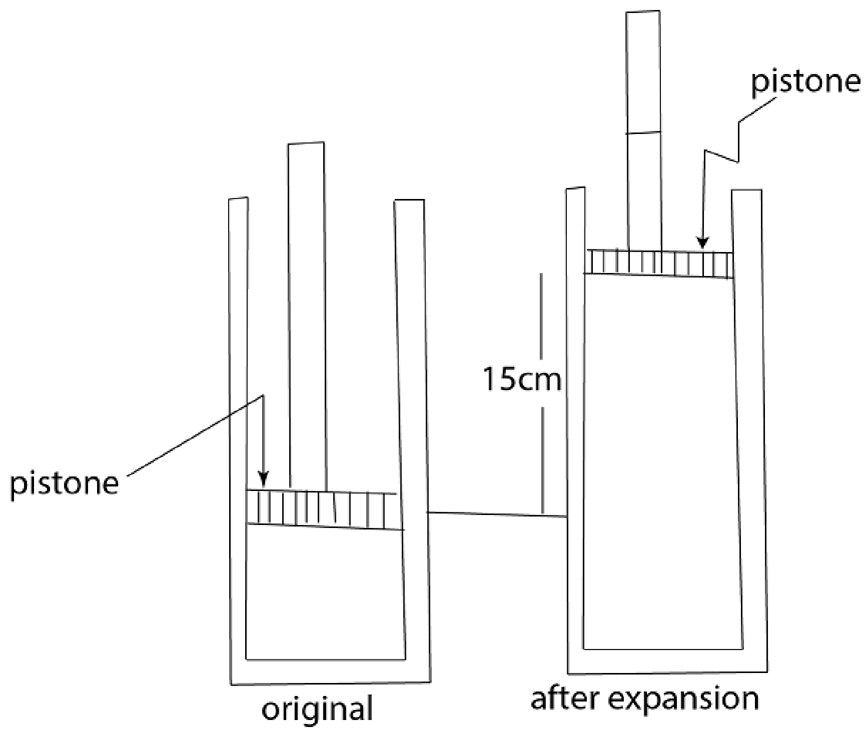
b) In the change from B to C the temperature of a gas rises from 630 K to 1500 K. The molar heat capacity at constant volume of the gas is 21 $JK^{-1}mol^{-1}$. Calculate the internal energy of the gas.

c) How much work is done by the gas in changing from B to C?

d) In the change from C to D, the gas expands to its original volume; the temperature at D is 680 K. Calculate the pressure at D.

Problem ***100***

The figure below shows a sample of gas enclosed in a cylinder by a frictionless piston of area 100 cm^2. The cylinder is now heated, so that 250J of energy is transferred to the gas, which then expands against atmospheric pressure of 1.00 x $10^5 Nm^{-2}$. And pushes the piston 15.0 cm along the cylinder as shown



Calculate:

(a) The external work done by the gas

(b) The increase in internal energy of the gas.

Problem ***101***

When 1.50kg of water is converted to steam (at 100 °C) at standard atmospheric pressure of 1.01 $\times 10^5 Nm^{-2}$, 3.39MJ of heat are required. During the transformation from liquid to vapor state, the increase in volume of the water is 2.50m^3. Calculate the work done against the external pressure during the process of vaporization. Explain what happens to the rest of the energy.

Problem 102

A fixed mass of gas is cooled, so that its volume decreases from 4.0 liters to 2.5 liters at a constant pressure of 1.0 $\times 10^5$ Pa.

Calculate the external work done by the gas.

Problem 103

The specific latent heat of vaporization of steam is 2.26 MJ Kg^{-1}. When 50cm^3 of water is boiled at standard atmospheric pressure of 1.01 $\times 10^5$ Pa, 83 $\times 10^3\ cm^3$ of steam are formed.

Calculate

(a) The mass of water boiled

(b) The heat input needed

(c) The external work done during vaporization

(d) The increase in internal energy

Given that density of water 1000 kgm^{-3}

Problem 104

56.0 $\times 10^{-3}$ kg of nitrogen is to be heated from 270 K to 310K. When this occurs in an insulated freely extensible container, 2.33 Kj of heat is required when contained in an insulated rigid container, 1.66KJ of heat is required. Calculate the principal molar heat capacities of nitrogen.

Problem 105

The specific heat capacity of a diatomic gas at constant volume is 0.410 KJ $Kg^{-1}k^{-1}$

Calculate

(a) The specific heat capacity of the gas at constant pressure.

(b) The specific gas constant for the gas.

Problem 106

The amount of heat required to raise the temperature of 3.00 mole of a polyatomic gas, at constant pressure, from 320 K to 370 K is 4.99 KJ.

Calculate

(a) C_P and C_V

(b) The value of γ

(c) The heat required to raise the temperature of 4.00 mole from 300 K to 400 K at constant volume

Problem 107

Argon has a molar heat mass of 40 $\times 10^{-3}$ kg and a principal molar heat capacity, at constant volume, of 12.5 $Jmol^{-1}k^{-1}$.

Calculate:

(a) The valve of γ

(b) The specific heat capacity at constant volume

(c) The amount of heat required to raise the temperature of 1.00kg of argon by 80 K at constant volume.

Problem 108

2.00 mole of nitrogen, at 300K are in an insulated, freely extensible container, and the pressure outside the container is 1.00 $\times 10^5 Nm^{-2}$. The principal molar heat capacity of nitrogen at constant pressure is 29.0 $Jmol^{-1}k^{-1}$.

Calculate:

(a) The heat required to raise its temperature to 340 K.

(b) The increase in volume of the gas during this process.

(c) The external work done

(d) The internal energy change

(e) The heat required to effect the temperature change at constant volume

Compare (d) and (e) and comment

Problem 109

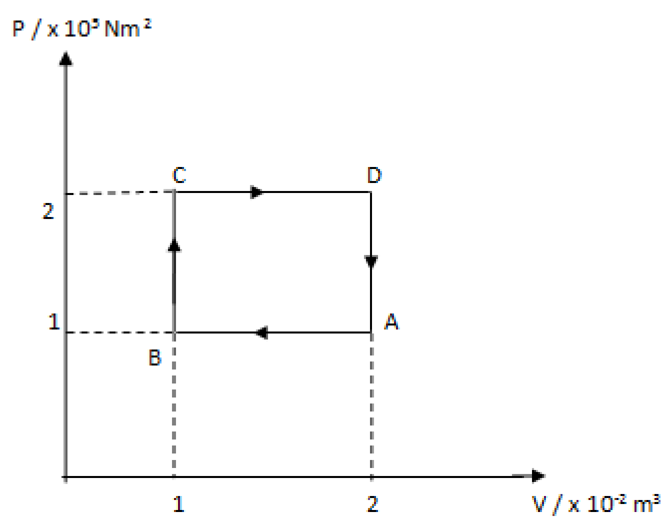
The piston of a bicycle pump is slowly moved in until the volume of air enclosed is one-fifth of the total volume of the pump and is at room temperature (290K). The outlet is then sealed and the piston suddenly drawn out to full extension. No air passes the piston. Find the temperature of the air in the pump immediately after withdrawing the piston assuming that air is a perfect gas with γ = 1.

Problem 110

A fixed mass of gas, initially at 7$^{\circ}C$ and a pressure of 1.00 $\times 10^5 Nm^{-2}$, is compressed isothermally to one – third of its original volume. Calculate the final temperature and pressure, assuming γ = 1.40

Problem 111

A fixed mass of gas is taken through the closed cycle A$\rightarrow$B$\rightarrow$C$\rightarrow$D$\rightarrow$A as shown in the figure below



(a) Calculate the work done in the cycle.

(b) How much heat transferred in the cycle?

(c) Is the heat absorbed or emitted by the gas?

Problem 112

One mole of water, occupying a volume of 1.8 $\times 10^{-5}\ cm^3$, is turned into steam in a boiler at a temperature of 373 K and a pressure of 1.0 $\times 10^5$ Pa. The volume of steam generated is 0.031 cm^3. The energy required is 41,000J.

Calculate the work done (in Joules) against the atmospheric pressure in the production of steam.

Problem 113

The figure below shows water changing into steam at constant pressure and held in a cylinder by a free – sliding piston.

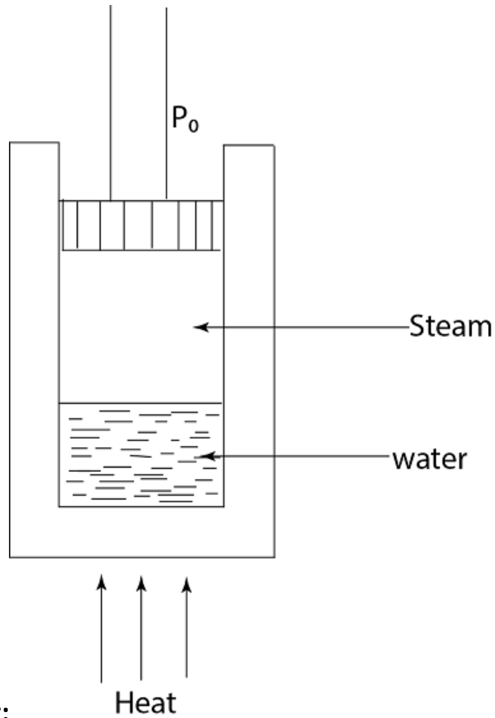
1.00 kg of water at 100 $°C$ is changing into steam at atmospheric pressure.

Calculate:

(a) The external work done

(b) The increase in internal energy

(c) What happens to the internal energy absorbed during the vaporization process?



Given that:

Density of water at 100 $^{\circ}C$ = 960 kgm^{-3}

Density of steam at 100 $^{\circ}C$ and at atmospheric pressure = 0.59

Atmospheric pressure, P_0 = 1.01 $\times 10^5$ Pa.

Specific latent heat of vaporization of water = 2.26 $\times 10^6 Jkg^{-1}$

Problem 114

At a temperature of 100 and a pressure of 1.01 $\times 10^5$ Pa, 1.00Kg of steam occupies 1.67 m^3, but the same mass of water occupies only 1.04 $\times 10^{-3}$ cm^3. The specific latent heat of vaporization o water at 100 $^{\circ}C$ is 2.26 $\times 10^6 Jkg^{-1}$. For a system consisting of 1.00kg of water changing to steam at 100 $^{\circ}C$ and 1.01 10^5 Pa, find :

(a) The heat supplied to the system

(b) The work done by the system

(c) The increase in internal energy of the system.

Problem 115

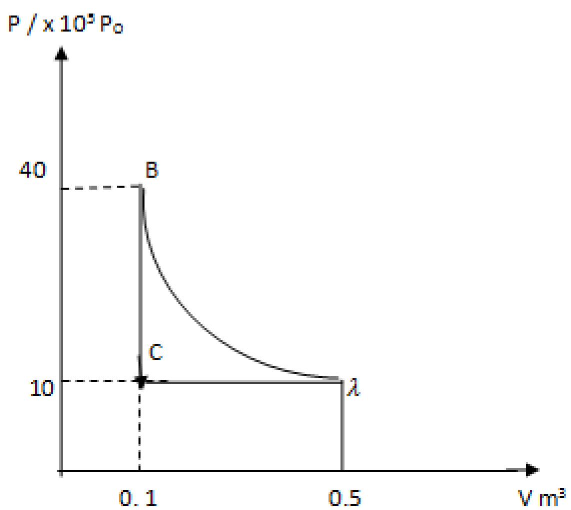
The ratio of the principal heat capacities of an ideal gas is γ, and the molar gas constant is R.
Show that the molar heat capacity at constant pressure of the gas is

$$C_p = \frac{\gamma R}{\gamma - 1}$$

Problem 116

The specific heat capacity at constant volume of a certain ideal gas is 6 x $KJg^{-1}K^{-1}$ and is independent of temperature.
Find the internal energy of 5.0 x 10^{-3} kg of the gas at 27 °C.

Problem 117



Helium may be assumed to behave as an ideal gas. A sample of 20 moles of the gas are taken through the cycle of changes ABC as shown in the figure above.

(i) Use the data from the figure to show that the change from A to B must take place at constant temperature.

(ii) The temperature for this change is 300K. What is the temperature of the gas at C?

(iii) What energy process takes place between B and C?

(iv) The change in internal energy of the sample in the process from B to C is 56KJ. Calculate the molar heat capacity at constant volume for helium.

(v) Calculate the work done during the change from C to A. State and explain whether work is done on or by the gas during this part of the cycle. Justify your answer.

(vi) Determine the value of molar heat capacity at constant pressure for helium. Show Clearly how you arrive at your answer.

(vii) Use the figure to estimate the net work done during one complete cycle.

Problem 118

A fixed mass of an ideal gas has a volume V_0 at an initial temperature of 300 K and an initial pressure of 1.2 $\times 10^5$ Pa.
It is made to undergo the following cycle of process.

A. Isothermal expansion from its initial volume V_0 to a volume 2 V_0
B. Expansion at constant pressure to a volume 4 V_0
C. Isothermal compression

D. Compression at constant pressure to its initial state.

(a) Sketch a cycle on a P – V diagram

(b) Determine:

(i) The pressure at the end of process A

(ii) The temperature at the end of process B

(iii) The volume at the end of process C

Problem 119

A fixed mass of an ideal gas at an initial temperature of 20 ^{o}C and at a pressure of 1.00 $\times 10^5$ Pa is compressed until its volume is one- quoter of its original volume.

Calculate the final temperature and pressure of the gas, assuming:

(a) The compression is isothermal

(b) The compression is adiabatic

Given that $C_P/C_V = \gamma = 1.40$

Problem 120

In a diesel engine, fuel oil is injected into a cylinder in which air has been heated by adiabatic compression to above the ignition temperature of the oil. The ignition temperature of a certain fuel is 630 $^{\circ}C$, and the air enters the cylinder, which has an initial volume of 5.0 $\times 10^{-4} m^3$ at a pressure of 1.0 $\times 10^5$ Pa and a temperature of 28 $^{\circ}C$

(a) What minimum compression ratio (the ratio of the initial to the final volume of the cylinder) is required to heat the air to the fuel ignition temperature?

(b) How much work is done in compressing the air?

Given that for air $\gamma = 1.40$

Problem 121

(a) A cylinder fitted with a piston which can move without friction contains 0.05 mole of a mono atomic ideal gas at a temperature of 27 $^{\circ}C$ and a pressure of 1.0 $\times 10^5$ Pa.

Calculate:

(i) The volume of the gas.

(ii)The internal energy of the gas

(b) The temperature of the gas in (a) above is raised to 77 $^{\circ}C$, the pressure remaining constant.

Calculate:

(i) The change in internal energy

(ii)The external work done

(iii) The total heat energy supplied
Given that molar gas constant = 8.3 $J mol^{-1} k^{-1}$.

Problem 122

(a) Give one practical example of each of the following:
(i) A process in which heat is supplied to a system without causing an increase in

temperature.
(ii) A process in which no heat enters or leaves a system but the temperature changes.

(b) What happens to the energy added to an ideal gas when it is heated:

(i) At constant volume?

(ii) At constant pressure?

(c) Deduce an expression for the difference between the specific heat capacities of a gas at constant pressure and at constant volume.

(d) If the ratio of the principal specific heat capacities of a certain gas is 1.40 and its density at S.T.P is 0.09kgm^{-3}, calculate the values of the specific heat capacity at constant pressure and at constant volume. Standard atmospheric pressure = 1.01 x $10^5\ Nm^{-2}$

Problem 123

A steel pressure vessel of volume 2.2 $\times 10^{-2}\ m^3$ contains 4.0 $\times 10^5$ Pa and temperature 300 K. An explosion suddenly releases 6.48 $\times 10^4$ J of energy, which raises the pressure instantaneously to 1.0 $\times 10^6$ Pa. Assuming no loss of heat to the vessel, and ideal gas behaviour,

Calculate:

(a) The maximum temperature attained

(b) The two principal specific heat capacities of the gas.

What is the velocity of sound in this gas at a temperature of 300 K?

Problem 124

(a) Explain why an ideal gas can have infinity number of molar heat capacities and define the principal values.

(b)A thermally – insulated tube through which a gas may be passed at constant pressure contains an electric heater and thermometers for measuring the temperature of the gas as it enters and as it leaves the tube. 3.0 $\times 10^{-3}\ m^3$ of gas of density 1.8 kgm^{-3} flows into the tube in 90 seconds and, when electrical power is supplied to the heater at a rate of 0.16W, the temperature difference between the out let and inlet is 2.5 K. Calculate a value for the specific heat capacity of the gas at constant pressure.

Problem 125

(a) Explain clearly and concisely why, for a fixed mass of a perfect gas:

(i) The internal energy remains constant when the gas expands isothermally.

(ii) The heat capacity at constant pressure is greater than the heat capacity at constant volume.

(b) A vessel of volume 1.0 $\times 10^{-2} m^3$ contain an ideal gas at a temperature of 300 K and pressure 1.5 $\times 10^5$ Pa. Calculate the mass of gas, given that the density of the gas at temperature 285 K and pressure 1.0 $\times 10^5$ Pa is 1.2 kgm^{-3}

(c) 750 J of heat is suddenly releases in the gas, causing an instantaneous rise of pressure to 1.8 $\times 10^5$ Pa. Assuming ideal gas behavior, and no loss of heat to the containing vessel, Calculate the temperature rise, and hence the specific heat capacity at constant volume of the gas.

Problem 126

(a) What is an adiabatic change?

A vessel of volume 8.00 $\times 10^{-3} m^3$ contains an ideal gas at a pressure of 1.14 $\times 10^5$ Pa.

A stopcock in the vessel is opened and the gas expands adiabatically, expelling some of its original mass, until its pressure is equal to that outside the vessel 1.01 $\times 10^5$ Pa. The stopcock is then closed and the vessel is allowed to stand until the temperature returns to its original value; in this equilibrium state, the pressure is 1.06 $\times 10^5$ Pa.

(i) Explain why there was a temperature change as a result of the adiabatic expansion.

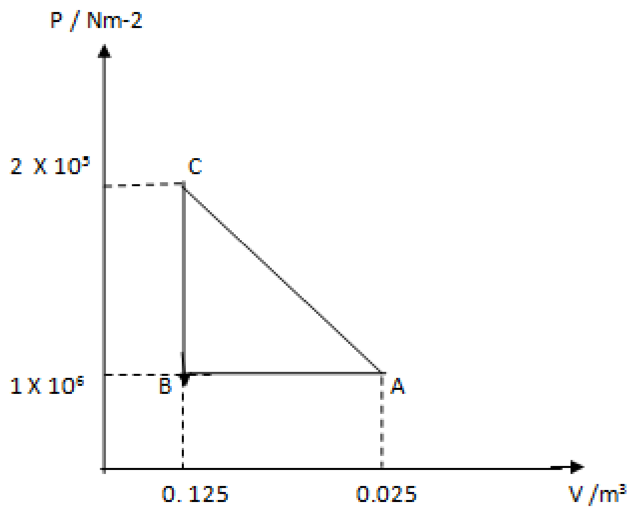
(ii) Find the volume which the mass of gas finally left in the vessel occupied under the original conditions.

(iii) Sketch a graph showing the way in which the pressure and volume of the mass of gas left in the vessel changed during the operations described above:

(iv) What is the value of γ, the ratio of the principal heat capacities of the gas.

(v) What can you deduce about the molecules of the gas? Give your reasons.

Problem 127



The diagram above represents an energy cycle whereby a mole of an ideal gas is firstly cooled at constant pressure $(A \rightarrow B)$ then heated a constant volume $(B \rightarrow C)$ and returned to its original state $(C \rightarrow A)$.

(a) Calculate the temperature of the gas at A, at B and at C

(b) Calculate the heat given out by the gas in the process A $\rightarrow$ B

(c) Calculate the heat absorbed in the process B $\rightarrow$ C

(d) Calculate the net amount of heat transferred in the cycle.

Given that R = 8.3 $Jmol^{-1}k^{-1}$. and

$C_V = \frac{5}{2} R$

Problem 128

The specific latent heat of vaporization of particular liquid at 130 $^{\circ}C$ and a pressure of 2.60 $\times 10^5$ Pa is 1.84 $\times 10^6\ Jkg^{-1}$.

The specific volume of the liquid under these conditions is 2.00 $\times 10^{-3}\ m^{-3}kg^{-1}$. And that of the vapor is 5.66 $\times 10^{-1}\ m^3kg^{-1}$.

Calculate:

(a) The work done, and

(b) The increase in internal energy when 1.00 kg of the vapor is formed from the liquid under these conditions.

Problem 129

(a) Explain what is meant by a reversible change.

(b) A mass of 0.35 kg of ethanol is vaporized at its boiling point of 78 °C and a pressure of 10 $\times 10^5$ Pa. At this temperature. The specific latent heat of vaporization of ethanol is 0.95 $\times 10^{-6} Jkg^{-1}$ and the densities of the liquid and vapor are 790 kgm^{-3} and 1.6 kgm^{-3} respectively. Calculate:

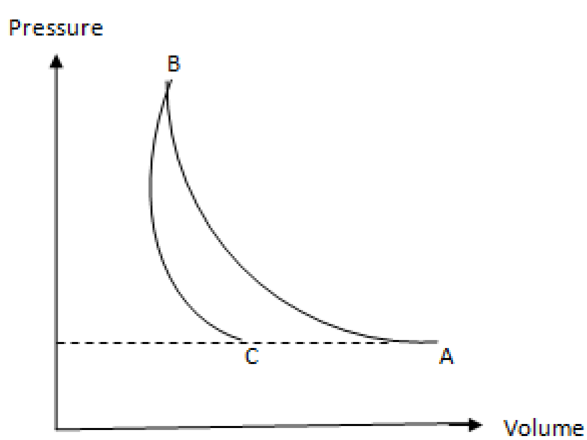
(i) The work done by the system

(ii)The change in internal energy of the system

Explain in molecular terms what happens to the heat supplied to the system.

Problem 130

The graph below relates the pressure and volume of a fixed mass of an ideal gas which is first compressed isothermally from A to B and then allowed to expand adiabatically from B to C.



For each of the changes shown on the graph, state and explain whether:

(a) The temperature of the gas changes

(b) There is heat transfer to or from the gas

(c) Work is done on or by the gas

Problem 131

An ideal gas at 17 $^{\circ}C$ has a pressure of 760mmHg, and is compressed.

(a) Isothermally,

(b) Adiabatically until its volume is halved, in each case reversibly. Calculate in each case the final pressure and temperature of the gas, assuming C_p = 2100$JKg^{-1}K^{-1}$ and C_V = 1500$JKg^{-1}K^{-1}$.

Problem 132

(a) Show that for an ideal gas the curves relating pressure and volume for an adiabatic change have a greater slope than those for an isothermal change, at the same pressure.

(b) A gas in a cylinder initially at a temperature of 17 $^{\circ}C$ and a pressure of 1.01 $\times 10^5 Nm^{-2}$, is to be compressed to one-eighth of its volume. What would be the difference between the final pressures if the compression were done.

(i) Isothermally

(ii) Adiabatically?

Given that γ = 1.40

Problem 133

Given that the volume of a gas at S.T.P is 2.24 $\times 10^{-2}\ m^3mol^{-1}$ and that standard pressure is 1.01$\times 10^5\ Nm^{-2}$, calculate the molar gas constant R and use it to find the difference between the quantities of heat required to raise the temperature of 0.01kg of oxygen from 0 $^{\circ}C$ to 10 $^{\circ}C$ when.

(a) The pressure is kept constant

(b) The volume is kept constant

(Given that relative molecular mass of oxygen = 32)

Problem 134

(a) By considering the expansion of an ideal gas contained in a cylinder and enclosed by a piston, show that the work done in a small expansion is equal to the pressure times the volume change.

(b) An ideal gas, at a temperature of 290 K and a pressure of 1.0 $\times 10^5\ Nm^{-2}$, occupies a volume of 1.0 $\times 10^{-3}\ m^3$. Its density conditions is 0.30 kgm^{-3}. It expands at constant pressure to a volume of 1.5 $\times 10^{-3}$. Calculate the energy added.

(c) The gas is now compressed isothermally to its original volume. **Calculate.**

(i) Its final pressure and temperature

(ii)The difference between its final and initial internal energies.

Given that specific heat capacity at constant volume of this gas =

7.1 $= 10^2 JKg^{-1}K^{-1}$.

Problem 135

A litre of air, initially at 20 $°C$ and at 760mmHg pressure, is heated at constant pressure until its volume is doubled. Find

(a) The final temperature

(b) The external work done by the air in expanding
(c) The quantity of heat supplied.

Assume that the density of air at S.T.P is 1.293 Kgm^{-3} and that the specific heat capacity of air at constant volume is 714 $JKg^{-1}K^{-1}$.

Problem 136

a) Deduce an expression for the difference between the specific heat capacities of an ideal gas.

(b) If the specific heat capacity of air at constant pressure is 1013$JKg^{-1}K^{-1}$ and the density at S.T.P is 1.29Kgm^{-3}, estimate a value for the specific heat capacity of air at constant volume.

Problem 137

(a) What is the importance of the ratio of the specific heat capacities of an ideal gas?

(b) A mass of air occupying initially a volume 2 $\times 10^{-3}m^3$ at a pressure of 760mmHg and a temperature 20 °C is expanded adiabatically and reversibly to twice its volume, and then compressed isothermally and reversibly to a volume of 3 $\times 10^{-3}m^3$. Find the final temperature and pressure, assuming the ratio of the specific heat capacities of air to be 1.40.

Problem 138

Air initially at 27 °C and at 750mmHg pressure is compressed isothermally until its volume is halved. It is then expanded adiabatically until its original volume is recovered. Assuming the changes to be reversible find the final pressure and temperature take γ = 1.40

Problem 139

When water at 100 °C and pressure of 101 kPa changes to steam under the same conditions, its volume increases by a factor of 1670 given the density of water is 960 Kgm^{-3} at 100 °C and 101 kPa, and its specific latent heat of vaporization is 2.26 $\times 10^6 JKg^{-1}$,

Calculate

(a) The heat supplied to convert 1 kg of water at 100 °C to steam at the same temperature.

(b) The work done when 1 kg of water turns to steam at 101kPa pressure.

(c) The increase of internal energy.

Problem 140

A fixed mass of ideal gas is contained in a cylinder. The cylinder volume can be varied by moving a piston in or out. The gas has an initial volume 0.01m^3 at 100 kPa pressure and its temperature is initially

300K. The gas is cooled at constant pressure until its volume is 0.006 m^3. Sketch a pressure against volume graph to show the change.

Calculate:

(a) The final temperature of the gas.

(b) The work done on the gas.

(c) The number of moles of gas.

(d) The change of internal energy of the gas.

(e) The heat transfer from the gas (Assume R =8.3 $Jmol^{-1}k^{-1}$)

Problem 141

Two identical cylinders X and Y contain equal volumes of ideal gas at the same temperature and pressure. The volume of each cylinder can be varied by moving a piston in or out for the cylinder. The gas in each cylinder is then compressed to half its initial volume: X is compressed isothermally whereas Y is compressed adiabatically. Show the changes on a compressed adiabatically. Show the changes on a pressure against volume diagram and compare the energy changes for the two gases.

Problem 142

A motor car tyres has a pressure of four atmospheres at a room temperature of 27 °C. If the Tyre suddenly bursts, calculate the temperature of the escaping air. Value of γ for air is 1.4

Problem 143

A molecule of a gas at 27 °C expands isothermally until its volume is doubled. Find the amount of work done and heat absorbed.

Problem 144

A cylinder fitted with a movable piston contains 3 moles of hydrogen at standard temperature are made and pressure. The walls of the cylinder are late by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume? Given γ = 1.4

Problem 145

A quantity of air $(\gamma = 1.4)$ at 27 °C is compressed

(i) Slowly and

(ii) Suddenly to one third of its volume. Find the change in temperature in each case.

Problem 146

A Tyre pumped to a pressure of 6 a.t.m suddenly burst. The room temperature is 15 $^{\circ}C$. Calculate the temperature of escaping air.

Take γ = 1.4

Problem 147

A litre of air, initially at 20 $^{\circ}C$ and at 760mmHg pressure, is heated at constant pressure until its volume is doubled.

Find:

(i) The final temperature

(ii) The external work done by the air in expanding

(iii)The quantity of heat supplied.Assume that the density of air at N.T.P is 1.293 kgm^{-3} and C_V = 714$Jkg^{-1}K^{-1}$

Problem 148

A gas is suddenly compressed to one-half of its volume. Calculate the rise in temperature, the original temperature being 27 $^{\circ}C$. Take γ = 1.5

Problem 149

A certain volume of dry air at N.T.P is allowed to expand four times its original volume under.

i) Isothermal conditions

(ii) Adiabatic conditions

Calculate the final pressure and temperature in each case. Take γ = 1.4

Problem 150

10 moles of hydrogen gas at NTP are compressed adiabatically so that its temperature becomes 400 $^{\circ}C$. How much work is done by the gas? Also find the increase in internal energy of the gas.

Given R = 8.4 $Jmol^{-1}K^{-1}$ and γ = 1.4

Problem 151

Calculate the work done when one mole of a perfect gas is compressed diabatically. The initial pressure and volume of the gas are $10^5 Nm^{-2}$ and 6 litres respectively. The final volume of the gas is 2 litres. Molar specific heat of the gas at constant volume is $^{3R}/_{2}$

Problem 152

A cylinder contains 1 mole of oxygen at a temperature of 27 $^{\circ}C$. The cylinder is provided with a frictionless piston maintains a constant pressure of 1 a.t.m on the gas. The gas is heated until its temperature rises to 127 $^{\circ}C$.

(i) How much work is done by the gas in the process?

(ii) What is the increase in internal energy of the gas?

(iii) How much heat was supplied to the gas?

Given that C_P = 7.03 cal $mol^{-1}C^{-1}$ and

R = 1.99 cal $mol^{-1}C^{-1}$

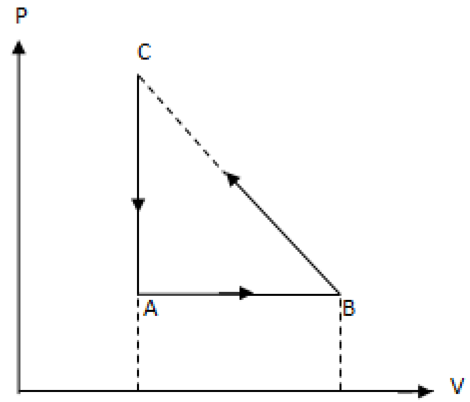
Problem 153

Two moles of helium gas $\left(\gamma = ^{5}/_{3}\right)$ are initially at a temperature 27 $^{\circ}C$ and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled then it undergoes adiabatic change until its temperature returns to its original value.

(i) Sketch the process the P – V diagram

(ii) What are the final volume and pressure of a gas?

(iii) What is the work done by the gas? Gas constant R = 8.3 $Jmol^{-1}K^{-1}$



Problem 154

Consider the cyclic process ABC on a sample of 2.0 mole of an ideal gas as shown in the figure below

The temperature of the gas at A and B are 300 K and 500 K respectively. A total of 1200 J of heat is withdrawn from the sample. Find the work done by the gas in part BC.

Given that R = 8.3J $mol^{-1}K^{-1}$

Problem 155

A cylinder contains 3 moles of oxygen at a temperature of 27 °C. The cylinder is provided with a frictionless piston which maintains a constant pressure of 1 atmosphere on the gas. The gas is heated unless its temperature rises to 127 °C.

(i) How much heat is supplied to the gas?

(ii) What is the change in internal energy of the gas?

(iii) How much work is done by the gas in the process? Given that C_P = 7.03 cal $mol^{-1}C^{-1}$

Problem 156

An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $T/2$

i) What is the value of γ for the gas?

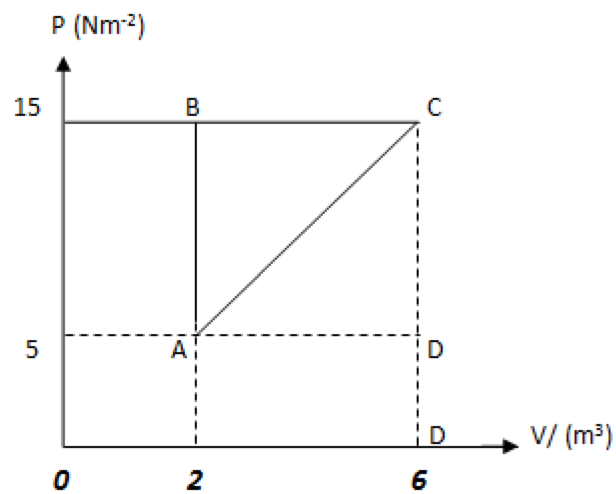
(ii ii) Obtain the work done by the gas during expansion as a function of initial pressure P and volume.

Problem 157

What amount of heat is to be transferred to nitrogen in an isobaric heating so that the gas may perform 2 J of work?

Problem 158

In the figure below an ideal gas changes its state from state A to C by two paths ABC and AC



(i) Find the path along which work done is less

(ii)The internal energy of gas at A is 10 J and the amount of heat supplied to change its state to C through the path AC is 200J . Calculate the internal energy at C.

(iii) The internal energy of gas at state B is 20J. Find the amount of heat supplied to the gas to go from A to B.

Problem 159

As a result of isobaric heating by ΔT = 72 K, one mole of a certain ideal gas obtains an amount of heat Q = 1.60 KJ.

Find:

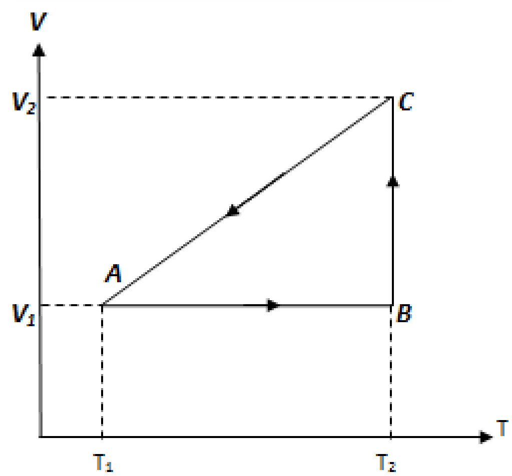
(i) The work done by the gas

(ii) The increment in its internal energy

(iii) The value of γ

Problem 160

The figure below shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.

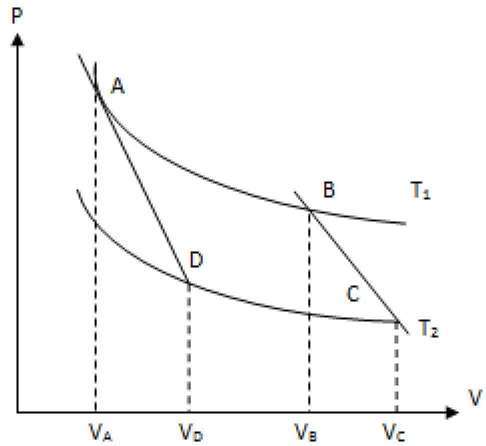


Problem 161

In a thermodynamic process the pressure of a fixed mass of a gas is changed in such a manner that the gas releases 20 J of heat and 8 J of work is done on the gas. If the initial energy of the gas was 30 J. What will be its final internal energy?

Problem 162

Two different adiabatic paths for the same gas intersect two isothermal at T_1 and T_2 as shown in the P – V diagram below. How does V_A/V_A compare with V_B/V_C?



Problem 163

At 27 °C two moles of an ideal mono atomic gas occupies a volume V. The gas expands adiabatically to a volume 2V. Find:

(i) The final temperature of the gas

(ii)The change in its internal energy

(iii) The work done by the gas during the process.

Given that R = 8.31 J $mol^{-1}K^{-1}$

Problem 164

A gas $(\gamma = 1.4)$ of 2m^3 volume and at a pressure of 4 $\times 10^5 Nm^2$ is compressed adiabatically to a volume 0.5m^3. Find its new pressure. Compare it with the pressure obtained if compression were isothermal. Calculate the work done in each process.

Problem 165

(a) Explain what is meant by temperature gradient.

(b) An ideally lagged compound bar 25 cm long consists of a copper bar 15 cm long joined to an aluminum bar 10 cm long and of equal cross-sectional area. The free end of the copper is maintained at 100 °C and the free end of the aluminium at 0 °C. Calculate the temperature gradient in each bar when steady state conditions have been reached. (Thermal conductivity of copper = 390 W $m^{-10}C^{-1}$. Thermal conductivity of aluminium = 210 W $m^{-1}°C^{-1}$.)

Problem 166

(a) If a copper kettle has a base of thickness 2.0mm and area3.0 x $10^{-2}m^2$, estimate the steady difference in temperature between inner and outer surfaces of the base which must be maintained to enable enough heat to through so that the temperature of 1.00 kg of water rises at the rate of 0.25 Ks^{-1}. Assume that there are no heat losses, the thermal conductivity of copper = 3.8 $\times 10^2$ W $m^{-1}K^{-1}$ and the specific heat capacity of water = 4.2 $\times 10^3$ J $kg^{-1}K^{-1}$.

(b) After reaching the temperature of 373 K the water in (a) above is allowed to boil under the same conditions for 120 seconds and the mass of water remaining in the kettle is 0.948 kg. Deduce a value for the specific latent heat of vaporization of water (neglecting condensation of the steam in the kettle

Problem 167

A cubical container full of hot water at a temperature of 90 $^\circ C$ is completely lagged with an insulating material of thermal conductivity 6.4 $\times 10^2$ W $m^{-1} {}^\circ C^{-1}$. The edges of the container are 1.0m. Estimate the rate of flow of heat through the lagging if the external temperature of the lagging is 40 $^\circ C$. Mention any assumptions you make in deriving your result.

Problem 168

A thin-walled hot-water tank having a total surface area 5 m^2, contains 0.8 m^3 of water at a temperature of 350 K. It is lagged with a 50mm thick layer of material of thermal conductivity 4 $\times 10^{-2}$ W $m^{-1}K^{-1}$. The temperature of the outside surface of the lagging is 290 K. What electrical power must be supplied to an immersion heater to maintain the temperature of the water at 350 K? Assume the thickness of the copper walls of the tank to be negligible) What is the justification for the assumption that the thickness of the copper walls of the tank may be neglected? (Thermal conductivity of copper = 400 W $m^{-1}K^{-1}$.

If the heater were switched off, how long would it take for the temperature of the hot water to fall 1 K?

(Density of water $=$ 1000 kg m^{-3}; specific heat capacity of water = 4170 J $kg^{-1}K^{-1}$.)

Problem 169

(a) Sketch graphs to illustrate the temperature distribution along a metal bar heated to one end when the bar is (a) lagged, and (b) unlagged. In each case assume the temperature equilibrium has been reached. Explain the difference between the two graphs.

(b) A window pane consists of a sheet of glass of area 2.0 m^2 and thickness 5.0mm. if the surface temperatures are maintained at 0 $^\circ C$ and 20 $^\circ C$, calculate the rate of flow of heat through the pane assuming a steady state is maintained. The window is now double glazed by adding a similar sheet of glass so that a layer of air 10mm thick is trapped between the two panes. Assuming that the air is still calculate the ratio of the rate of flow of heat through the window in the first case to that in the second.

(Conductivity of glass = 0.80 W $m^{-1}K^{-1}$, conductivity of air = 0.025 W $m^{-1}K^{-1}$.)

Problem 170

An iron pan containing water boiling steadily at 100 $°C$ stands on a hot-plate and heat conducted through the base of the pan evaporates 0.090 kg of water per minute. If the base of the pan has an area of 0.04m^2 and a uniform thickness of 2.0 $\times 10^{-3}$ m, calculate the surface temperature of the pan.

(Thermal conductivity of iron = 66 W $m^{-1}K^{-1}$. Specific latent heat of vaporization of water at 100 $°C$ = 2.2 $\times 10^6$ J kg^{-1})

Problem 171

(a) A sheet a glass has an area of 2.0 m^2 and a thickness 8.0 x 0^{-3} m. The glass has a thermal conductivity of 0.80 W $m^{-1}K^{-1}$. Calculate the rate of heat transfer through the glass when there is a temperature difference of 20 K between its faces.

(b) A room in a house is heated to a temperature 20 K above that outside. The room has 2 m^2 of windows of glass similar to the type used in (a) above. Suggest why the rate of heat transfer through glass is much less than the value calculated above.

Problem 172

(a) Explain why two sheets of similar glass each 4mm thick separated by a 10mm layer of air. Assuming the thermal conductivity of glass to be 50 times greater than that of air calculate the ratio.

(b) A double-glazed window consists of two panes of glass each 4mm thick separated by a 10mm layer of air. Assuming the thermal conductivity of glass to be 50 times greater than that of air calculate the ratio.

(i) Temperature gradient in the glass to temperature gradient in the air gap.

(ii)Temperature difference across one pane of the glass to temperature difference across the air gap.

Problem 173

(a) Outline an experiment to measure the thermal conductivity of a solid which is a poor conductor, showing how the result is calculated from the measurements.

(b) Calculate the theoretical percentage change in heat loose by conduction achieved by replacing a single glass window by a double window consisting of two sheets of glass separated by 10mm of air.

Problem 174

The silica cylinder of a radiant wall heater is 0.6m long and has a radius of 5mm. If it is rated at 1.5 kW estimates its temperature when operating. State two assumptions you have made in making your estimate.

(The Stefan constant, σ = 6 $\times 10^{-8}$ W $m^{-2}K^{-4}$).

Problem 175

(a) Explain what is meant by black body radiation

(b) A blackened metal sphere of diameter 10mm is placed at the focus of a concave mirror of diameter 0.5m directed towards the sun. If the solar power incident on the mirror is 1600 W m^{-2}. Calculate the maximum temperature in which the sphere can attain. State the assumptions you have estimated.

(The Stefan's constant, = 6 $\times 10^{-8}$ W $m^{-2}K^{-4}$).

Problem 176

If the mean equilibrium temperature of the Earth's surface is T and the total rate of energy emission by the sun is E Show that

$$T^4 = \frac{E}{16\sigma\pi R^2}$$

Where σ is the Stephan constant and R is the radius of the Earth's orbit around the sun.

(Assume that the Earth behaves like a black body)

Problem 177

An unlagged thin-walled copper pipe of diameter 2.0 cm carries water at a temperature of 40 K above that the surrounding air. Estimate the power loss per unit length of the pipe if the temperature of the surroundings is 300K and the Stefan constant, σ, is 5.67 $\times 10^{-8}$ W $m^{-2}K^{-4}$).

State two important assumptions you have made.

Problem 178

The solar radiation falling normally on the surface of the Earth has an intensity 1.40 k Wm^{-2}. If this radiation fell normally on one side of a thin, freely suspended blackened metal plate and the temperature of the surroundings was 300 K, calculate the equilibrium temperature of the plate. Assume that all heat interchange is by radiation.

(The Stefan constant = 5.67 $\times 10^{-8}$ W $m^{-2}K^{-4}$).

Problem 179

A steel rod has length 1.5m and radius 1 cm. One end of the rod is maintained at 100 $^{\circ}C$ and the other end is at 0 $^{\circ}C$. Find the quantity of heat conducted through the rod in 2 minutes. The thermal conductivity of steel is 50.4 W/m K.

Problem 180

A glass window pane of a room has dimensions 2m $\times$ 0.5 m x 0.002m. The temperature on its two sides are 300 K and 295 K respectively. Find the quantity of heat conducted out of the room in 10 minutes if the room has two windows, each having two such panes. $\left(K_{glass} = 0.84w/m\,K\right)$

Problem 181

In Searle's method. A metal rod of length 50cm and area of cross-section 8 cm^2 is used. The flow of water through the tube is adjusted at 20 grams per minute. The stead temperature of 65 $^{\circ}C$ and 55 $^{\circ}C$ respectively are shown by the two thermometers instead in the rod. The separation between the thermometers is 4 cm. The out flowing water shows a rise of 6 $^{\circ}C$. Find the thermal conductivity of the metal.

Problem 182

In Searle's experiment for the measurement of thermal conductivity of a metal, a road having a cross-sectional area of 10cm^2 is used. The flow of water through the cooling tube is adjusted at 150gm/minute. When a steady state is reached, two thermometers, inserted in the road at a distance of 5cm from each other, record temperature of 60 $^{\circ}C$ and 50 $^{\circ}C$ respectively. If the rise in temperature of the water flowing through the cooling tube is 5 $^{\circ}C$, find the thermal conductivity of metal.

Problem 183

The temperature inside an air-conditioned room is maintained at 20 °C when the outside temperature is 30 °C. Calculate the quantity of heat conducted per minute through a glass window pane of area 0.25 m^2 and thickness 5mm if the thermal conductivity of glass is 0.84 W/mK.

Problem 184

The temperature inside the room is 15 °C and that of outside is 5 °C. How much heat will be lost by conduction per hour through one square meter of the wall if its thickness is 25cm [K for the material of the wall = 2.5 w/mK]

Problem 185

Calculate the amount of heat conducted per minute through a glass window pane of length 50cm, breadth 20cm and thickness 0.5 cm, if there is a steady temperature difference of 10 °C on its two sides (Thermal conductivity of glass = 0.002 CGS units)

Problem 186

One end of a copper rod 20 cm long and 5 cm in diameter is maintained at 50 °C while the other end is kept at a constant temperature of 20 °C. Calculate the quantity of heat conducted through the rod in 10 seconds if the thermal conductivity of copper is 0.92 CGS units

Problem 187

A large glass window has an area of 10 cm^2 and thickness of 3mm. If the temperature in side and outside the room is 20 °C and -10 °C respectively, calculate the quantity of heat flowing per second through the window. Thermal conductivity of glass = 1.5 $\times 10^{-4}$ MKS units)

Problem 188

In Searle's method, rod of length 30cm and cross-sectional area 5 cm^2 is used and flow of water is adjusted at 60 grams per minute. Steady temperature of 60 °C and 50 °C respectively are shown by two thermometers inserted in the rod 8cm apart. If the water coming out of the spiral shows 5 °C rise in temperature, calculate the thermal conductivity of the metal.

Problem 189

The thermal conductivity of brass is 0.26 cal/s cm °C. In Searle's experiment a brass rod having a cross-sectional area of 10 cm^2 is used. When the steady state is reached, the temperature recorded by the two thermometers, inserted in the rod at a distance of 4 cm from each other, differ by 5 °C.

If the rate of flow of water through the cooling tube is 0.5 gm/s. find the rise in temperature of water.

Problem190

A hollow cube of metal having mean length 10cm and thickness 0.25cm is filled with ice at 0 ℃ and is surrounded by water at 80 ℃. How much ice will melt in ten minutes?

Latent heat of ice = 80 kCal/g.

WAVE MOTION-1

Definition

- A wave is a periodic disturbance which propagate energy from a point in to another.
- Waves transfer energy from point to another point without carrying matter.

CLASSIFICATION OF WAVES

Waves may be classified into two classes:

1. Mechanical waves
2. Electromagnetic waves

MECHANICAL WAVES

Definition

A mechanical wave is a disturbance which is transported through a medium due to particle interaction.
or
Is the wave which require material medium for transfer of energy from one point to another.

Example

i. waves on a spring
ii. water waves
iii. sound waves

iv. waves on stretched string (e.g. in musical instruments)

- mechanical waves require a material medium to transfer energy.

- when a mechanical waves travel through a medium the particle that make up the medium are disturb from their rest or equilibrium positions.

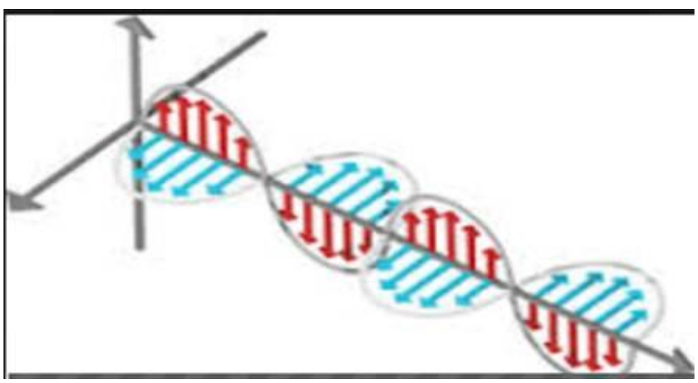
ELECTROMAGNETIC WAVES

Definition

Electromagnetic waves are the waves that consists of particles moving in electric and magnetic field. or
Electromagnetic waves are the waves which does not require the material medium during the propagation of energy from one point to another.

-The electric and magnetic field oscillates at right angle to each other and to the direction of propagation

DIAGRAM OF ELECTROMAGNETIC WAVES



- Electromagnetic waves do not require a material medium to transfer energy.

- They can travel through a vacuum.

Example:

i. visible light
ii. radio waves
iii. infra-red radiation
iv. ultraviolet radiation
v. gamma radiation
vi. X-rays

WAVES FORM

-A wave form is a shape of a wave or pattern representing a vibration.

-It can be illustrated by drawing a graph of the periodically varying quantity against distance for one complete wavelength.

TYPES OF WAVE

Waves may be divided into two forms , which are:-

1. Stationary / Standing waves
2. Progressive waves

1. STATIONARY / STANDING WAVE

Are the types of wave in which the wave profile is not moving

2. PROGRESSIVE WAVE

Are the type of waves in which the wave profile is moving

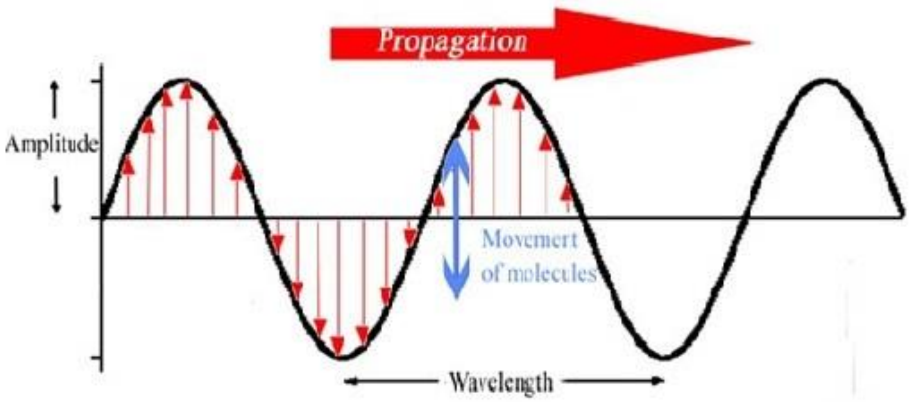
TYPES OF PROGRESSIVE WAVE

i) Transverse waves
ii) Longitudinal waves
iii) Mechanical wave
iv) Electromagnetic wave

TRANSVERSE **WAVE**

Definition

A transverse wave is the one that make the particle of the medium to vibrate in a direction perpendicular to the direction of movement of the wave



Example

i. Water waves
ii. Wave on a string
iii. Electromagnetic waves

- A transverse wave is propagated by a series of crest C and trough (valley).

CREST

-Is an elevation of the medium above its equilibrium state when a transverse wave passes through the medium.

TROUGH (VALLEY)

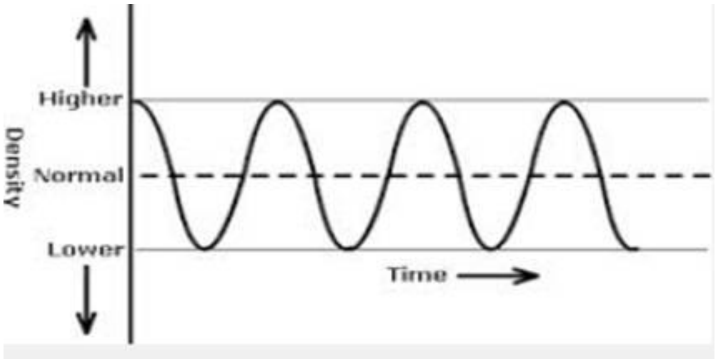
-Is the depression of the medium below its equilibrium state when transverse wave passes through the medium.

LONGITUDINAL WAVES

Definition:

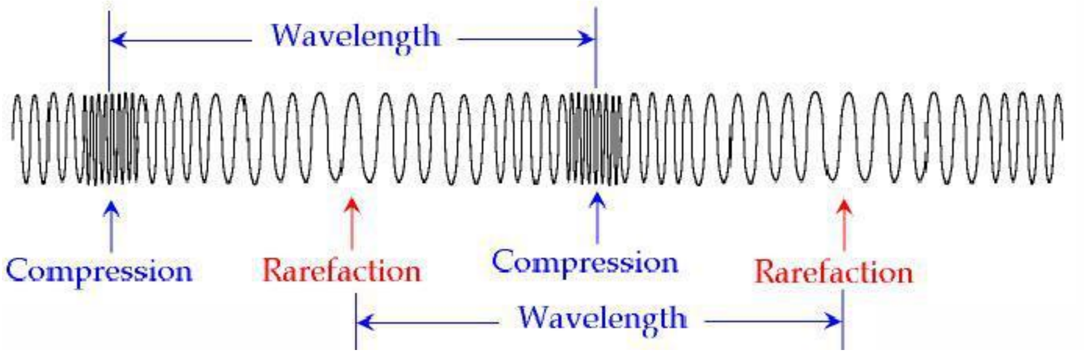
A longitudinal wave is the one which the particles of the material medium vibrate in a direction parallel to the direction of the wave motion.

DIAGRAM OF LONGITUDINAL WAVES



Example

- Sound waves
- A longitudinal wave is propagated by a series of compression and rarefaction



Where C = Compression

R = Rarefaction

COMPRESSION

- Is a region of high pressure in a longitudinal wave.

RAREFACTION

- Is a region of low pressure in longitudinal wave.

DIFFERENCE BETWEEN TRANSVERSE AND LONGITUDINAL WAVES

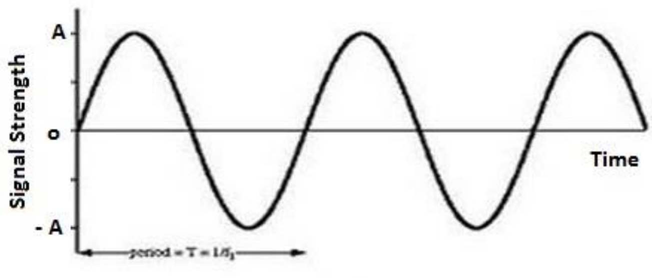
TRANSVERSE WAVES	LONGITUDINAL WAVES
1)The particle of the medium vibrate perpendicular to the direction in which the waves advance	1) The particle of the medium vibrate in the sa in which the wave advance.
2) It consists of crest and troughs.	2) it consists of compression and rarefaction

3) It can propagate only in solid and at the surface of liquids	3) it can propagate in all type of media (solid, li
4) There is no pressure variation	4) The pressure and density are maximum at com minimum at rarefactions.

GENERAL WAVE DIAGRAM

- The crest and trough (valleys) in transverse wave can be likened to compression and rarefaction respectively in a longitudinal wave.

- Thus we may represent any wave by a single diagram as shown bellow:-



(a) sine wave

Where y = wave displacement

X = direction of waves travel

T = Time of propagation

0 = speed of wave

<u>**TERMS APPLIED IN WAVE**</u>

The chief characteristic of a wave are:-

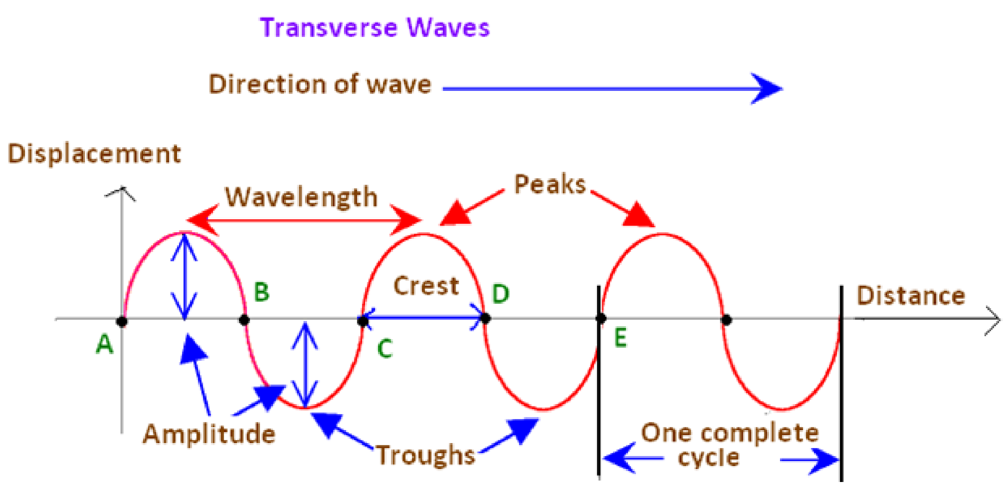
i. Amplitude
ii. Wavelength
iii. Frequency
iv. Speed/ Velocity of propagation
v. v. Displacement

AMPLITUDE

- Symbol, A

-This is maximum displacement of the disturb particle from its equilibrium position.

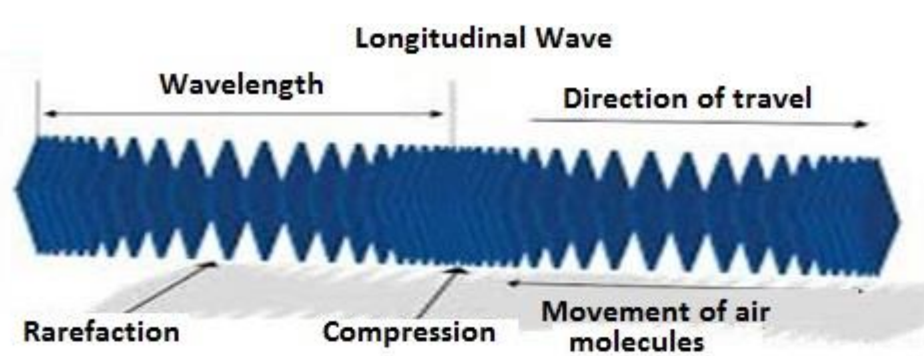
- The SI unit of amplitude is the meter(m)



WAVE LENGTH

- Symbol, (Greek letter "lambda"),λ

- This is the distance between two successive points of equal phase in a wave.



Alternative definition

- The wave length of a wave is the distance between two successive or adjacent crest or trough.
- It is a distance that the wave travels in the complete cycle.

UNIT OF LAMBDA, λ

- The S.I unit of wavelength is the meter,m
- Other unit in common use are:

i) nanometer (nm)

ii) angstrom (Å)

NOTE

1 nm = 10^{-9}m

1Å = 10^{-10}m

FREQUENCY

-Symbol, f

-This is the number of complete disturbance (cycles) passing a given point in unit time.

-f $= {no\ of\ complete\ cycle}/{time}$

UNIT OF , f

- By definition

f $= {no\ of\ complete\ cycle}/{time}$

= Cycles/sec

= Hertz (HZ)

- Hence the S.I unit of frequency is the Hertz (HZ).

1HZ = 1 cycle/ sec

Other unit in common use are:

i. Kilohertz (KHZ)

ii. Megahertz (MHZ)

NOTE:

$1KHZ = 10^3$ HZ

$1MHZ = 10^6$ HZ

SPEED OF PROPAGATION

-Symbol, V

- This is the distance covered by the wave in unit time.

V = Distance covered / Time

$V = X/t$

UNIT OF V

Definition

$V = X/t = metre/second = ms^{-1}$

-Hence the S.I unit used is meter/sec (ms^{-1})

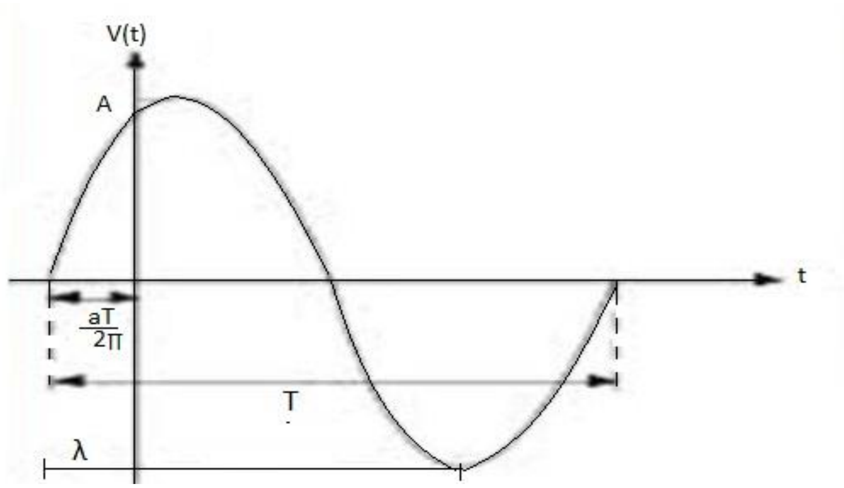
PERIOD

- Symbol, T

-This is the time taken for the wave to make one complete cycle (oscillation)

- It is the time taken for the wave to travel through a distance equal one wavelength

-The S.I unit for period is the second (S)



RELATIONSHIP BETWEEN ƒ AND T

-From the definition of *f*

$$f = {}^{no\ of\ complete\ cycle}/_{time}$$

-This given an alternative unit of ƒ i.e. per second (S^{-1})

1HZ = 1 S^{-1}

RELATIONSHIP BETWEEN V and f

- From the definition of V

$$V = \frac{X}{T}$$

For complete one cycle x =λ then t = T (=period)
v = λ/T
But f = 1/T
v = λf
where
v = velocity
λ = wavelength
T = Period

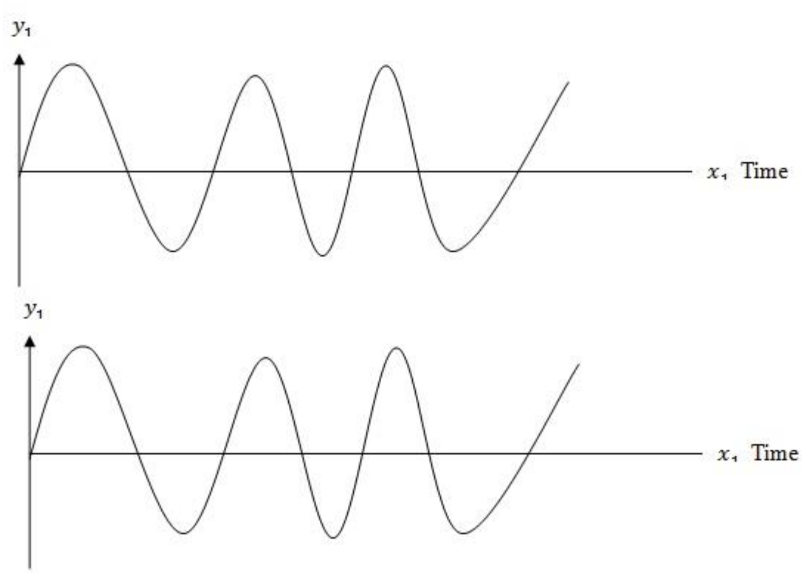
PHASE OF WAVES MOTION

- This is an angular displacement of a given wave particle in time t.

- This term is usefully in comparing two wave motions.

- Two waves are said to be in phase if their maximum and minimum values occur at the same instant otherwise there is said to be a phase difference

Two wave in phase;

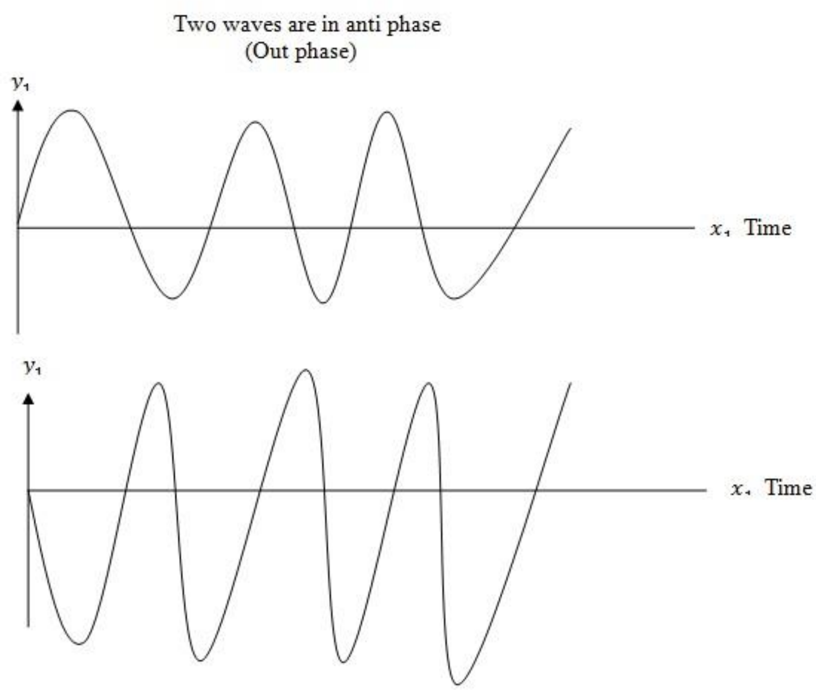


PHASE DIFFERENCE / PHASE ANGLE

- **Symbol, Ø**

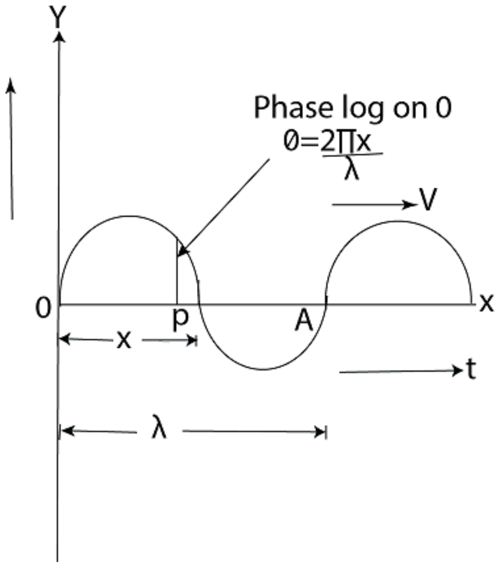
- This is the difference in phase between two wave motion

- If the phase difference is radians then the two waves are in ant phase (out of phase)



EXPRESSION OF PHASE DIFFERENCE

- Consider two particle O and P along the path of the waves



- Let O be origin of the wave

- A particle at P a distance X right of O will vibrate in different phase with the particle at O.

- Let Ø be phase different between O and P

- Now

2^{π} rad $\longrightarrow \lambda$

$\phi \longrightarrow X$

$\Rightarrow \phi \times \lambda = 2\pi X/\lambda$

$\therefore \emptyset = {}^{2\pi X}/_{\lambda}$

PROGRESSIVE/ TRAVELING WAVE

-A progressive wave / traveling waves are that wave in the wave profile move along with the speed of wave profile across the medium

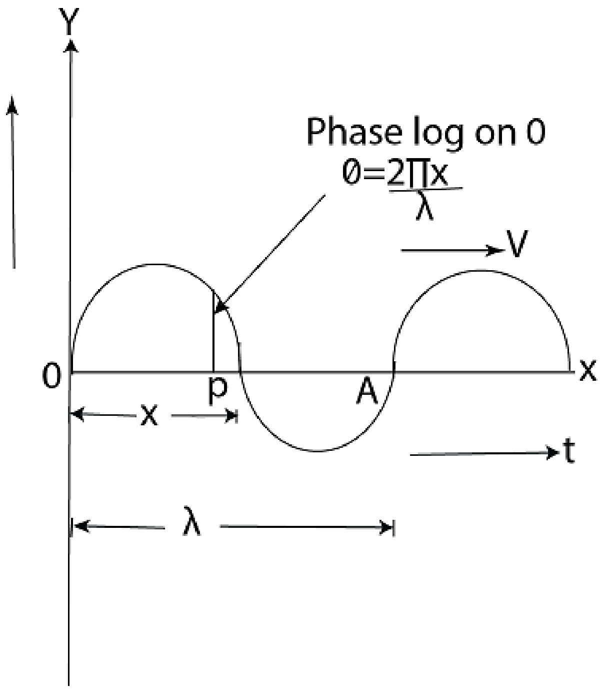
- Both transverse and longitudinal waves are progressive wave i.e. the wave profile move along with the speed of the wave across the medium.

-The oscillation of any progression wave repeat at equal interval of distance called wavelength and at equal interval of time called period.

-The vibration proceed to progressive waves are of the same amplitude and frequency.

THE PROGRESSIVE/ TRAVELING WAVE EQUATION

- Consider a progressive / traveling wave which is moving from left to right with a velocity V.



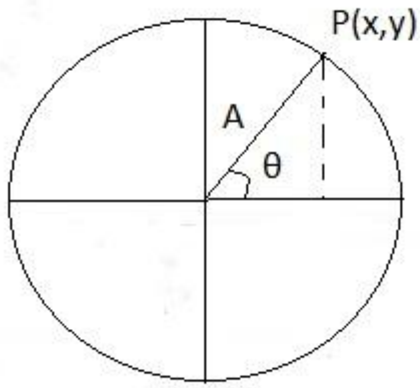
- Let O be origin of the wave

-The displacement y of the particles at O

- If the wave generate travel from left to right then a particle at P, distance X from O will lag behind the particles at O by a phase angle Ø given by

$\emptyset. \quad = {}^{2\pi X}/_{\lambda}$ ....(i)

- For the displacement y of the particle at P we have: Consider the particle vibrating with Amplitude A



Y is a displacement t time t

Sin θ = y/a

y = a sin θ

But θ = wt

y = A sin wt

This is displacement at time t when initial displacement is zero. But if initial displacement is y' = A sin Φ

Displacement y = A sin wt + A sin Φ

y = A sin (wt ± Φ)

Substitute equation (i) in this equation

$$y = A\sin\left(\omega t \pm {}^{2\pi x}/_{\lambda}\right)$$

Where

A = Amplitude

w = Angular velocity

t = time

Φ = phase difference

If the wave is traveling in the opposite direction i.e. in the negative X – direction then the equation becomes

$$Y = A\sin(\omega t + kx)$$

If the wave is traveling in the same direction i.e in the positive x- direction the equation becomes

$$y = A sin(\omega t - {}^{2\pi x}/_{\lambda})$$

where k = 2π/ λ
f = frequency
T = period

Problem 01

a) What is a wave?

b) Y = A sin $\{2000\pi t - {}^{\pi x}/_{17}\}$ is the equation

Of a plane progressive wave where x and y are in cm and t is time in second.

Determine:

i. Frequency
ii. Wavelength
iii. Velocity

Problem 02

A certain travelling wave has equation

$Y = A \sin(\omega t + kx)$

a) Deduce whether the wave is traveling in the positive or negative x-direction.

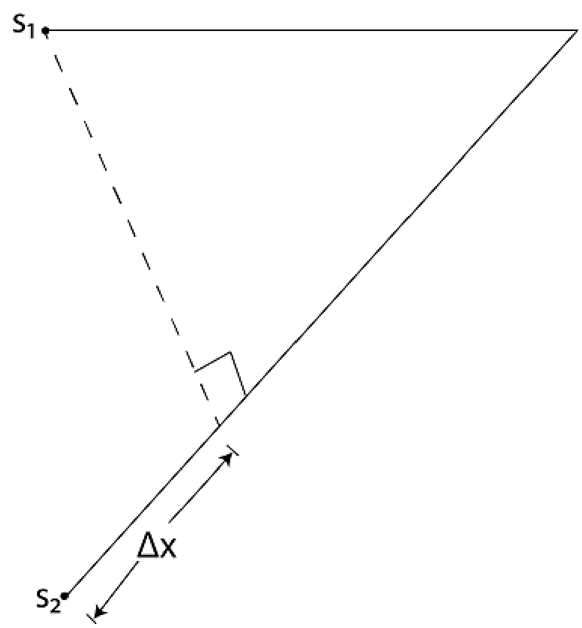
b) If $\omega = 6.6 \times 10^3$ rad s^{-1} and $k = 20m^{-1}$ calculate the speed of the wave in the medium

PATH DIFFERENCE

- This is difference in path length between two waves which meet at a point or between two points in a given wave.
- It is the distance corresponding to phase difference

CASE 1

- Consider waves from two sources S_1 and S_2 which meet at point P.



Path difference, â^†x = S_2 P- s_1 p

CASE 2

- Consider two point P_1 and P_2 in the path of the wave from source S

Path difference, â^†x $= SP_2 - SP_1$

- from the expression of phase difference

$$Ø = \frac{2\pi x}{\lambda}$$

- when x = â^†x = path difference, Ø

$$â^†Ø = \frac{2\pi â^†x}{\lambda}$$

Problem 03

A train of plane wave sound propagating in air has individual particles of air executing simple harmonic motion such as that their displacement y from equilibrium position at any time t is given by

$Y = 5x10^{-6} \sin (800\pi t + Ø)$

Where y is in cm t is in second and Ø is a phase term. Calculate

a) the wavelength of the wave

Given that velocity of sound air = $340ms^{-1}$

Problem 04

The speed of wave in a medium is $960ms^{-1}$. If 3600 waves are passing through a point in one minute, calculate the wavelength.

Problem 05

A progressive wave of frequency 500HZ is traveling with the velocity of $360ms^{-1}$. How far apart are two point 60^0out of phase?

Problem 06

A plane progressive wave of frequency 25HZ, amplitude 2.5×10^{-5} and initial phase zero propagate along the negative x- direction with a velocity of $300ms^{-1}$.

At any instant what is the phase difference between oscillations at two points 6 cm apart along the line of propagation?

Problem 07

The equation y = A sin ($\omega t - kx$) represent a plane wave traveling in a medium along the x direction y being the displacement at the point x at time t.

a) Deduce whether the wave is traveling in the positive x- direction or in the negative x- direction

b) If a = 1.0 x 10^{-7}m, $\omega = 6.6$ x $10^3 s^{-1}$ and k = $20 m^{-1}$ calculate

i) The speed of the wave

ii) The maximum speed of a particle of the medium due to the wave

SUPERPOSITION OF WAVES

- The principle state "If two or more wave arrive at a point simultaneously then the resultant displacement at that point is the vector sum of the displacement due to individual waves".

$$y = y_1 + y_2 + \cdots \dots \dots \dots \dots \dots \dots \dots + y_n$$

- Where y = resultant displacement

$y_1, y_2, \dots \dots \dots \dots \dots \dots . . y_n$ are the displacement due to individually waves.

Problem 08

Two waves travelling together along the same line are given by:

Y_1 = 5 sin ($\omega + \frac{\pi}{2}$) and

Y_2 = 5 sin ($\omega + \pi/3$)

Find:

i) The resultant equation of motion

ii) The resultant amplitude

iii) The initial phase angle of the motion

IMPORTANT CASE OF SUPERPOSITION

- There are three important cases:

a. stationary waves

b. beats

c. interference

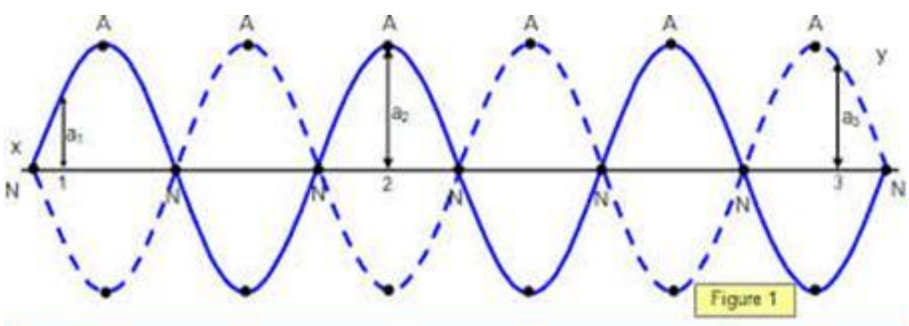
STATIONARY WAVES/STANDING WAVES

Definition

A stationary / standing wave is a form of wave in which the profile of the wave does not move through the medium but remains stationary.

HOW A STATIONARY WAVE IS FORMED

-A stationary wave result when a progressive / travelling wave is reflect back along its own path.



-The incident and reflect wave then interfere to produce a stationary wave

-Thus we can say a stationary / standing wave is formed when two progressive waves of the same wavelength frequency and amplitude are made to move in opposite direction in the same medium.

-A stationary / standing wave has two important points:

i) Nodes(N)

ii) Antinode (A)

NODE (N)

- This is a point of minimum disturbance in a stationary wave system

ANTINODE (A)

- This is a point of maximum disturbance in a stationary wave system

THE EQUATION AT STATIONARY / STANDING WAVE

- Consider two progressive waves of the same wavelength, frequency and amplitude to be moving in opposite direct within the same medium such that when they meet a stationary wave is formed.

-Let the waves be y_1 and y_2 such that

$Y_1 = A \sin (\omega t + kx)$ *and*

$Y_2 = A \sin (\omega t - kx)$

-According to the principle of super position of wave the resultant displacement y is given by:

$Y = y_1 + y_2$

$Y = A \sin (\omega t + kx) + a \sin (\omega t - kx)$

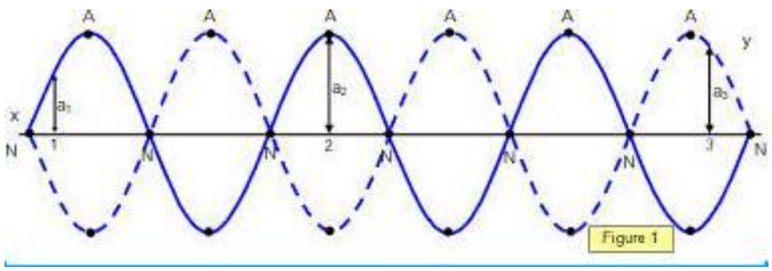
$Y = A (\sin (\omega t + kx) + \sin(\omega t - kx))$

$$Y = 2A \sin \left(\frac{\omega t + kx + \omega t - kx}{2}\right) \cos \left(\frac{\omega t + kx - \omega t + kx}{2}\right)$$

$Y = 2A \sin \omega t \cos kx$

PROPERTIES OF A STATIONARY/STANDING WAVE

1) The distance between a node and its neighboring antinode is one quarter of a wavelength



2) All point along the wave have different amplitude.

3) Point between successive nodes are in phase.

DIFFERENCE BETWEEN PROGRESSIVE AND STATIONARY WAVES

PROGRESSIVE WAVE	STATIONARY WAVE
1)The wave advance with a constant speed	1) The wave does not advance but remain confined in a particular region
2) The amplitude is the same for all the particles in the path of the wave	2) the amplitude varies according to position being zero at the nodes and maximum at the anti nodes
) All particles within one wavelength have difference phase	3)phase of all particles between two adjacent nodes is the same
4) Energy is transmitted in the direction of the wave	4)energy is associated within the wave but there is no transfer of energy across any section of the medium

Problem 09

a) Explain the difference between progressive waves and stationary wave

b) Progressive wave and stationary wave each has the same frequency of 250HZ and the same velocity of 30ms^{-1}. Determine

(i)The phase difference between the vibrating points on the progressive wave which are 10cm apart

(ii) The distances between nodes in stationary wave
(iii) The equation of the progressive wave if its amplitude is 0.03m

Problem 10

Two waves travelling in opposite directions producing a standing wave. The individual functions are given by:

$$y_1 = 4sin(3x - 2t)cm$$

$$y_2 = 4sin(3x + 2t)cm$$

- Where x and y are in cm

($i)$ Find the maximum displacement of motion at $x = 2.3cm$

($i)$ Find the positions of nodes and antinodes

Problem 11

The transverse displacement of a string clamped at its two ends given by:

$$y = 0.06sin\left(2\pi/_3\right)\times cos(120\pi t)$$

Where x, y are in metre and t is in the seconds

($i)$ Does the function represent a travelling or stationary wave?

($ii)$ Interpret the wave as superposition of two waves travelling in opposite directions

What are the wavelength, frequency and speed of propagation of each wave?

Problem 12

A standing wave on the string has nodes at $x = 0; x = 4cm; x = 8$cm and x$= 12cm.$What is the wavelength of the travelling wave?

Problem 13

The equation of standing wave is given by

$$y = 12cos\left(\pi x/_{5}\right)sin\left(20\pi t\right)$$

- Where x and y are in cm and t in second

Find: (i) Frequency and wavelength

(ii) Velocity and amplitude of progressive wave?

SOUND WAVE

Definition

Sound is a longitudinal wave motion which is conveyed through an elastic medium from a vibrating body to a listener.

AUDIBLE RANGE

-This is a range of frequency which can be detected by human ear.

- The human ear is very sensitive and can detect very faint sounds.

-The human ear can detect sounds in the frequency range of 20HZ to 20000HZ

-With increased age upper limit of frequency range fall quite considerably

-However, the ear is most sensitive sound with frequency around 3000HZ

INFRASONIC SOUND

-This is a sound with frequency below 20HZ.

ULTRASONIC SOUND

-This is sound with frequency above 20000HZ.

-Some animals including dogs, cats, bats and dolphins can detect ultrasonic sound with frequencies' as high as 100000HZ.

PROPAGATION OF SOUND WAVE

-Sound waves require a material medium to be transmitted through.

- It cannot be transmitted through a vacuum.

-Sound is transmitted by vibration of particles.

-One particle vibrates to transfer energy to the next until the sound reaches another point.

- If the particles are closer together, sound will travel faster.

- Hence sound travel with different velocities in different materials.

IN SOLID

-The velocity (v) of sound in the form of a rod wire depends on:

(i) Young's modulus E of the solid.

(ii) Density ρ of the solid.

-One uses dimension analysis to get relationship

$V \propto E^x \rho^y$

-Which gives

$$V = \sqrt{E/\rho}$$

IN FLUIDS

-The velocity (v) of the sound in fluid (liquid gas) depends on:

(i) Bulk modulus β

(ii) Density ρ of the fluid

- Again we use dimension analysis to get relationship

$$V = \sqrt{\beta/\rho}$$

NEWTON'S FORMULA

- Newton assumed that the propagation of sound in a gas takes place under isothermal condition (constant temperature condition)

in isothermal conditional Boyle`s law applies and hence: PV = K

differentiating this equation:

$$pdv + Vdp = 0$$

$$Pdv = -Vdp$$

$$P = -V\,{}^{dp}/_{dv}$$

$$\therefore P = -{}^{dp}/\left(\frac{dv}{v}\right)$$

- Where dp = change in pressure

$${}^{dv}/_{v} = {}^{change\ in\ volume}/_{orginal\ volume}$$

- **By definition:**

$${}^{dp}/_{\frac{dv}{v}} = \beta = Bulk\ modulus$$

Under isothermal condition

$$\therefore \beta = P$$

From the expression of velocity of sound in fluids:

$$V = \sqrt{\beta/\rho}$$

$$\therefore V = \sqrt{P/\rho}$$

Applying this equation to the case of air at S.T.P

Given

$P =$ 1.013 x 105Pa

$\rho = 1.29kgm - 3$ V = 280ms^{-1}

-This value is less than the actual value, which is a about 33oms^{-1}

-This large discrepancy shows that Newton`s formula is not correct.

LAPLACE`S CORRECTION

- Laplace pointed out that Newton`s assumption of isothermal propagation is not correct.

REASON

-The compressions and rarefactions follow each other so rapidly that there is no time for the compressed layer (Which is at a higher temperature) and the rarefied layer (Which is at a lower temperature) to equalize their temperatures with the surroundings.

-As a result the sound propagates under adiabatic and not under isothermal conditions.

LAPLACE` S EQUATION

-Under adiabatic condition of air:

Of air $PV^{\gamma} =$ constant

$$PV^{\gamma} = k \ldots \ldots \ldots \ldots \ldots \ldots \ldots (i)$$

Where

$P = Pressure$

$V = Volume$

$\gamma =$ ratio of specific capacities

$\gamma = {}^{C_p}/_{C_v}$

Differentiate equation (i) above

$Pd(v) + v^{\gamma} dp = 0$

$P\gamma v^{\gamma} dV + v^{\gamma} dp = 0$

$P\gamma \frac{v^{\gamma}}{v} dV + v^{\gamma} dp = 0$

$\gamma P \frac{v^{\gamma}}{v} dV = -v^{\gamma} dp$

$\frac{\gamma P}{v} = -{}^{v^{\gamma} dp}/_{v^{\gamma} dV}$

$\gamma P = -{}^{dp}/_{\frac{dv}{v}}$

Under adiabatic condition

$\therefore \beta = \gamma P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (ii)$

From expression of velocity of sound in

$$V = \sqrt{{}^{\beta}/_{\rho}}$$

$$\therefore V = \sqrt{\gamma P / \rho}$$ ....(iii)

- Applying this equation to the case of air at S.T.P.

GIVEN

V = 1.4

P = 1.013 X 10^5Pa

P = 1.29kgm^{-3}

V = $\frac{\sqrt{1.4 \ x \ 1.013 \ x \ 105}}{1.29}$

V = 330ms^{-1} which agree with the experimental value.

EXPRESSION OF VELOCITY OF SOUND IN AIR/GAS IN TERM OF ABSOLUTE TEMPERATURE

- From ideal gas equation

PV $= nRT$

$P = {nRT}/{V}$

v

n = no of moles
n = mass, m/Molar mass,M
n = m / M
P = m/M . $\frac{RT}{v}$

P = m/v . $\frac{RT}{M}$

Molar mass $= {m}/{M}$

v ${m}/{V} = \rho = Density$

$$P = \frac{\rho RT}{M} \quad ------ 4$$

Substitute equation (4) in equation (iii)

$$V = \sqrt{\frac{\gamma\rho RT}{M} * \frac{1}{\rho}}$$

γ

$$V = \sqrt{\frac{\gamma RT}{M}} \quad ---------- \text{(v)}$$

where R = molar gas constant = 8.31 Jmo$l^{-1}k^{-1}$

T = absolute temperature

M = molar mass of the gas

- This equation shows that the velocity of sound in air/gas is independent of the gas/air pressure.

FACTORS AFFECTING THE VELOCITY OF SOUND IN AIR / GAS

(1) TEMPERATURE OF A GAS / AIR

- From the equation (5) above:

$$V = \sqrt{\frac{\gamma RT}{M}}$$

or a given gas $\delta, R \ and \ M$ are constant

❖ $V \ \alpha \sqrt{T}$

- Thus, the velocity of sound in a gas / air is directly proportional to the square root of absolute temperature.

$\rightarrow V = K\sqrt{T}$

- If V_1 is the velocity of sound in air at a temperature T_1 and V_2 is the velocity at a temperature T_2 then

$$V_1 = K\sqrt{T_1} \text{ and } V_2 = K\sqrt{T_2}$$

$$\frac{V_1}{V_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$ -------------- (vi)

(2) <u>MOLAR MASS OF THE GAS</u>

-From equation (v) above :

$$V = \sqrt{\frac{\gamma RT}{M}}$$

-If δ, R and T are constant then

$$V \alpha \frac{1}{\sqrt{M}}$$

-The velocity of sound in a gas is inversely proportional to the square root of molar mass of the gas.

-Sound travels faster in lighter gases like hydrogen or Helium than in heavier gases such as carbon-dioxide or ammonia.

(3) <u>HUMIDITY</u>

- Humidity air is less dense than dry air, thus sound travels faster in humid air than in dry air.

(4) <u>WIND SPEED AND DIRECTION</u>

- Sound travels faster in the direction of the wind than in the direction opposite to it.

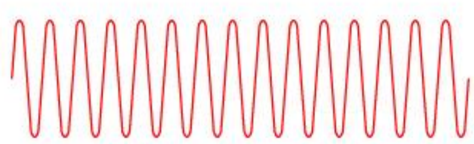
<u>Problem 14</u>

-The longitudinal wave speed in gases is given by:

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad where \ \gamma = \frac{C_p}{C_v}$$

Example

- Sound produce by a tuning fork, flute, piano etc.



-There are no sudden changes in loudness

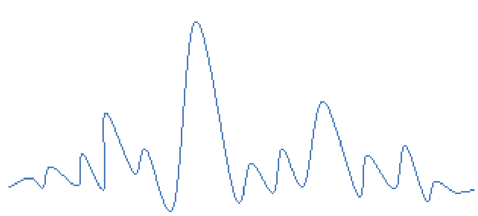
Noise

- Is an unpleasant, discontinuous and non-uniform sound produced by irregular succession of disturbance.

Example

-Sound produced by falling brick.

-Sound produced by clapping of two wooden blocks etc



-There are sudden changes in loudness.

CHARACTERISTICS OF A MUSICAL NOTES

There are three fundamental characteristics of a musical notes

(1) Pitch (frequency)

(2) Loudness (amplitude)

(3) Quality /timbre

PITCH

- This is the characteristics of musical note by which we can distinguish a shrill sound from a grave (hoarse) sound.

-The pitch of the sound depends upon frequency of the vibration of the source

-If the frequency of a sound is high, its pitch is also high and sound is said to be shrill.

- If the frequency of the sound is low, its pitch is low and the sound is said to be grave or flat.

Example

(i) The voice of children and ladies is shrill because of higher pitch.

(ii) The voice of an old man is horse because of low pitch.

(iii) The sound produced by the mosquito is of higher pitch and is therefore shrill.

LOUDNESS

- The loudness of the musical note is the intensity of the sound as perceived by the human ear.

- Loudness is determined by the amplitude of the sound and vice versa.

-The larger the amplitude, the louder the sound and vice versa.

QUALITY / TIMBRE

- This is the characteristics of the musical note which enable us to differentiate sounds of the same pitch and loudness produced by different instruments.

- It depends on the waveforms of the sound.

-The same note played on two different instruments does not sound the same and hence different waveforms are obtained which consist of

(i) Main notes (fundamental note)

(ii) Overtones

i) FUNDAMENTAL NOTE

- This is a component of a musical note with the lowest frequency called fundamental note frequency.

FUNDAMENTAL NOTE FREQUENCY

- This is the lowest frequency that a vibrating string or pipe can produce.

ii) OVERTONE

-This is constituent of a musical note other than the fundamental note.

-The frequencies of overtones are multiples of the fundamental note frequency.

Example

If f_0 is the fundamental note frequency, then the frequency of overtones are $2f_0$, $3f_0, 4f_0 \dots \dots \dots \dots \dots \dots$ etc.

NOTE

-The fundamental note is also called 1^{st} harmonic.

HARMONIC

Definition

- A harmonic is a musical note whose frequency is an integral multiple of the fundamental note frequency.

-The wave form of a note depends upon the presence of overtones / harmonics.

-Hence the quality of a musical note depends upon the number of overtones / harmonics an instrument produces.

-Different instrument emit different overtones / harmonics and hence the quality of sound produced is different.

Examples.

(a) A note played on piano has larger number of overtones / harmonics compared to that played to a flute.

- Hence musical sound from a piano is more rich ($i.e$ of better quality) than that of a flute.

(b) When stringed instruments (eg violin, guitar etc) are played, they are plucked near one end instead of in the middle.

- It is because plucking near the end produces more overtones / harmonics and gives a richer sound.

MUSICAL INSTRUMENTS

Definition

A musical instrument is a device constructed or modified for the purpose of making musical.

- **They include**

(i) Percussion instruments.

(ii) String instruments.

(iii) Pipe instruments / wind instrument

PERCUSSION INSTRUMENTS

- These are instrument which produce musical sounds by being struck with an implement or by any other action which sets the object into vibration.

Examples

i. Drum
ii. Cymbals
iii. Tambourine
iv. Marimba
v. Xylophone

STRING INSTRUMENTS

- String instruments consist of a tightly stretched wire fixed at both ends.

Wire fixed at both ends A and B

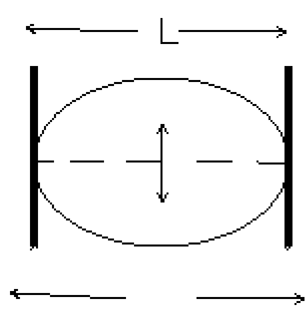
- When then wire is struck (piano), bowed (violin) or plucked (guitar) a stationary wave pattern is formed.

MODE OF VIBRATION OF A STRING

- A string fixed at both ends can be set into vibration with different modes called harmonics.

1^{st} HARMONIC (FUNDAMENTAL NOTE)

- If the string is plucked in the middle, the simplest mode of vibration which can be set up on it's the 1^{st} harmonic (fundamental note).



- where l = length of string
- if λ_0 is the wavelength of the fundamental note, then:

$$\frac{1}{2}\lambda_0 = l$$

$\lambda_0 = 2l$ ------------------ (1)

- Let f_0 be fundamental note frequency
- If V is the speed of a transverse wave along a string then:

V = $\lambda_0 f_0$

$f_0 = \frac{V}{\lambda_0}$ ---------------- (2)

- Substitute equation (1) in equation (2)

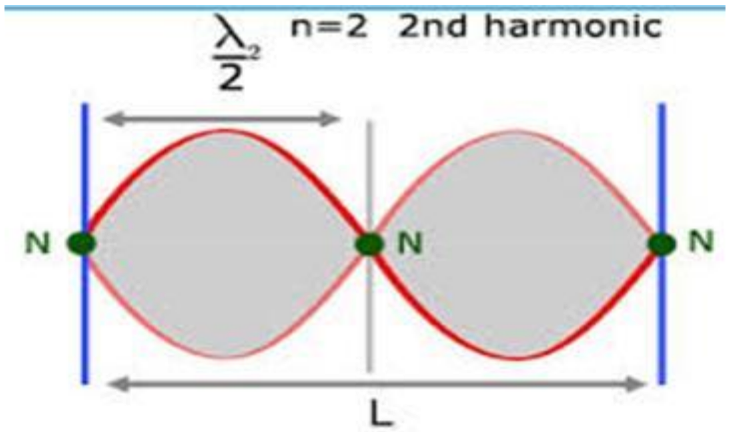
$f_0 = \frac{V}{2L}$ ------------------- (3)

OR

$2f_0 = \frac{V}{l}$

. 2^{nd} HARMONIC (1^{st} OVERTONES)

- By the plucking the string at a point a quarter of its length from one end, it can vibrate in two segments



- Let λ_1 be wavelength of the 1st overtone
- Let f_1 be frequency of the 1st overtone.
- If V is the speed of a transverse wave along a string then:
- Substitute equation (4) in this equation
- But from equation *

f_o = v/2L

f_o= 1/2 * v/L

But f_1 = v/L

f_o = 1/2 * f_1

f1 = $2f_o$ .... Which is a multiple of fundamental note frequency.

N.B: Any overtone is a multiple of fundamental note frequency.

NOTE:

- A string can be made to vibrate with several modes simultaneously depending on where it is plucked

THE VELOCITY OF A TRANSVERSE WAVE IN A STRING

- Consider a transverse wave that travels along a string:
- Let l = length of a string

m = mass of the string

T = tension in the string

- The velocity V of a transverse wave in a string depends on l, m and T.
- One uses dimensional analysis to get the relationship.
- Which gives

$$V = k\sqrt{Tl/m}$$

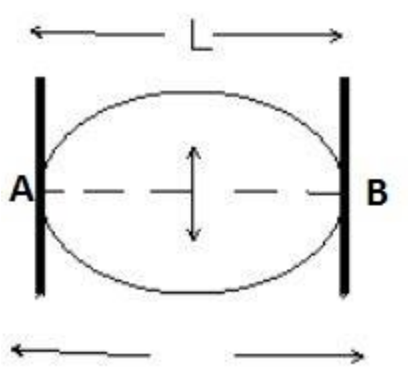
$$V = k\sqrt{T/\frac{m}{l}}$$

$$V = \sqrt{T/\mu}$$

- Where $m/l = \mu =$ mass per unit length of a string = linear mass density and K = I

THE FUNDAMENTAL NOTE FREQUENCY FORMULA

- Consider a string of a length l fixed between two points A and B
- Let the string to be plucked as its mid- point so that a fundamental note is produced



$$1/2\,\lambda = l$$

$\therefore \lambda = 2l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (1)$

- Where λ = wavelength of the note emitted
- Let f be frequency of the note emitted
- If V is the velocity of transverse wave in a string then.

$$V = \lambda f$$

$$f = {}^{V}/_{\lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (2)$$

- Substitute equation (1) in the equation (2)

$$f = {}^{V}/_{2l}$$

$$f = {}^{1}/_{2l} \times V$$

- **But** $V = \sqrt{{}^{T}/_{\mu}}$

$$\therefore \boldsymbol{f = {}^{1}/_{2l}\sqrt{{}^{T}/_{\mu}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)}$$

LAWS OF VIBRATION OF A STRETCHED STRING.

- From the fundamental note frequency formula, we have three laws of vibration of a stretched string.

LAW 1

The frequency of a vibrating string is inversely proportional to its length

LAW 2

The frequency of a vibrating string is directly proportional to the square root of tension on the wire.

LAW 3

The frequency of a vibrating string is inversely proportional to the square root of mass per length of the string.

THE SONOMETER

-This is an instrument which is used to set the frequency variation of a vibrating string in relation to its length, tension and mass per unit length.

- The instrument consists of a hollow wood box with two movable bridges A and B.

-On top of these bridges a sonometer wire is passed.

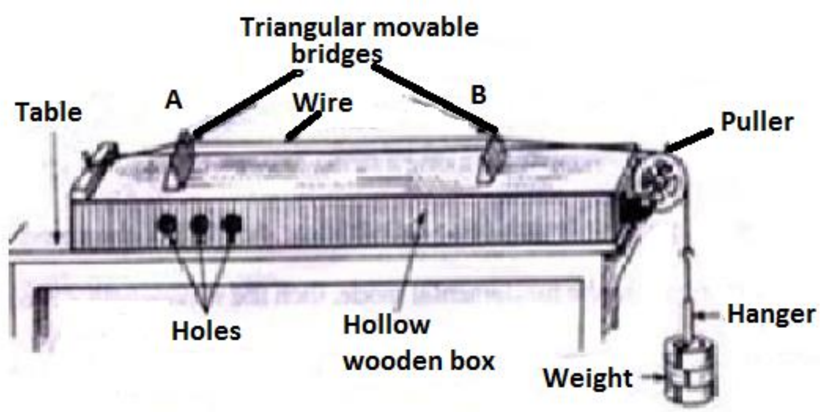
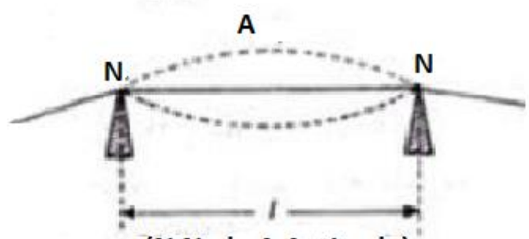


Fig 5.1 SONOMETER



(N:Node.A:Antinode)

To investigate the relationship between f and l the bridges are moved so that different lengths between them emit their fundamental frequencies when plucked at the centre.

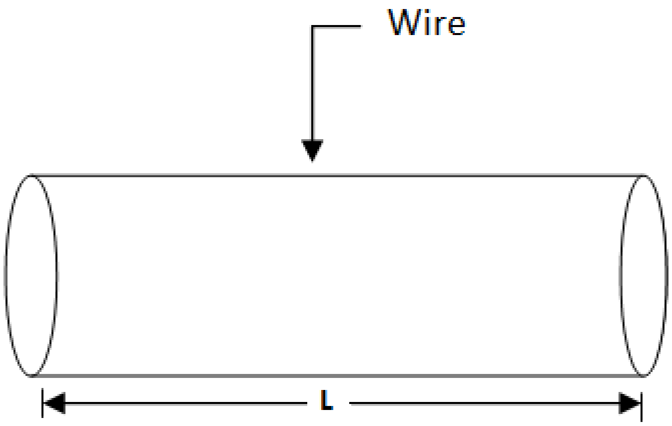
- To check that $f \alpha \sqrt{T}$, the same length of wire is subjected to different tensions by changing the hanging mass.

-The relation between f and $\frac{1}{\sqrt{\mu}}$ requires the use of wire of different diameter and materials but of the same vibrating length under the same tension.

NOTE:

$$\mu = \frac{mass\ of\ wire}{length\ of\ wire}$$

$$\mu = \frac{volume\ of\ wire \times density}{length\ of\ the\ wire}$$



Volume of wire = Al = $\frac{\pi d^2 l}{4}$

$$\mu = \frac{\left(\frac{\pi d^2 l}{4} \times \rho\right)}{l}$$

$$\mu = \frac{\pi d^2}{4}\rho$$

Where d = diameter of the wire

P = density of the material of which the wire is made

<u>FORCED VIBRATIONS AND RESONANCE</u>

Definition

- Forced vibrations are vibrations that occur in a system as a result of impulses received from another system vibrating nearby.

<u>Examples</u>

- When a turning fork is sounded and placed on a bench or hollow box, the sound produced is quite loud all over the room.

- It is because the bench or box acts like an extended source (or many point sources) which are set into forced vibrations by the vibrating fork.
- The response of the system that is sent into forced vibration is best when the driving frequency is equal to the natural frequency of the responding system.
- The responding system is then said to be in resonance with the driving frequency.

Definition

Resonance is a condition in which a body or system is set into oscillation at its own natural frequency as a result of impulses received from some other system which is vibrating at the same frequency.

Problem 17

A sonometer wire of length 0.50m and mass per unit length 1.0 kgm^{-1} is stretched by a load of 4kg. if it is plucked at its mid-point, what will be:

(a) The wavelength and

(b) The frequency of the note emitted?

Take g = 10Nkg^{-1}

Problem 18

Two sonometer wires A, of diameter 7.0$\times 10^{-4}$m and B of diameter 6.0 $\times 10^{-4}$m, of the same material are stretched side by side under the same tension. They vibrate at the same fundamental frequency of 256HZ. If the length of B is 0.91m, find the length of A. Calculate the number of beats per second which will occur if the length of B is reduced to 0.90m.

Problem 19

A sonometer wire of the length 1.0m emits the same fundamental frequency as a given turning fork. The wire is shortened by 0.05m, tension remaining unaltered and 10 beats per second are heard when the wire and fork are sounded together.

(i) What is the frequency of the fork?

(ii) If the mass per unit length of the wire is 1.4$\times 10^{-3} kgm^{-1}$, what is the tension.

Problem 20

A 160cm long string has two adjacent resonance at 85HZ frequencies respectively. Calculate:

(i) The fundamental frequency

(ii) The speed of the wave.

PIPE INSTRUMENTS

- Stationary waves in a column of air in a pipe are the source of sound in pipe instruments.
- To set the air into vibration a disturbance is created at one end of the pipe.

TYPES OF PIPE INSTRUMENTS

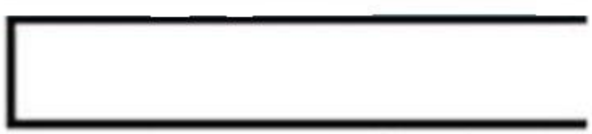
- There are two type of pipe instruments:

  (i) Closed pipe

  (ii) Open pipe

STATIONARY WAVES IN CLOSED PIPE

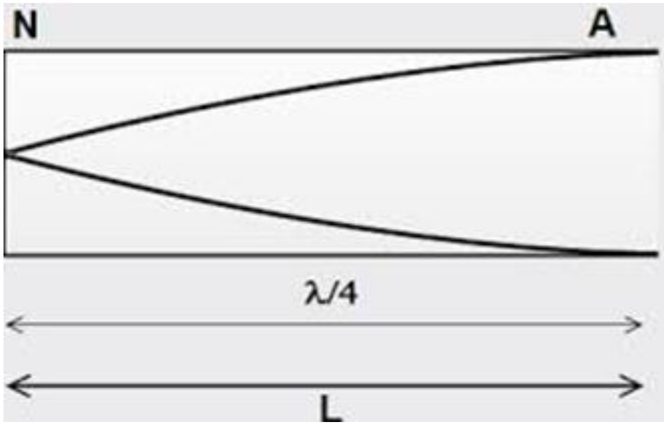
- A closed pipe is the one which is closed at one end and open at the other end.



closed–open cylindrical pipe

- The open end is always a displacement antinode (A) and the closed end is a displacement node (N).
- Different modes / Harmonics are obtained when the air inside a closed pipe is sent into vibration.

.1^{st} **HARMONIC / FUNDAMENTAL NOTE**



Where l = length of the pipe

$\lambda_0 =$ Wavelength of the fundamental note

➔ $\frac{1\lambda_0}{4} = l$

$\lambda_0 = 4l$ ----------------------- (1)

- Let f_0 be the fundamental note frequency
- If V is the velocity of the sound in air, then

V = $\lambda_0 f_0$

$$f_0 = \frac{V}{\lambda_0}$$

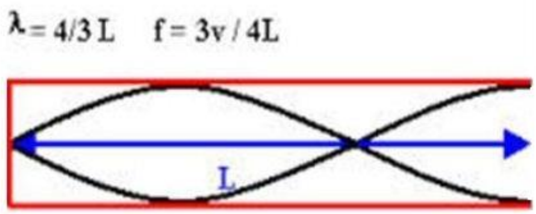
- Substitute eqn (1) in this eqn

$f_0 = \frac{V}{4l}$ ---------------------------- (2)

OR

$4f_0 = \frac{V}{l}$

- Overtones are encourage by blowing hardly



- Let λ_1 be wavelength of the 1^{st} overtone

i.e $\frac{3\lambda_1}{4} = l$

$\lambda_1 = \frac{4l}{3}$ ------------------------- (3)

- Let f_0 be the frequency of the 1^{st} overtone.
- If V is the speed of sound in air, then

V = $\lambda_1 f_1$

$f_1 = \frac{V}{\lambda_1}$

- Substitute equation (3) in this equation

$f_1 = \frac{V}{\frac{4l}{3}}$

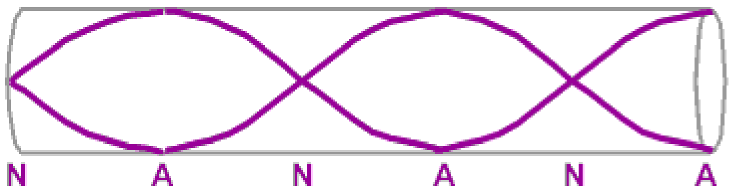
$f_1 = \frac{3}{4} \times \frac{V}{l}$

- But from equation (*) $\frac{V}{l}$ = $4f_0$

- $f_1 = \frac{3}{4} \times 4f_0$

$f_1 = 3f_0$ ------------------ which is the 3^{rd} harmonic

SECOND OVERTONE



- Let λ_2 be wavelength of a 2^{nd} overtone

i.e $5\frac{\lambda_2}{4} = l$

$\lambda_2 = \frac{4l}{5}$ ----------------------- (4)

- Let f_2 be the frequency of the 2^{nd} overtone
- If V is the speed of sound in air, then

V = $\lambda_2 f_2$

$f_2 = \frac{V}{\lambda_2}$

- Substitute equation (4) in this equation

$f_2 = \frac{V}{\frac{4l}{5}}$

$f_2 = \frac{5}{4} \times \frac{V}{l}$

- But from equation * $\frac{V}{l} = 4f_0$

- $f_2 = \frac{5}{4} \times 4f_0$

$\{f_2 = 5f_0\}$ ------------------- which is the 5^{th} harmonic

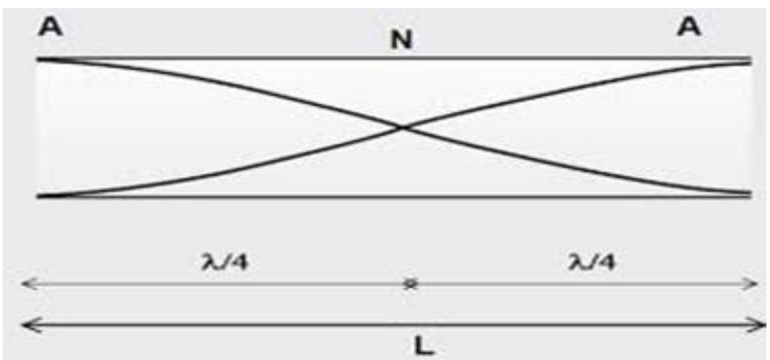
- Thus a closed pipe produces odd number harmonics

i.e. $f_0, 3f_0, 5f_0, 7f_0, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$

STATIONARY WAVES IN OPEN PIPES

- Here both of pipes are opened and displacement antinode
- Different modes/harmonics are obtain when the air inside the open pipe is set into vibration

1st HARMONIC/FUNDAMENTAL NOTE



- Where L = length of pipe

$\lambda_0 = wavelength\ of\ the\ fundamental\ note$

$\frac{\lambda_0}{2} = l$

$\{\lambda_0 = 2l\}$ ------------------------- (1)

- Let f_0 be frequency of the fundamental note.
- If V is the speed of sound in air, then

V = $\lambda_0 f_0$

$f_0 = \frac{V}{\lambda_0}$

- Substitute equation (1) into equation

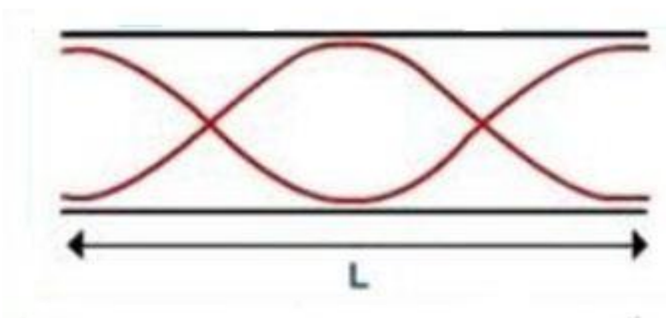
$f_0 = \frac{V}{2l}$ --------------------- (2)

OR

$2f_0 = \frac{V}{l}$ --------------------- (*)

- Overtones are encouraged by the blowing hardly

FIRST OVERTONE



- Where $\lambda_1 =$ wavelength of the 1^{st} overtone

L = length of the pipe

$\lambda_1 = l$ ----------------------- (3)

- Let f_1 be the frequency of the 1^{st} overtone
- If V is the speed of the sound in air, then

V = $\lambda_1 f_1$

$f_1 = \frac{V}{\lambda_1}$

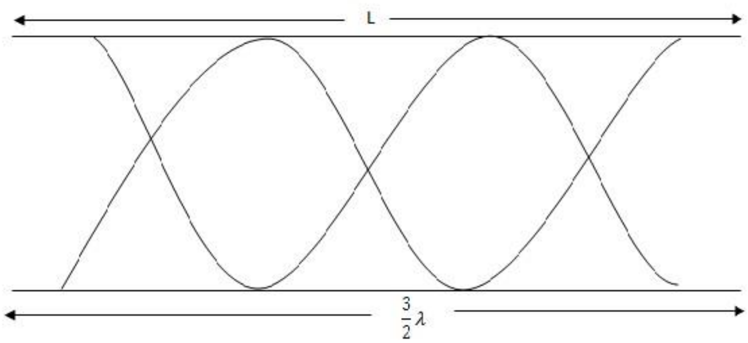
- Substitute equation (1) into this equation

$$f_1 = \frac{V}{l}$$

- But from equation $\frac{V}{l} = 2f_0$

* $\{ f_1 = 2f_0 \}$ ----------------------- which is the 2^{nd} harmonic

SECOND OVERTONE



- Let λ_2 be the wavelength of the 2^{nd} overtone

$$\frac{3}{2}\lambda_2 = l$$

$$\lambda_2 = \frac{2}{3}l \text{ ----------------------- (4)}$$

- Let f_2 be frequency of the 2^{nd} overtone.

- If V is the speed of sound in air, then

$V = \lambda_2 f_2$

$f_2 = v/\lambda_2$

$$f_2 = \frac{V}{\gamma_2}$$

- Substitute equation (4) in this equation

$$f_2 = \frac{V}{\frac{2l}{3}}$$

$$f_2 = \frac{3}{2} * \frac{V}{l}$$

- From equation * $\frac{V}{l} = 2f_0$

$$f_2 = \frac{3}{2} * 2f_0$$

[$f_2 = 3f_0$] ------------------- which is the 3^{rd} harmonic

- Thus, for an open pipe the frequency of harmonics are:

$$f_0, 2f_0, 3f_0, 4f_0, \dots\dots\dots\dots\dots\dots\dots\dots\dots$$

END CORRECTION OF A PIPE

- In practice the air just outside the open end of a pipe is set into vibration and the displacement antinode of a stationary wave occurs a distance "C" called end correction beyond the open end.

- The effective length of the air column is therefore slight greater than the length of the pipe.

Example

(a) For a closed pipe

➔ $\frac{1}{4}\lambda_0 = l + C$

$\lambda_0 = 4l + 4C$

(b) For an open pipe

➔ $\frac{1}{2}\lambda_0 = l + 2C$

$\lambda_0 = 2l + 4C$

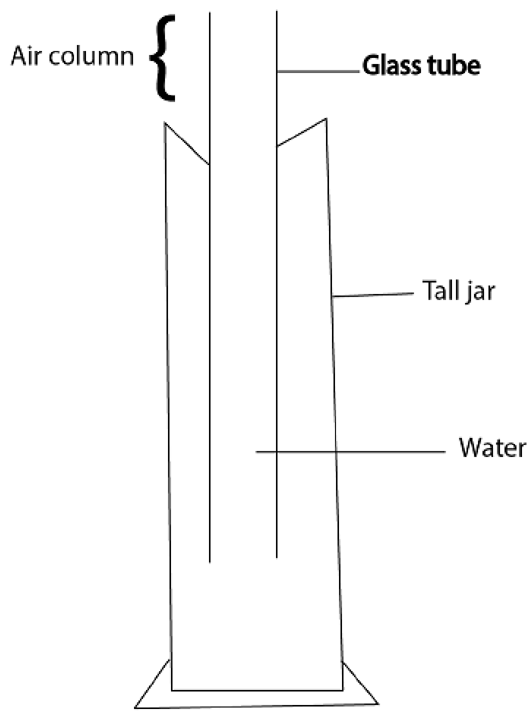
Definition

-The end correction C of a pipe is that small length of a stationary wave which protrudes just outside the open end of a pipe instrument where the air inside it is set into vibration.

RESONANCE IN A CLOSED PIPE

Apparatus

- A resonance jar is used.
- It consists of a glass tube which stands in a tall jar full of water.



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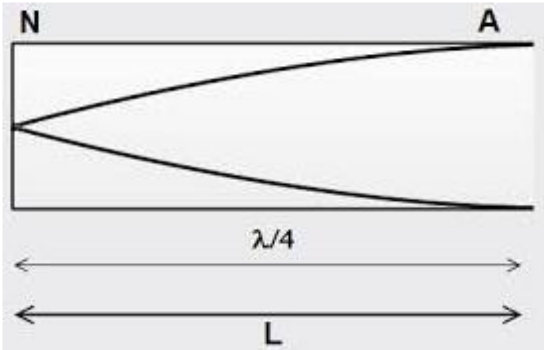
- The length of the air column is varied by raising or lowering the glass tube.

WORKING

- Starting with a very short air column, a vibrating fork is held over the mouth of the tube and the length of the column is then gradually increased.
- Strong resonance occurs when the column reaches a certain critical length l_1 (say).
- This is called the first position of resonance.
- At this position the air in the tube vibrates at its fundamental note/first harmonic



Where

- λ = Wavelength of sound produced
  C = end correction
- $\frac{1}{4}\lambda = l_1 + C$

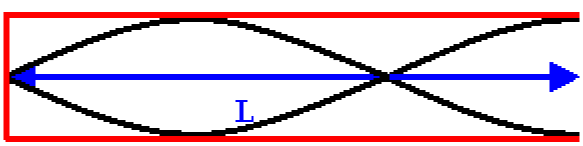
$\lambda = 4L_1 + 4C$ --------------------- (1)

- If the length of the air column is now increased still further a second position of resonance is obtained when the column is approximately three times as long as (l_2 say).

  - At this position the air in the tube vibrates with its 3^{rd} harmonic/first overtone.

Third harmonic

λ = 4/3 L f = 3v / 4L



- $3\lambda = l_2 + C$
  - $3\lambda = 4l_2 + 4C$ --------------------- (2)

- Equation (2) – equation (1)

$$\begin{cases} 3\lambda = 4l_2 + 4C \\ \lambda = 4l_1 + 4C \end{cases}$$
-

$3\lambda - \lambda = 4l_2 - 4l_1$

$2\lambda = 4(l_2 - l_1)$

$\lambda = 2(l_2 - l_1)$ ---------------------- (3)

- If the frequency f of the fork is known then the speed V of sound in air can be found.

V = λf

- Substitute equation (3) in this equation

V = 2 $(l_2 - l_1)f$

V = 2f $(l_2 - l_1)$ --------------------- (2)

Problem 21

What are the two successive response lengths of a closed pipe containing air at 27^0C for a turning

Take the speed of sound in air at 0^0C to be $330m/s$

Problem 22

Two open organ pipes of lengths 50cm and 51cm respectively give beats of frequency 6.0HZ when sounding their fundamental notes together. Neglecting end correction, what value does this give for the velocity of sound in air?

Problem 23

A Cylindrical pipe of length 28cm closed at one end is found to be at resonance when a tuning fork of frequency 864HZ is sounded near the open end. Find the mode of vibration of the air in the pipe and calculate the value of the end correction (speed of sound in air = 340mms^{-1})

BEATS

Definition

Beats are the periodic increase and decrease in loudness heard when two notes of slightly different frequency is sounded at the same time.

- The two notes producing beats must be of the same amplitude.
- The periodic increase and decrease in loudness is a result of successive occurrence between the two notes, as the repeatedly become in phase and then out in phase with each other.

MATHEMATICAL TREATMENT OF BEATS

- Let $y_1, and\ y_2$ be the individual displacement two notes whose frequencies are $f_1, and\ f_2$ respectively.
- Let "a" be the amplitude of each note.

$y_1 = a\sin(2\pi f_1 t)$ And

$y_2 = a\sin(2\pi f_2 t)$

- Applying the principle of superposition of waves, the resultant displacement y is given by:

Y = $y_1 + y_2$

Y = $a \sin(2\pi f_1 t) + a\sin(2\pi f_2 t)$

$$y = 2a\ sin\left[\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right] \text{Cos}\left[\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right]$$

$$y = 2a\ sin\left[\frac{2\pi(f_1 + f_2)t}{2}\right]\ cos\left[\frac{2\pi(f_1 - f_2)t}{2}\right]$$

$$y = 2a\ cos\left[\frac{2\pi(f_1 - f_2)t}{2}\right] \text{sin}\left[\frac{2\pi(f_1 + f_2)t}{2}\right]$$

But 2a cos$\left[\frac{2\pi(f_1 - f_2)t}{2}\right]$ = A = Amplitude of the resultant wave.

$$y = A\ sin\left[\frac{2\pi(f_1 + f_2)t}{2}\right]$$

- This equation shows that resultant wave has an effective frequency equal to the average frequency of the two sources.

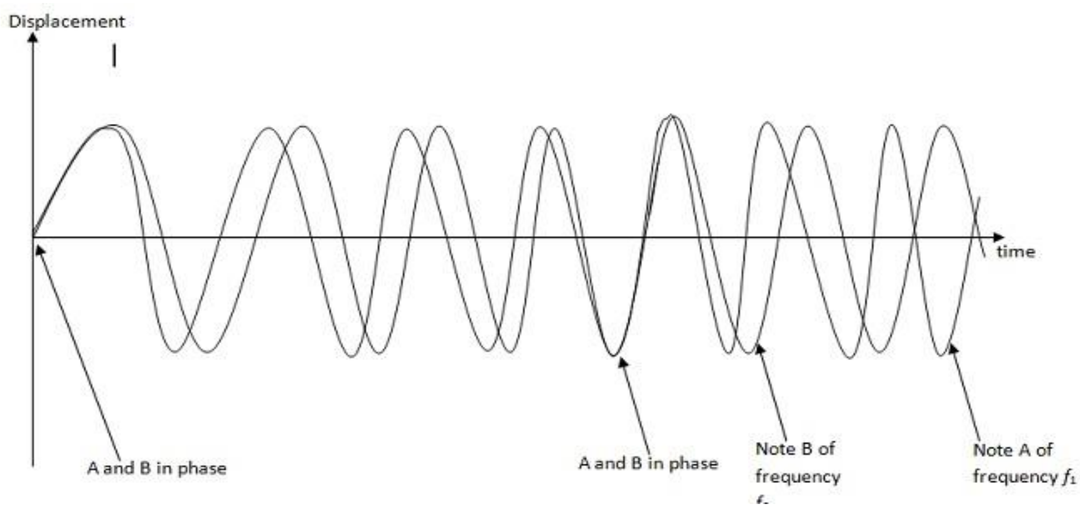
i.e. $\frac{f_1 + f_2}{2}$

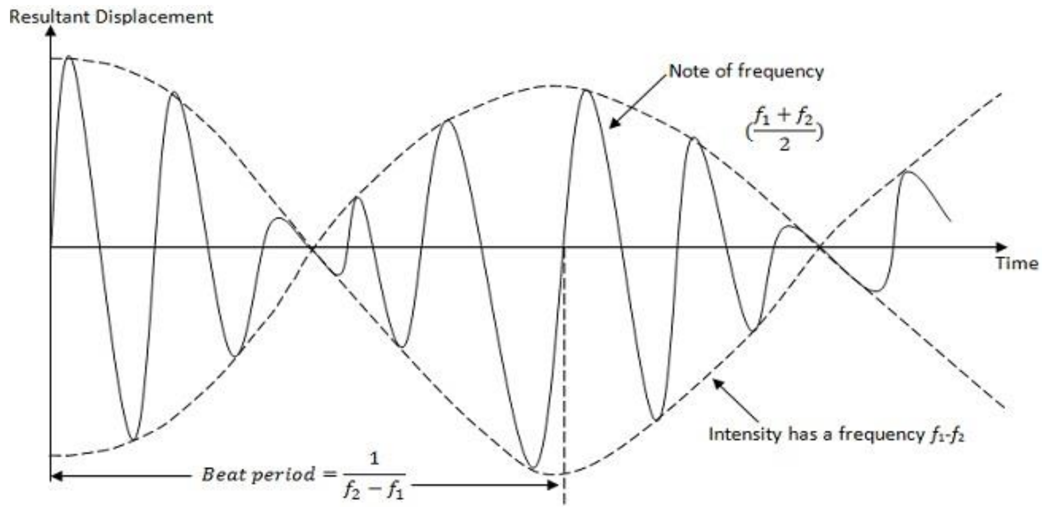
- The amplitude A of the resultant wave is

A = $2a\ cos\left[\frac{2\pi(f_1 - f_2)t}{2}\right]$

- This equation shows that the amplitude varies in time with a frequency given by:-

$$\frac{f_1 - f_2}{2}$$





BEAT FREQUENCY (B.f)

- This is the number of beats heard per second when two notes of slightly different frequency are sounded at the same time.

- It is given by difference between the frequencies of the two notes.

B.f = $f_1 - f_2 \ \ if \ \ f_1 > f_2$

OR

B.f = $f_2 - f_1 \ \ if \ \ f_2 > f_1$

NOTE

Beat period T = $\frac{1}{B.f}$

APPLICATION OF THE PHENOMENON OF BEATS MEASUREMENT OF THE UNKNOWN FREQUENCY

- The phenomenon of beats can be used to determine the unknown frequency of some wave motion by causing the wave to beat with a wave of the same kind whose frequency is known.

- Suppose a note of unknown frequency f_1 is made to produce beats with a note of known frequency f_2.

- If f_2 is not very different from f_1 then it is possible to count the number of beats that occur in some given time, and hence determine the beat frequency (B.f).

B.f = $f_1 - f_2$ if $f_1 > f_2$

$$f_1 = f_2 + B.f$$

OR

B.$f = f_2 - f_1$ if $f_2 > f_1$

$$f_1 = f_2 - B.f$$

- In order to discover which of f_1 and f_2 is the higher frequency, one of the frequencies, f_2 say is changed slightly.
- This can be done by loading f_2 $with$ a small pieces of plasticine / wax and the effect on the beat frequency is noted.

Problem 24

The beat frequency of two notes of nearly equal frequency is 6HZ. If one is loaded with wax, the beat frequency becomes 4HZ. What is the frequency of the other note of the one note loaded is 250HZ?

Problem 25

Two turning forks are sounded together and their beats frequency is 4HZ. One of the turning forks has a frequency of 320HZ. When the other turning fork is loaded with plasticine, the beat frequency becomes 6HZ. What is the frequency of the other tuning fork before being loaded with plasticine?

Problem 26

Calculate the speed of sound in a gas in which two waves of wavelength 1.00m and 1.01m produce 30 beats in 10 seconds.

Problem 27

Two similar sonometer wires of the same materials produce2 beats per second. The length of one is 50cm and that of the other is 50.1cm. calculate the frequency of the two wires.

DOPPLER EFFECT

Definition

Doppler Effect is the apparent change in the observed frequency of a wave as a result of relative motion between the source and the observer.

Example

- There is sudden decrease in pitch (frequency) heard by a personal standing in a railway station as a sounding train siren passes by him.

CASE 1

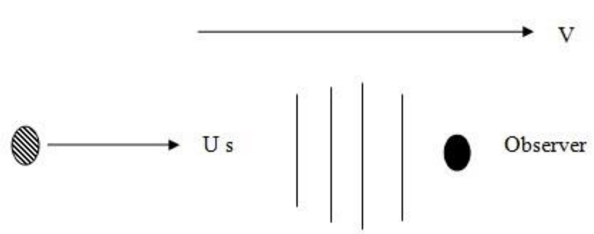
SOURCE MOVING

- Motion of the source affect the wavelength of the wave and hence the apparent wavelength (λ_0) is given by:

$$\lambda_0 = \frac{velocity\ of\ sound\ relative\ to\ observer}{true\ frequency}$$

(i) SOURCE MOVING TOWARDS A STATIONARY OBSERVER

- Consider a source S of sound wave to be moving towards a stationary observer O



- Where $U_s =$ velocity of the source

  V = velocity of sound

- Velocity of the wave relative to observer at O = V - U_s
- The apparent wavelength λ_0 reaching the observer at O is

$$\lambda_0 = \frac{V - U_s}{f} \text{ --------------------- (1)}$$

- Where f = true frequency of the source
- Let f_0 be apparent frequency
- From V = $\lambda_0 f_0$

$f_0 = \frac{V}{\lambda_0}$ ---------------- (2)

- Substitute equation (1) in equation (2)

$$f_0 = \frac{V}{\frac{[V - U_s]}{f}}$$

$$f_0 = \left[\frac{V}{V - U_s}\right] f$$

(ii) SOURCE MOVING AWAY FROM A STATIONARY OBSERVER

- Consider a source S of sound wave to be moving with velocity U_s away from a stationary observer O

-Where V = Velocity of sound wave

- Velocity of wave relative to observer at 0 = V + U_S

- The apparent wavelength γ_0 reaching the observer at 0 is :

$\lambda_0 = \frac{V + U_s}{f}$ ------------------- (4)

- The apparent frequency $f_0 = \frac{V}{\lambda_0}$ ------------------- (5)
- Substitute equation (4) in equation (5)

$$f_0 = \frac{V}{\left[\frac{V+U_s}{f}\right]}$$

$$f_0 = \left[\frac{V}{V+U_s}\right]f \quad \text{------------------- (6)}$$

CASE 2

OBSERVER MOVING

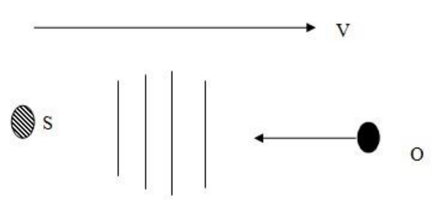
- The motion of the observer affects the velocity of the waves he receives.
- In this case the wavelength is unchanged and is given by:

$$\lambda = \frac{V}{f} \quad \text{--------------------------- (7)}$$

- The apparent frequency f_0 is given by

$$f_0 = \frac{Relative\ velocity\ of\ waves}{wavelength}$$

(i) OBSERVER MOVING TOWARDS A STATIONARY SOURCE



- Where $U_0 =$ Velocity of observer

  V = Velocity of wave

- Velocity of wave relative to observer = V + U_0

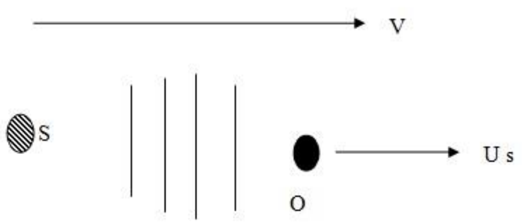
- The apparent frequency f_0 of the wave is

$$f_0 = \frac{relative\ velocity\ of\ the\ waves}{wavelength}$$

$$f_0 = \frac{V + U_0}{[V/f]}$$

$$f_0 = \frac{(V+U_s)f}{v}$$ ------------------------ (8)

(ii) OBSERVER MOVING AWAY FROM STATIONARY SOURCE



- Velocity of waves relative to observer

= V - U_0

- The apparent frequency f_0 is given by:

$$f_0 = \frac{relative\ velocity\ of\ waves}{wavelength}$$

$$f_0 = \frac{V - U_0}{[\frac{V}{f}]}$$

$$f_0 = \frac{(V - U_0)f}{v}$$ ---------------------- (9)

CASE 3

SOURCE AND OBSERVER ARE MOVING

(i) SOURCE AND OBSERVER ARE APPROACHING

- Velocity of wave relative to observer is:
  $V_o = V + U_o$

- Apparent wavelength λ_0 is

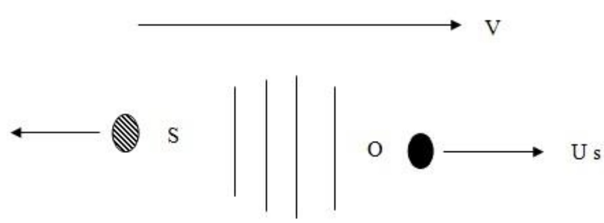
$$\lambda_0 = \frac{V - U_s}{f}$$

- The apparent frequency f_0 is given by

$$f_0 = \frac{V_0}{\lambda_0} = \frac{V+U_0}{\left[\frac{(V-U_s)}{f}\right]}$$

$$f_0 = \left[\frac{V+U_0}{V-U_s}\right] f$$ ----------------- (10)

(ii) **<u>SOURCE AND OBSERVER ARE MOVING AWAY FROM EACH OTHER</u>**



- Velocity of wave relative to observer is:

$$V_0 = V - U_0$$

- Apparent wavelength $\gamma_0 \, is$

$$\lambda_0 = \frac{V+U_s}{f}$$

- Apparent frequency f_0 *is given by*

$$f_0 = \frac{V_0}{\lambda_0}$$

$$f_0 = \frac{V-U_0}{\frac{V+U_s}{f}}$$

$$f_0 = \left[\frac{V-U_0}{V+U_s}\right]f \quad \text{------------------------ (11)}$$

Problem 28

Calculate the frequency of the beats heard by a stationary observer when a source of sound of frequency 100HZ moves directly away from him with a speed of 10.0ms^{-1} towards a vertical wall. Given that speed of sound in air =340ms^{-1}.

Problem 29

Two whistles A and B each has a frequency of 500HZ. "A" is stationary and "B" is moving towards the right (away from A) at a velocity of 200fts^{-1}. An observer is between the two whistles. Moving towards the right with a velocity of 100fts^{-1}. Take the velocity of sound in air as 1100fts^{-1}.

(a) What is the frequency from A as heard by the observer?

(b)What is the frequency from B as heard by the observer?

(c)What is the beat frequency heard by the observer?

Problem 30

A source of sound waves "S" emitting waves of frequency 1000HZ, is traveling towards the right in still air with a velocity of 100fts^{-1}. At the right of the source is large, smooth, reflecting surface moving towards the left at a velocity of 400fts^{-1}

(a) How far does the emitted wave in 0.01 second?

(b) What is the wave length of the emitted waves in front of (i.e at the right of) the source?

(c) How many waves strike the reflecting surface in 0.01sec? Take the velocity in air as 1100fts^{-1}

Problem 31

A car sounding a horn produce a note of 500HZ, approaches and then passes a stationary observer O at a steady speed of 20ms^{-1}. Calculate the apparent frequency in each case. Velocity of sound = 340m/s.

Problem 32

A cyclist and railway train are approaching each other. The cyclist is moving at 10m/s an the train at 20m s^{-1}. The engine driver sounds a warning siren at a frequency of 480HZ. Calculate the frequency of the note heard by the cyclist.

(i) Before and

(ii) After the train has passed by

Given that speed of the sound in air = 340ms^{-1}

Problem 33

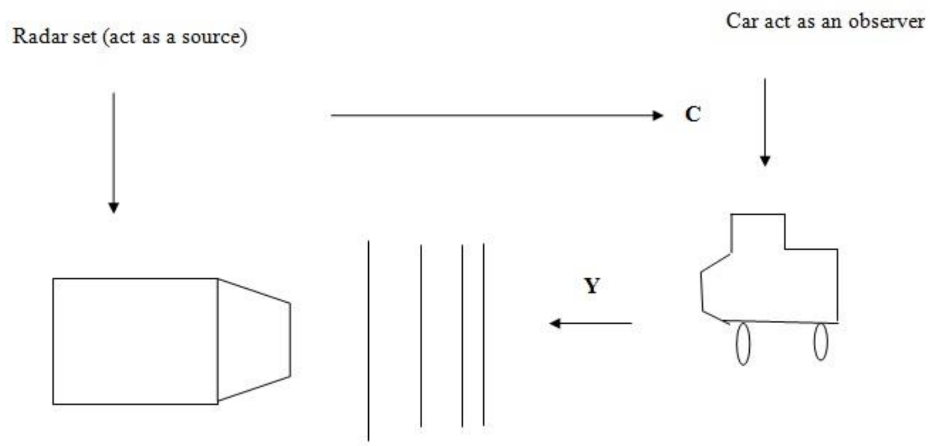
An observer standing by a railway track notices that the pitches of an engine whistle change in the ration of 5:4 on passing him. What is speed of the engine?

THE RADAR SPEED TRAP

- This is an instrument used to determine the speed of a moving car.
- The instrument sent out microwaves of frequency f (about 10.7GHZ) towards a moving car.
- The speed of moving car can be found by measuring the shift in frequency of microwaves reflected by it.

Consider a car moving with speed V towards a stationary source of microwave of frequency f.

- The car act as an observer moving towards a stationary source and the wave as received by the car have a frequency f.



- f= is given by

$$f' = \frac{relative\ velocity\ of\ waves}{wavelength}$$

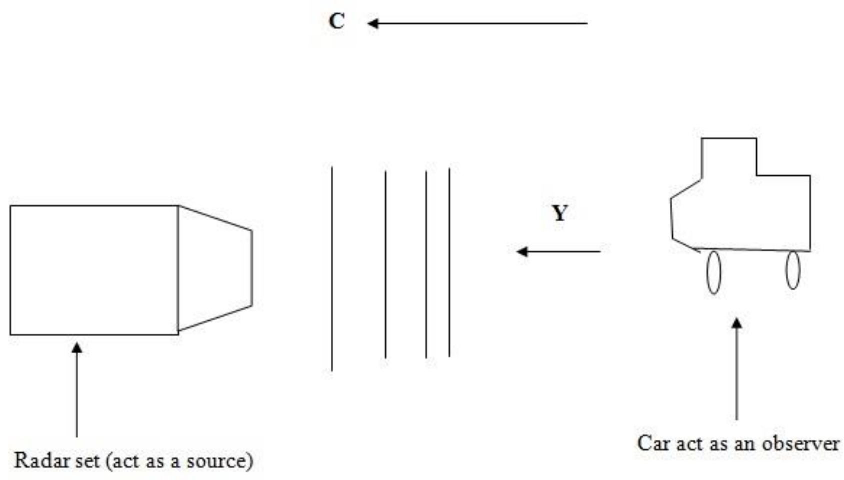
$$f' = \frac{C+V}{\lambda}$$

But $\lambda = \frac{c}{f}$

$$\rightarrow f' = \frac{C+V}{\left[\frac{C}{f}\right]}$$

$$f' = \left[\frac{C+V}{C}\right] f \text{ ------------------ (1)}$$

- Where C = speed of microwaves in free space
- Wave of this frequency (f') are reflected back to the source, so that the car is now acting as a source moving with velocity V and the radar set is acting as a stationary observer.



- Let f'' be frequency of waves on reaching the radar set.

$$f'' = \frac{c}{\lambda_0}$$

But $\lambda_0 = \frac{c-V}{f'}$

$$f'' = \frac{c}{\left[\frac{c-V}{f'}\right]}$$

$$\therefore f'' = \left[\frac{c}{c-V}\right] f' \text{ ---------------------- (2)}$$

- Substitute equation (1) in equation (2)

$$f'' = \left[\frac{c}{c-V}\right]\left[\frac{c+V}{c}\right] f$$

$f'' = \left[\frac{C+V}{C-V}\right] f$ --------------------- (3)

- The fractional change in frequency $\frac{\Delta f}{f}$ is given by

$$\frac{\Delta f}{f} = \frac{f'' - f}{f}$$

$$\frac{\Delta f}{f} = \frac{f''}{f} - 1$$ ------------------- (4)

- Substitute equation (3) in equation (4)

$$\frac{\Delta f}{f} = \frac{\left(\frac{c+v}{c-v}\right)f}{f} - 1$$

$$\frac{\Delta f}{f} = \frac{C+V}{C-V} - 1$$

$$\frac{\Delta f}{f} = \frac{C+V-(C-V)}{C-V}$$

$$\frac{\Delta f}{f} = \frac{C+V-\ C+V}{C-V}$$

$$\frac{\Delta f}{f} = \frac{2V}{C-V}$$

- Since V << C, therefore C – V ≈ C

$$\frac{\Delta f}{f} = \frac{2V}{C}$$

❖ $V = \frac{C\Delta f}{2f}$ --------------------- (5)

- Where â^†f = beat frequency of the waves transmitted and received by the radar set.

- Thus, by causing the incoming signal to beat with the transmitted signal and knowing the frequency of the transmitted signal, we can find V from equation (5) above:

Problem 34

Calculate the beat frequency produced if car travels a radar speed trap at 30ms^{-1}, the operating frequency of the speed trap being 10.7GHZ. Take the velocity of light to be 3 $\times 10^8 ms^{-1}$.

Problem 35

Calculate the change in frequency of the radar echo received from an aeroplane moving at 250ms^{-1} if the operating wavelength of the radar set is 1 metre.

DOPPLER EFFECT IN LIGHT

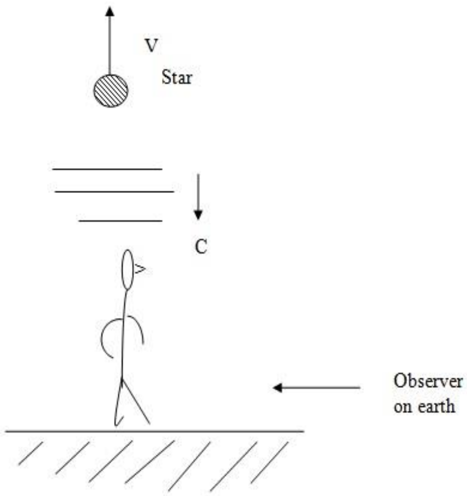
- The Doppler effect in light is used to measure the speed of distant objects and planets.
- Suppose a source of light emits waves of frequency f and wavelength λ.
- If C is the velocity in light in free space then:

$$\lambda = \frac{c}{f} \quad \text{----------------------- (1)}$$

EXPRESSIONS OF APPARENT WAVELENGTH AND FREQUENCY

When the source of light is moving away from the stationary observer

- Consider a source of light example star is moving with a velocity V away from the earth



- The apparent wavelength λ' to an observer on the earth in line with star's motion is

$$\lambda' = \frac{relative\ velocity\ of\ waves}{true\ frequency}$$

$$\lambda' = \frac{C+V}{f}$$

From equation (1) above

$$\lambda = \frac{C}{f}$$

$$f = \frac{C}{\lambda} \text{ ------------------------ (3)}$$

Substitute equation (3) in equation (2)

$$\lambda' = \frac{C+V}{\frac{C}{\lambda}}$$

$$\lambda' = \left[\frac{C+V}{C}\right]\lambda$$

$$\lambda' = \left[1 + \frac{V}{C}\right]\lambda \text{ ---------------------- (*)}$$

➔ $\lambda' = \lambda + \frac{v\lambda}{c}$

❖ $\lambda' - \lambda = \frac{v\lambda}{c}$ -------------------- (4)

- When $\lambda' - \lambda = \Delta\lambda$ = shift in wavelength

❖ $\Delta\lambda = \frac{v\lambda}{c}$ -------------------------- (5)

- It is clear from equation (4) that λ' is greater than λ.
- Thus, when a star is moving away from the earth, the apparent wavelength increases. They say that light is RED-SHIFTED i.e. it is shifted to longer wavelengths.

V	I	B	G	Y	O	R

Direction of increasing wavelength

- The apparent frequency f' is given by

$$f' = \frac{c}{\lambda'}$$

But from equation *

$$\lambda' = \left[1 + \frac{v}{c}\right]\lambda$$

➔ $f' = \frac{C}{\left[1+\frac{V}{C}\right]\lambda}$

➔ $f' = \frac{C/\lambda}{1+\frac{V}{C}}$

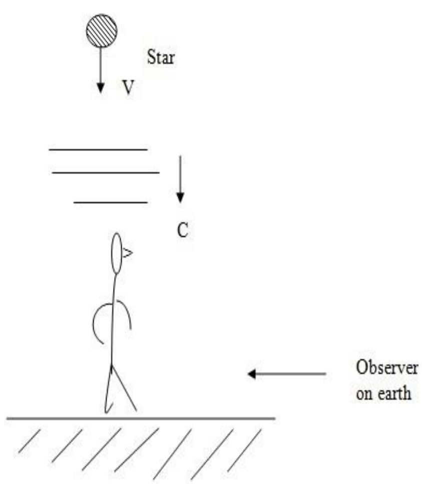
- Where $\frac{C}{\lambda} = f$

➔ $f' = \frac{f}{1+\frac{V}{C}}$

➔ $f' = f\left[1+\frac{V}{C}\right]^{-1}$

❖ $f' = f\left[1-\frac{V}{C}\right]^{-1}$ ---------------------- (6)

(iii) <u>When the source of light is moving towards the stationary observer</u>



- The apparent wavelength λ' is given by

$$\lambda' = \frac{relative\ velocity\ of\ waves}{true\ frequency}$$

$$\lambda = \frac{C-V}{f}$$

-From equation (1) $\lambda = \frac{C}{f} \rightarrow f = \frac{C}{\lambda}$

➔ $\lambda' = \frac{C-V}{\left[\frac{C}{\lambda}\right]}$

➔ $\lambda' = \left[\frac{C-V}{C}\right]\lambda$

❖ $\lambda' = \left[1-\frac{V}{C}\right]\lambda$ -------------------(7)

➔ $\lambda' = \lambda - \frac{V\lambda}{C}$

$$\lambda - \lambda' = \frac{V\lambda}{C}$$ ------------------------------ (8)

- Where $\lambda - \lambda' = \Delta\lambda =$ shift in wavelength.

❖ $$\Delta\lambda = \frac{V\lambda}{C}$$ -------------------------- (9)

- It is clear from equation (8) that λ' is less than λ.
- Thus, when a star is moving towards the earth, the apparent wavelength decreases.
- We say that light is blue shifted $i.e$ it is shifted to shorter wavelengths.
- The apparent frequency f' is given by:

$$f' = \frac{c}{\lambda'}$$

- But from equation (7) $\lambda' = \left[1 - \frac{v}{c}\right]\lambda$

$$f' = \frac{c}{\left[1-\frac{v}{c}\right]\lambda}$$

$$f' = \frac{c/\lambda}{1-\frac{v}{c}}$$

But C/$\lambda = f$

$$f' = \frac{f}{1-\frac{v}{c}}$$

$$f' = f\left[1 - \frac{v}{c}\right]^{-1}$$ --------------- (10)

APPLICATIONS OF DOPPLER EFFECT

(1) It is used in radar speed trap to determine the speed of a moving car.

(2) It is used for measurement of speed of star.

(3) It is used to determine the direction of motion of a star.

(4) It is used for measurement of plasma temperature.

NOTE

- There are some cases in which there is no Doppler effect in sound (i.e no changes in frequency)

(i) When the source of sound and the observer are moving in the same direction with the same speed.

(ii) When either the source or observer is at the center of the circle and the other is moving on the circle with uniform speed.

PHYSICAL OPTICS

- This deals with the study of phenomena that depend on the wave nature of light include:

(1) Interference

(2) Diffraction

(3) Polarization

THE NATURE OF LIGHT

Definition

- Light is a form of energy which stimulates the sense of vision.

- It can be transmitted in air / vacuum with a speed of $3 \times 10^8\ ms^{-1}$

HISTORICAL BACKGROUND

- Three different theories were put forward to explain the nature of light.

(1) NEWTON'S CORPUSCULAR THEORY OF LIGHT

- This theory suggested that light is a stream of particles emitted from a luminous object.
- This theory ignored the wave nature of light.

(2) HUYGEN'S THEORY

- This theory suggested that light travels from one point to another by wave motion.
- Thomas young produced evidence that light behaves as a wave motion by performing an experiment.

(3) EINSTEIN'S THEORY

- This theory suggested that light consist of a stream of particles carrying energy with them called PHOTONS.

- This is like corpuscular theory but with small difference.

NOTE

- Both particle theory and wave theory of light are accepted in solving and explaining different cases concerning propagation of light.

- Hence we have dual nature of light.

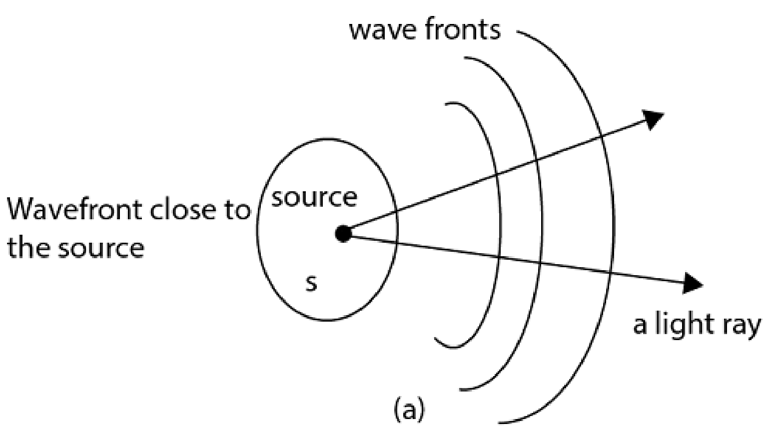
THE DUAL NATURE OF LIGHT

- Is the behavior of light in which two separate aspects can be isolated i.e wave nature and particle nature.

WAVE FRONTS AND RAYS OF LIGHT

Wave fronts

- Assume S to be a source of light waves in air



(a)

- The wave spread out equally in all directions.

- A line joining all adjacent points at which the disturbances are in phase is called a wave front

Definition

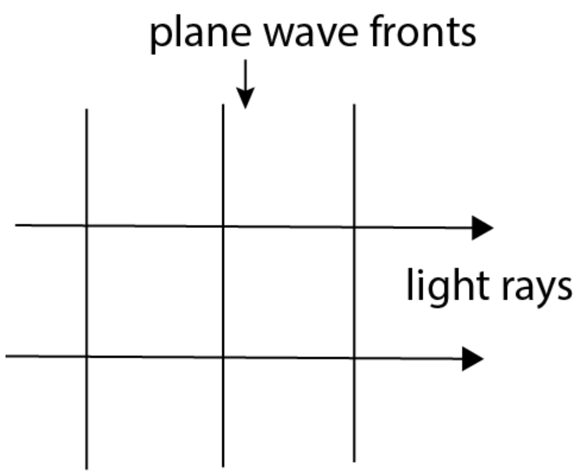
- A wave front is a locus of points having the same phase of oscillation.

-For a point source of light in a homogenous medium the wave fronts are spherical.

- A small part of a spherical wave front from a distant source will appear plane and therefore called plane wave front.

Example

- Sunlight reaches the earth with plane wave fronts



(b)

Rays of light

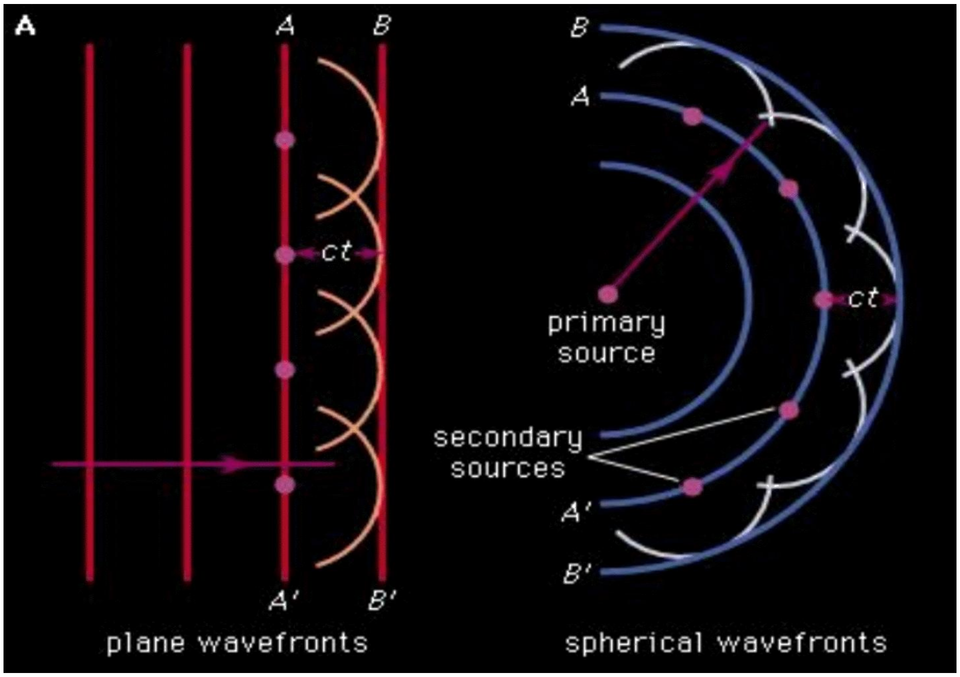
- A ray is the direction of the path taken by light.
- At any point along the path of light a ray is perpendicular to the wavefront.

HUYGEN'S PRINCIPLE

- Huygens's principle provides a geometrical method to determine the position of the wave front at a later time from its given position at any instant.
- The principle states:

(1) Each point on a wave front act as fresh source of secondary wavelets, which spread out with the speed of light in that medium.

(2) The new wave front at any later time is given by the forward envelop of the secondary wavelets at that time.

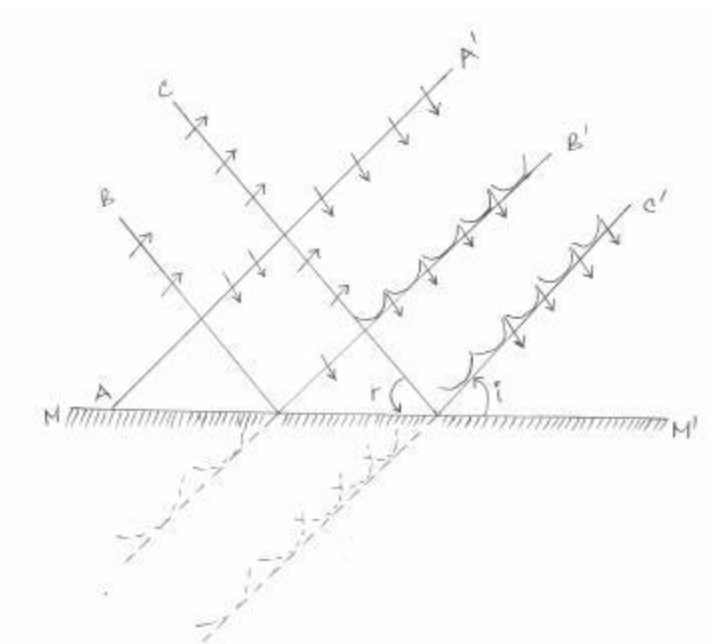


-Where AA' = original wave front

BB' = wave front after time the interval t

DERIVATION OF THE LAW OF REFLECTION OF LIGHT ON THE BASIS OF HUYGEN'S PRINCIPLE

-Consider a plane wave front AA' which is incident on the reflecting surface MM'



The position of the wave front after a time interval t may be found by applying Huygens's principle with points on AA' as centers.

- Those wavelets originating near the upper end of AA' spread out unhindered and their envelope given the portion of the new wave front OB'
- Those wavelets originating near the lower end of AA' strike the reflecting surface and get reflected 18^{0^0} out of phase.
- The envelope of these reflected wavelets is then the portion of the wave front OB.
- A similar construction gives the line CPC' for the wave front after another time interval, t.

RELATIONSHIP BETWEEN i and r

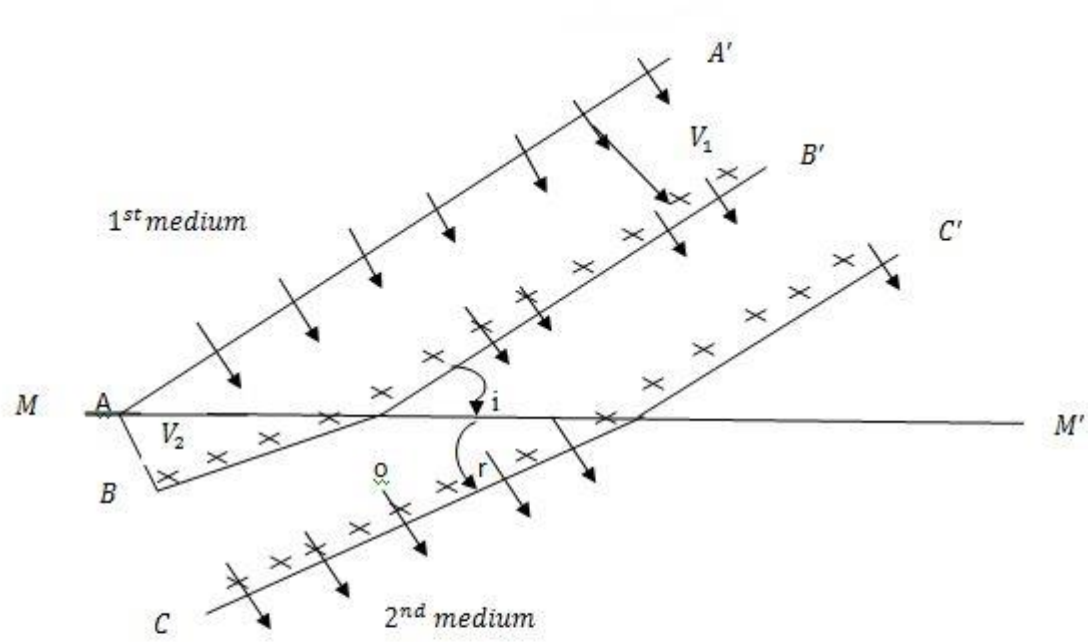
- Where i = angle of incidence

  r = angle of reflection

WAVE MOTION-2

DERIVATION OF SNELL'S LAW ON THE BASIS OF HUYGEN'S PRINCIPLE

-Consider a plane wave front AA' which is incident on a transparent medium MM'



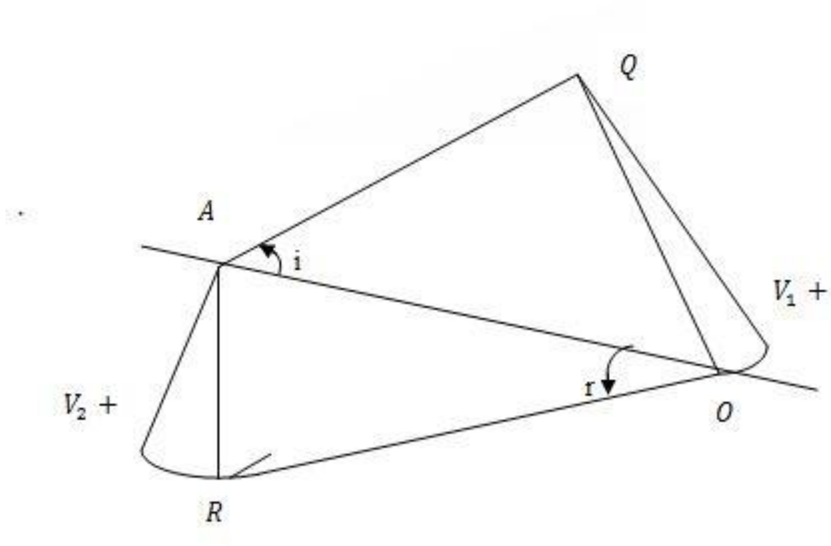
The position of a new wave front after time t may be found by applying Huygen's principle with points on AA' as centers.

Those wavelets originating near the upper end of AA' travel with a speed V_1 *in* 1^{st} medium and those wavelets originating near the lower end of AA' travel with a speed V_2 *in the* 2^{nd} medium.

Hence, after time = t:

- è Distance travelled in the 1^{st} medium = $V_1 t$
- è Distance travelled in the 2^{nd} medium = $V_2 t$

RELATIONSHIP BETWEEN i and r



Where i = angle of incidence

r = angle of reflection

From â^†AOQ

Sin i = $\frac{V_1 t}{AO}$ --------------------- (1)

From â^†AOR

Sin r = $\frac{V_2 t}{AO}$ ---------------- (2)

Dividing $\frac{eqn\ (1)}{eqn\ (2)}$

$$\frac{\sin i}{\sin r} = \frac{V_1 t/AO}{V_2 t/AO}$$

$$\frac{Sin\ i}{\sin\ r} = \frac{V_1}{V_2} = \text{constant} \text{ ---------------- } (3)$$

For a given pair of media, this ratio is constant which expresses Snell's law.

From V = λf

➔ $V_1 = \lambda_1 f \ \ and\ \ V_2 = \lambda_2 f$

- Equation (3) becomes

$$\frac{\sin\ i}{\sin\ r} = \frac{V_1}{V_2} = \frac{\lambda_1 f}{\lambda_2 f} = {}_1\eta_2$$

$$\frac{\sin i}{\sin r} = \frac{V_1}{V_2} = \frac{\gamma_1}{\gamma_2} = {}_1\eta_2 \text{ ---------------------(4)}$$

v

- Where $\lambda_1 =$ Wavelength in the 1^{st} medium

λ_2 = wavelength in the 2^{nd} medium

${}_1\eta_2$ = refractive index of the second medium with respect to the first.

Definition

-The refractive index (η) of material is the ratio of the sine of the angle of incidence to the sine of the angle of refraction.

Problem 36

Monochromatic light of a wavelength 800nm enters a glass plate of refractive index 1.5

Calculate;

(i) The speed of light in glass

(ii) The frequency of the light

(iii) The wavelength of light in glass

Given that C = $3 \times 10^{8} ms^{-1}$ and 1nm = 1×10^{-9} m

INTERFERENCE OF LIGHT WAVES

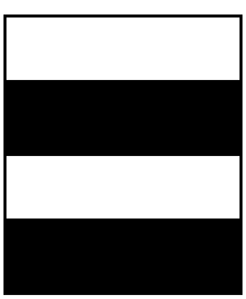
Interference is a situation in which two or more waves overlap in space. In a region where two or more light waves cross, superposition occurs giving reinforcement (addition) of the waves at some points and cancellation (subtraction) at others.

The resulting effect is called an interference pattern or system of fringes

Definition

Interference pattern/system of fringes is a set of light bands (fringes) and dark bands (fringes) formed on a screen when interference of light waves occurs.

INTERFERENCE PATTERN



------------- B

------------- D

------------- B

------------- D

------------- B

------------- D

------------- B

- Where B = Bright band (fringe)

  D = Dark band (fringe)

In order to bring about the interference of light waves the first task is to produce two coherence sources of light.

Coherent source of light are sources producing light of the same wavelength, frequency and amplitude.

- The sources have constant phase difference
- Two independent sources of light cannot be coherent.
- It is because the emission of light from any source is from a very larger number of atoms and the emission from each atom is random.
- Therefore, there is no stable phase relationship between radiations from two independent sources.
-

HOW TO PRODUCE COHERENT SOURCES OF LIGHT

- -Coherent sources of light are obtained by splitting up the light from a single source (primary source) into two parts (secondary source).

METHODS OF PRODUCING INTERFERENCE PATTERN

- Interference pattern can be produced by either of the following methods

(1) Young's double slit

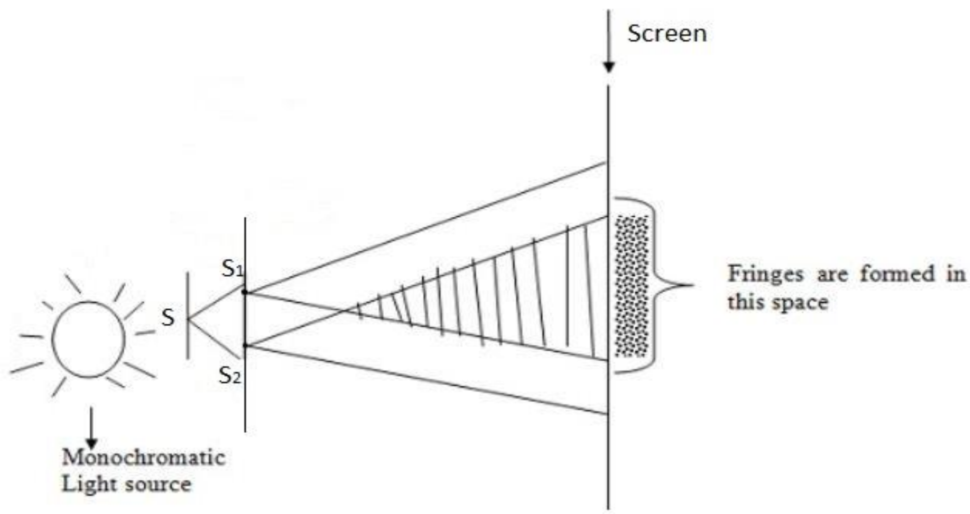
(2) Lloyd's mirror

(3) Fresnel's biprism

(4) Newton's rings

(5) Wedge fringes

In this experiment monochromatic light is passed through a slit "s" and the light emerging from this slit is used to illuminate two adjacent slits S_1 *and* S_2



By allowing light from these two slits to fall on the screen, a series of alternate bright and dark bands / fringes are formed on the screen.

The bright bands/ fringes represent constructive interference while the dark bands / fringes represent destructive interference.

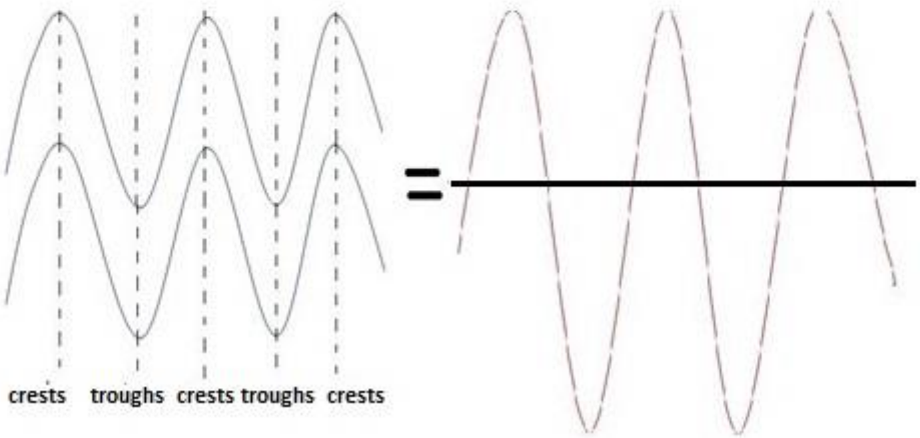
CONSTRUCTIVE INTERFERENCE

Constructive interference occurs when the interfering wave are in phase.

Waves are said to be in phase if their maximum and minimum values occur at the same instant.

Example

separated waves resultant displacement



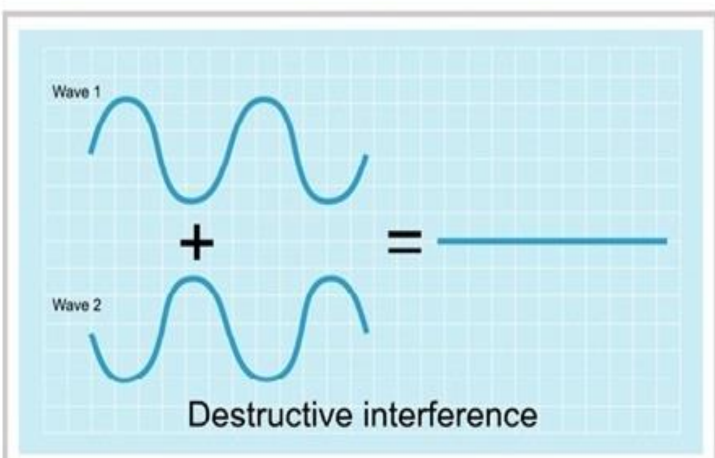
-When constructive interference occur the two interfering waves combine together to give a wave of larger amplitude and hence bright fringe is formed.

DESTRUCTIVE INTERFERENCE

Destructive interference occurs when the two interfering waves are <u>out of phase.</u>

Waves are said to be out of phase if the maximum of one wave and minimum the other wave are formed at the same instant.

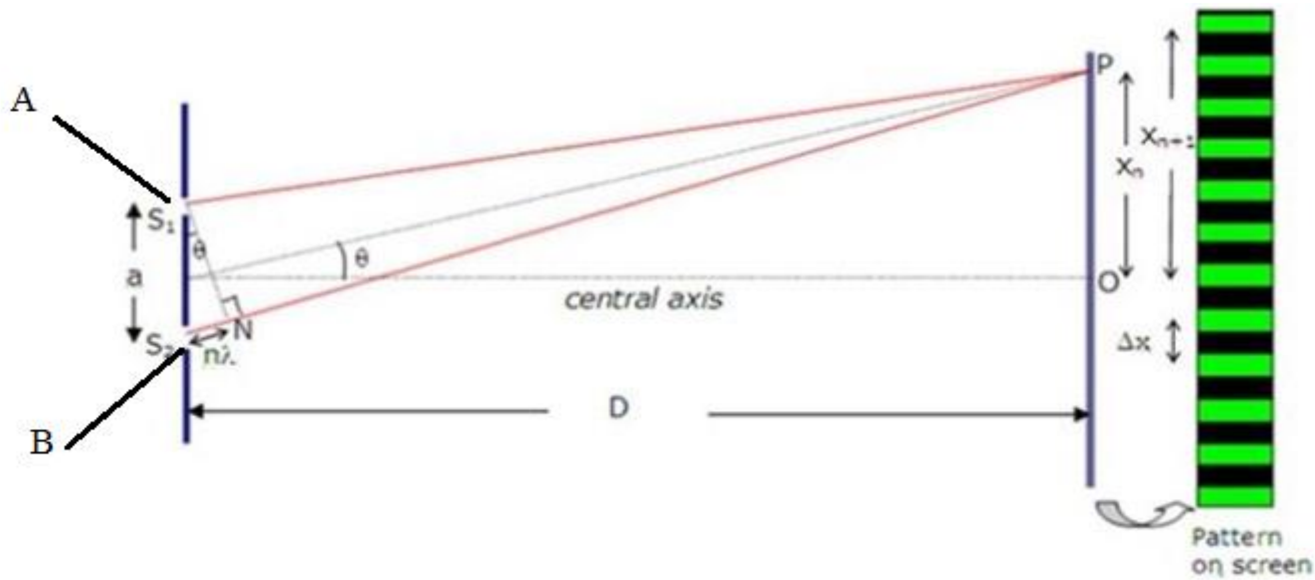
Example



When destructive interference occurs the two interfering waves cancel each other to produce nothing and hence a dark fringe is formed.

THEORY OF YOUNG'S DOUBLE SLIT EXPERIMENT

Consider the following below



Let A and B be two close slits separated by a distance, a

Let O be central bright fringe

Suppose the n^{th} bright fringe to be formed at point P a distance X_n from O.

CONCEPT

If the two coherent monochromatic light sources are A and B then a bright fringe can be seen at P only if the path difference (BP – AP) is a whole number of wavelength.

$BP - AP = n\lambda$

Where n = 0, 1, 2, 3, ………………..

(2) If the difference is an odd number of half wavelength then darkness is obtained at the point considered.

I.e. BP – AP = $\left[n+\frac{1}{2}\right]\lambda$

here n = 0, 1, 2, 3, ………………………

In the figure above, for a bright fringe to be seen at P, we have:

Path difference = BP – AP = $n\lambda$

- But, AP = NP

è Path difference = BP – NP = $n\lambda$

v BN = $n\lambda$ ------------ (1)

- Let θ be angular displacement of the n^{th} bright fright from central bright fringe.

- Consider âˆ†ABN and âˆ†PMO

è m(B$\acute{A}$N) = m(PMO) = θ

- From âˆ†BAN

Sin $\theta = \frac{BN}{AB}$

- Where $BN = n\gamma$ and AB = a

Sin θ = $\frac{n\lambda}{a}$

Since θ is very small angle sin θ ≈ θ (in radian)

v $\theta = \frac{n\lambda}{a}$ ------------- (2)

- From âˆ†PMO

è $\tan\theta = \frac{PO}{MO} = \frac{X_n}{D}$

- Where D = Distance from the slits to the screen

- Since θ is a very small angle tan $\theta \approx \theta$ (in radian)

$\theta = \frac{X_n}{D}$ ------------------- (3)

è equation (2) = equation (3)

$$\frac{n\lambda}{a} = \frac{X_n}{D}$$

$X_n = \frac{n\lambda D}{a}$ --------------------- (4)

- Where n = 0, 1, 2, 3, ……………….

- Equation (4) above gives the displacement of n^{th} bright fringe from central bright fringe at O.

NOTE.

- Central bright fringe, n= 0
- First bright fringe, n = 1
- Second bright fringe, n = 2

e.t.c

- Similarly, the displacement from O for a dark fringe is given by:

$X_n = \left[n + \frac{1}{2}\right] \lambda D$ -------------------- (5)

- First dark fringe, n = 0
- Second dark fringe, n = 1
- Third dark fringe, n = 2

e.t.c

DISTANCE BETWEEN TWO CONSECUTIVE BRIGHT / DARK FRINGES (FRINGE WIDTH)

Let X_n be displacement of n^{th} bright fringe from O.

$$\rightarrow X_n = \frac{n\lambda D}{a} \text{ ----------------- } (1)$$

- For the next bright fringe

$$X_{n+1} = (\frac{n+1}{a})\lambda D \text{ ---------------- } (2)$$

- The distance between two consecutive bright fringes (ω) is given by:

$$\omega = X_{n+1} - X_n$$

$$\omega = \frac{(n+1)\lambda D}{a} - \frac{n\lambda D}{a}$$

$$\omega = \frac{(n+1)\lambda D - n\lambda D}{a}$$

$$\omega = \frac{n\lambda D + \gamma D - n\lambda D}{a}$$

$$\omega = \frac{\lambda D}{a} \text{ -------------- } (3)$$

- The same formula applies for the separation between two consecutive dark fringes**.**

All fringes are of the same width.

SOME POINTS ABOUT YOUNG'S INTERFERENCE FRINGES

(1) If the source slit "S" is put very close to the double slits S_1 the fringe separation does not change but the fringe intensity will increases.

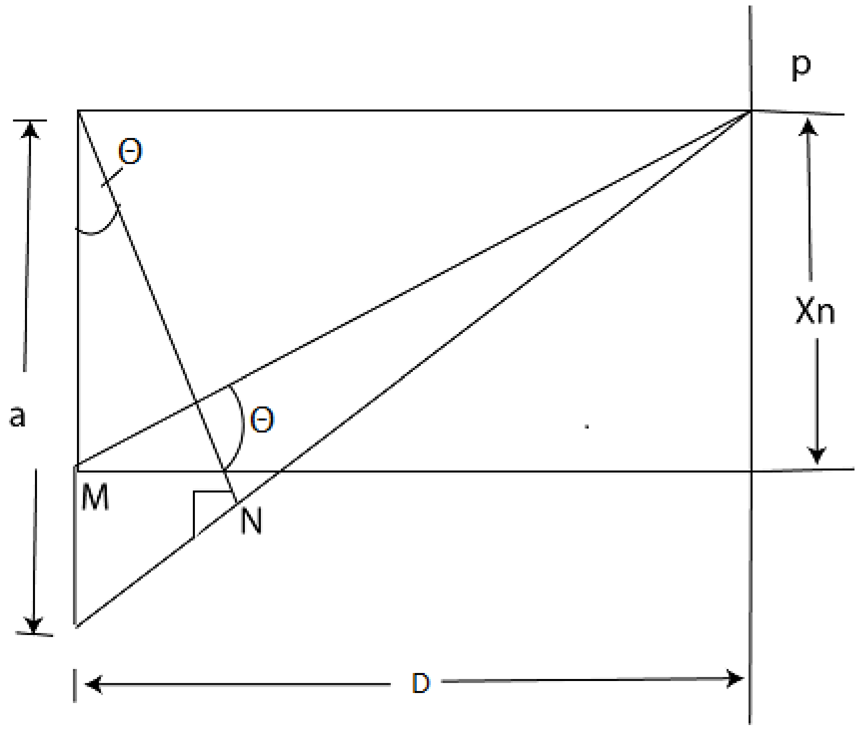
(2) If the double slits S_1 and S_2 are very close so that their separation distance approaches zero, then the fringe separation increases

(3) If one of the double slits is covered up, then no fringes can be seen on the screen.

(4) If the source slits S is made wider, then the fringes on the screen disappears.

(5) If white light is used instead of monochromatic light source then the central fringe at O is white but the other bright fringe on either sides of the central fringe are colored with violet near "O" and red far away from "O".

ANGULAR WIDTH OF A FRINGE



If P is the position of the first bright fringe then OP = $\omega =$ fringe width and θ = angular width of a fringe

$\tan\theta = \frac{\omega}{D}$

Since θ is very small, therefore tan $\theta \approx \theta$ (in radian)

$\theta = \frac{\omega}{D}$ ------------------- (1)

From the expression of fringe width ω

$\omega = \frac{\lambda D}{a}$

$\frac{\omega}{D} = \frac{\lambda}{a}$ -------------- (2)

Problem 37

Two slits are at a distance of 0.2mm apart and the screen is at a distance of 1m. The third bright fringe is found to be displaced 7.5mm from the central fringe. Find the wavelength of the light used.

Problem 38

A yellow light from a sodium vapour lamp of wavelength 5893$\mathring{A}$ is directed upon two narrow slits of 0.1cm apart. Find the position on the screen 100cm away from the slits.

Problem 39

In Young's experiment, the distance of the screen from the two slits is 1.0m. When light of wavelength 6000$\mathring{A}$ is allowed to fall on the slits the width of the fringes obtained on a screen is 2.0mm. Determine:

(i) The distance between the two slits and

(ii) The width of the fringes if the wavelength of the incident light is 4800$\mathring{A}$.

Problem 40

In double slit experiment, light has a frequency of $6 \times 10^{14} s^{-1}$. The distance between the centres of adjacent bright fringes is 0.75mm. What is the distance between the slits if the screen is 1.5m away?

Given that the speed of light in free space $3 \times 10^{8} ms^{-1}$.

Problem 41

In a Young's double slit experiment, two narrow slits 0.8mm apart are illuminated by the same source of yellow light ($\lambda = 5893\mathring{A}$). How far apart are the adjacent bright bands in the interference pattern observed on a screen 2m away?

Problem 42

In young's double slit experiment the angular width of a fringe formed on a distant screen is 0.1°. The wavelength of light used is 6000$\mathring{A}$. What is the spacing between the slits?

Problem 43

A beam of light consisting of two wavelength 6500$\mathring{A}$ and 5200$\mathring{A}$ is used to obtain interference fringes in a Young's double slit experiment.

(i) Find the distance of the third bright fringe on the screen from the central maximum to the wavelength 6500Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2mm and the distance between the plane of the slits and the screen is 120cm.

Problem 45

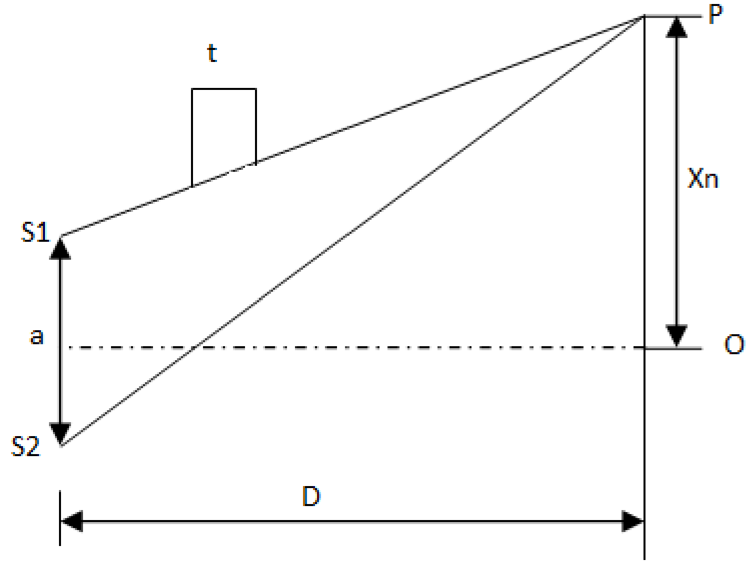
A Young's double slit arrangement produces interference fringe for sodium light ($\lambda = 5890Å$) that are 0.20° apart. What is the angular fringe separation if the entire arrangement is immersed in water? Given that refractive index of water = 4/3

Problem 46

In Young's double slit experiment the distance between the slits is 1mm and the distance of the screen from the slits is 1m. If light of wavelength 6000Å is used, then find the distance between the second dark fringe and the forth bright fringe.

FRINGE SHIFT

-When a thin transparent plate of thickness (t) and refractive index (μ) is introduced in the path of one of the interfering waves (say in the path S_1P.) then the effective path in air is increased by ($\mu - 1$)t due to the introduction of the plate.



- Effective path difference in air

$= S_2P - [S_1P + (\mu - 1)t]$

$= S_2P - S_1P - (\mu - 1)t$

- But $S_2P - S_1P = \frac{aX_n}{D}$

- Effective path difference in air

$= \frac{aX_n}{D} - (\mu - 1)t$

- For a bright fringe, path difference = $n\lambda$

è $n\lambda = \frac{aX_n}{D} - (\mu - 1)t$

è $n\lambda + (\mu - 1)t = \frac{aX_n}{D}$

v $X_n = \frac{D}{a}[(n\lambda + (\mu - 1)t]$ --------------- (1)

- For the next bright fringe, we have:-

$$X_{n+2} = \frac{D}{a}\,(n+1)\lambda = (\mu - 1)t$$

$$X_{n+1} = \frac{D}{a}\,[n\lambda + \lambda + (\mu - 1)t]$$ ----------------- (2)

- The fringe width ω is given by :

$$\omega = X_{n-1} - X_n$$

$$\omega = \frac{D}{a}\,[n\lambda + \lambda + (\mu - 1)t] - \frac{D}{a}\,[n\lambda + (\mu - 1)t]$$

$$\omega = \frac{D}{a}\,[n\lambda + \lambda + (\mu - 1)t - n\lambda - (\mu - 1)t]$$

v $\omega = \frac{\lambda D}{a}$ ------------------ (3)

- From equation (1)

$$X_n = \frac{D}{a}\,[n\lambda + (\mu - 1)t]$$

- In the absence of the plate t = 0

è $X_n = \frac{D}{a}\,[n\lambda + (\mu - 1) \times 0]$

❖ $X_n = \frac{n\lambda D}{a}$ ---------------------- (4)

- Let â ̂†X be the fringe shift due to introduction of the plate.

$$\Delta X = equation\ (1) - equation\ (4)$$

- â ̂†X = $\frac{D}{a}\,[n\lambda + (\mu - 1)t] - \frac{n\lambda D}{a}$

$$\Delta X = \frac{n\lambda D}{a} + \frac{D(\mu-1)t}{a} - \frac{n\lambda D}{a}$$

❖ $\Delta X = \frac{D}{a}(\mu - 1)t$ --------------------- (5)

- From equation (5)

$$\omega = \frac{\lambda D}{a}$$

v $\frac{D}{a} = \frac{\omega}{\lambda}$ -------------------------------- (6)

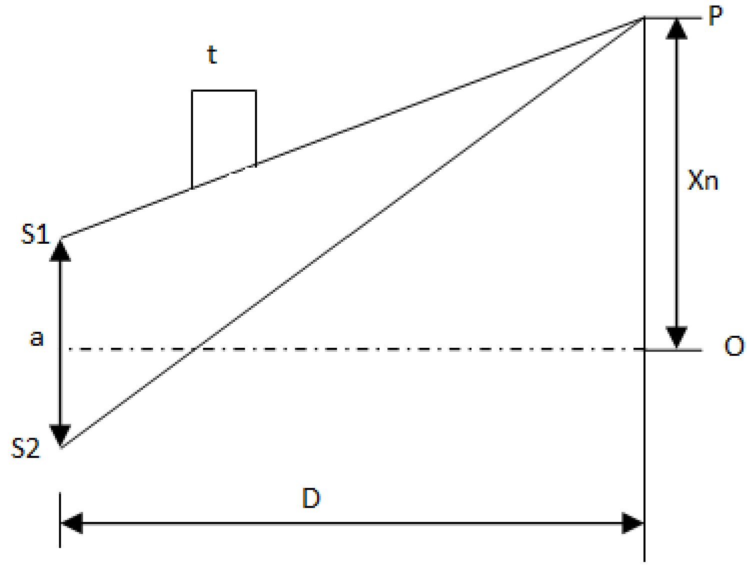
- Substitute equation (6) in equation (5)

$$\Delta X = \frac{\omega}{\lambda}(\mu - 1)t \text{ - - - - - - - - - - - - - . (7)}$$

Thus, with the introduction of the transparent plate in the path of one of the slits, the entire fringe pattern is displaced through a distance ΔX given by either equation (5) or equation (7) towards the side on which the plates is placed.

Problem 47

In the figure below S_1 *and* S_2 are two coherent light sources in a Young's two slit experiment separated by a distance 0.5mm and O is a point equidistant from S_1 *and* S_2 at a distance 0.8m from the slits. When a thin parallel sided piece of glass (G) of thickness 3.6 $\times 10^{-6}$ m is placed near S_1 as shown, the central fringe system moves from O to point P. Calculate OP. (the wavelength of light used = 6.0 $\times 10^{-7}$ m).



CONDITIONS NECESSARY FOR SUSTAINED INTERFERENCE OF LIGHT WAVES

- Sustained interference is an interference pattern in which the positions of maxima and minima remain fixed.
- In order for light wave to produce an interference pattern which can be observed on a screen the following condition must be satisfied.

(1)The light should be monochromatic.

If this is not so, fringe of different color will overlap.

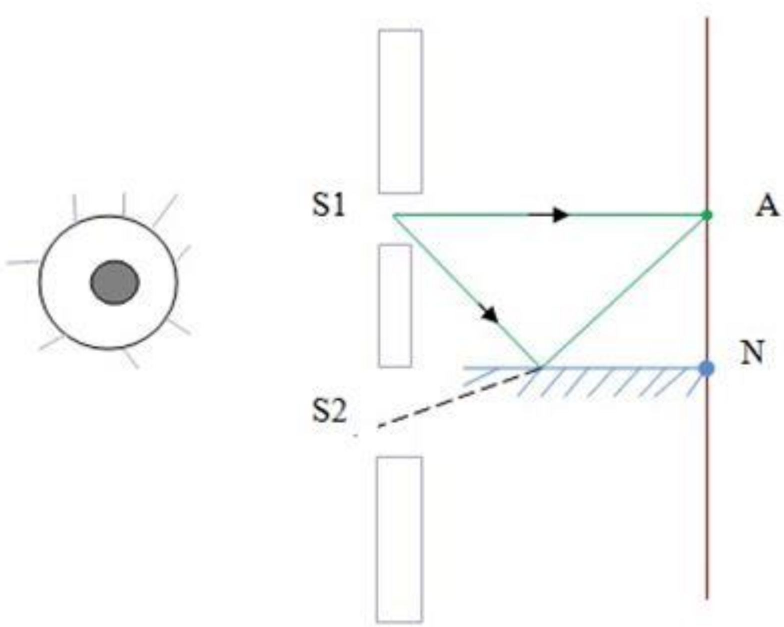
(2)The two sources producing interference must be coherent.

(3)The two interfering waves must have the same plane of polarization.

(4)To observe interference fringe clearly. It necessary that the fringe width is sufficiently large. This is possible if:

i) The two coherent sources are parallel and close to each other

ii) The distance between slits and screen is reasonably large.



A slit S_1 illuminated by monochromatic light and placed closed to a plane mirror MN

Interference occurs between direct light from the slit and light reflected from the mirror MN

The reflected ray MA appear to be coming from a virtual image S_2 of S1

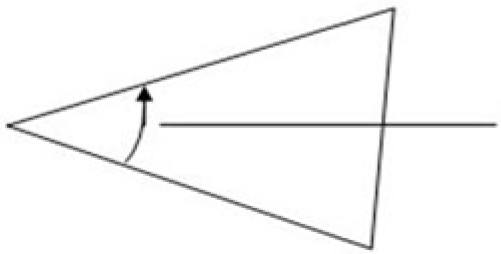
S_1 and S_2 act as two coherent sources of, the expression giving the fringe spacing is the same as for the double slit

$$\left[\omega = \frac{\lambda D}{a}\right]$$

Except that dark fringe will be obtained at N instead of a bright fringe

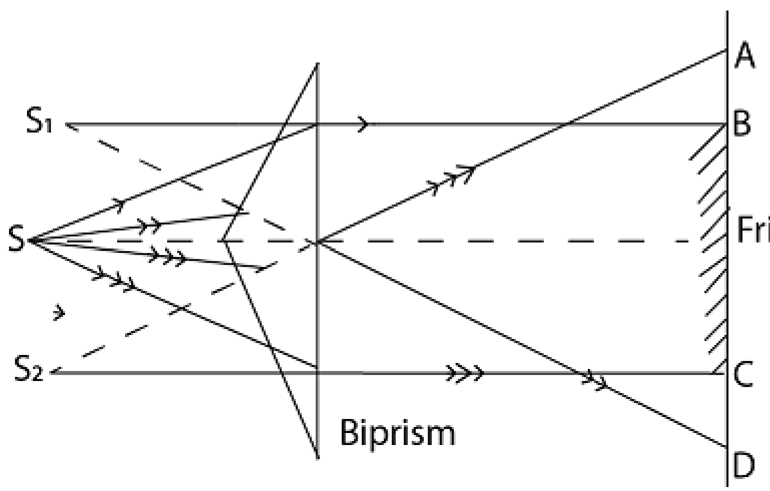
Bi prism

This is a glass prism with an obtuse angle that functions as two acute angle prisms placed.



A double image of a single object is always formed by means of this prism

The device was used by Fresnel to produce two coherent beams for interference experiment



Monochromatic light from a narrow slit S falls on a bi prism as shown in the figure above.

Two virtual images S_1 and S_2 of S are formed by refraction at each half of the bi prism and these acts as coherent source which are close together.

An interference pattern, similar to that given by the double slit but brighter, is obtained in the shaded region where the two refracted beans overlap.

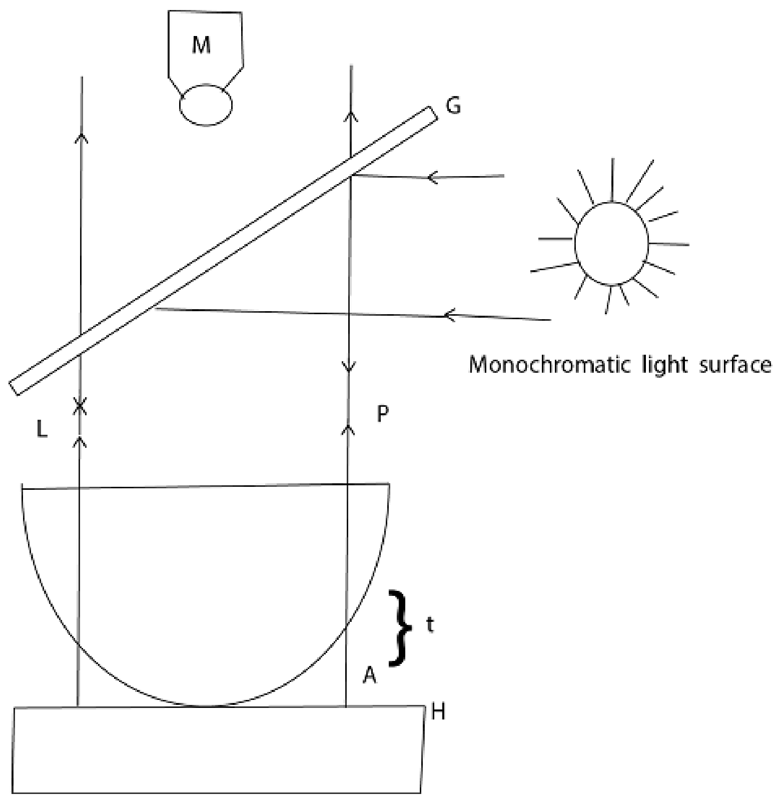
The theory and the expression for the fringes spacing are the same as for young's method

$$\left[\omega = \frac{\lambda D}{a}\right]$$

NEWTON'S RINGS

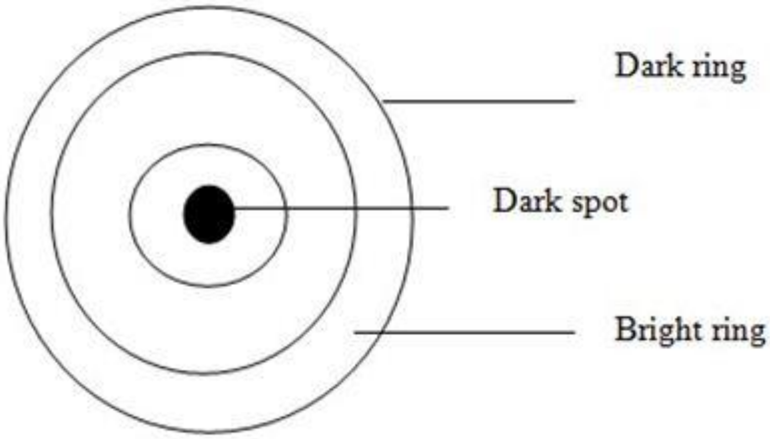
This is an interference effect discovered by Newton's

Interference fringes are formed by placing a slightly Plano-convex lens on a flat glass plate H



A glass plate G reflects the light that comes from the monochromatic light source towards the lens L downwards

Both reflected rays are going to be analyzed by the traveling microscope M and at the point of contact of the lens a dark spot is seen surrounded by a series of alternate bright and dark rings. These are called Newton's rings.



Definition

Newton's ring is a circular interference fringes formed between a lens and a glass plate with which the lens is contact.
Concept

Consider the air film PA t

Some of the incident light is reflected at P towards the microscope M

Some of the light passes to point A where it is reflected by glass plate H towards the microscope M.

For n^{th} dark ring Path difference = 2t = nλ

When n 0, 1, 2, 3,

Central dark spot, n 0

At constant pressure, $\frac{k}{P\pi d^2\sqrt{2}}$

i.e. At constant pressure the mean free path of a gas molecule

$\lambda \propto T$

At constant pressure the mean free path of a gas molecule is directly proportional to its absolute temperature T.

$\lambda = KT$

Where K constant of proportionality

If λ_1 and λ_2 are mean free path of a gas molecule at temperature T_1 and T_2 respectively:

And $\lambda_1 = KT_1$

$\lambda_2 = KT_2$

2. Pressure P of the gas

From equation (3)

$$\lambda = \frac{KT}{P\pi d^2\sqrt{2}}$$

$$\lambda = \frac{KT}{\pi d^2\sqrt{2}}.\frac{1}{P}$$

At constant temperature, ${KT}/{\pi d^2\sqrt{2}} = constant$

$$\therefore \lambda = \frac{1}{P}$$

At constant temperature the mean free path of a gas molecule is inversely proportional to pressure.

$P\lambda = K$

Where K constant of proportionality

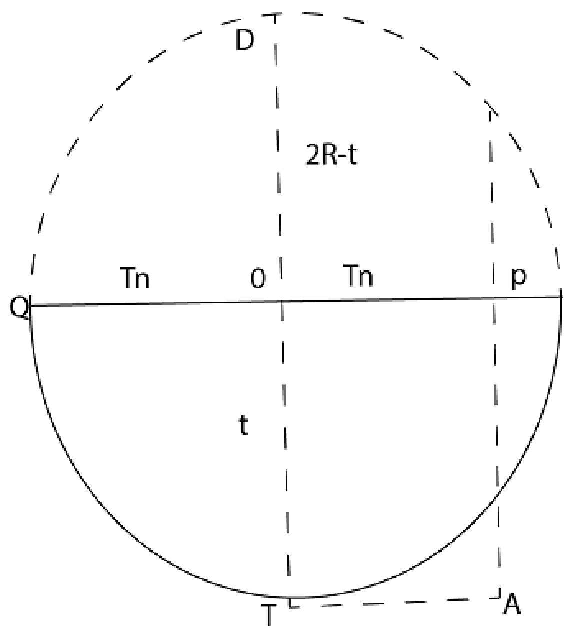
Generally, we may write

$P_1\lambda_1 = \lambda_2 P_2$

$$equation\ (i) / equation\ (ii) \qquad \lambda_1 / \lambda_2 = KT_1 / KT_2$$

MEASUREMENT OF WAVE LENGTH BY NEWTON'S RINGS EXPERIMENT

è The effect can be used to determine the wavelength of a light source if the radius of curvature R, of the lower surface of the lens is known:-



- Where r_n = Radius of the n^{th} ring

t = PA = air film

R = Radius of curvature of the lens

è Diameter. D = 2R→ $\overline{OD} = 2R - t$

Applying the theorem of intersecting chords

$$\overline{TO} \times \overline{OD} = \overline{OQ} \times \overline{0P}$$

$$t \times (2R - t) = r_n \times r_n$$

$$2tR - t^2 = r_n^2$$

Assuming the lens to have large radius of curvature R then t is very small and hence t^2 is very very small $i.e. t^2 \to 0$

2tR = r_n^2

2t = $\frac{r_n^2}{T}$ = pass difference

- **For a bright ring:** Path difference = 2t = ($n + \frac{1}{2})\lambda = \frac{r_n^2}{R}$

v

$$\left[n + \frac{1}{2}\right]\lambda = \frac{r_n^2}{R} \quad \text{----------------------- (1)}$$

- Where n = 0, 1, 2, 3,…………
- First bright ring, n = 0

Second bright ring, n = 1 etc.

For a dark ring:

Path difference = 2t = $n\lambda = \frac{r_n^2}{R}$

$$n\lambda = \frac{r_n^2}{R}$$

- Where n = 0, 1, 2, 3, ………..
- Central dark spot, n = 0

- First dark ring, n = 1
- Second dark ring, n = 2 etc.

Problem 48

A set of Newton's ring produced by monochromatic light is viewed through a traveling microscope. The diameter of the n^{th} and $(n + 5)^{th}$ dark rings are found to be 0.143cm and 0.287cm respectively. The radius of the curvature of Plano convex lens used is 64.5cm.

Determine the wavelength of the light used.

Problem 49

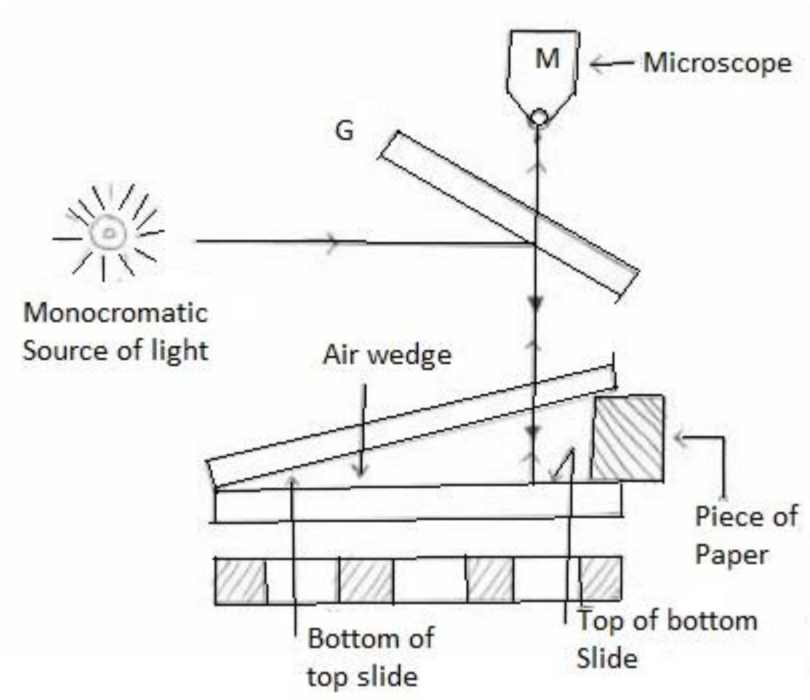
Calculate the radius of curvature of a Plano convex lens used to produce Newton's rings with a flat glass plate if the diameter of the tenth dark ring is 4.48mm, viewed by normally reflected light of wavelength 5.0×10^{-7} m. What is the diameter of the twentieth bright ring?

WEDGE FRINGES

This is another form of interference produced by a thin wedge shaped film of air, the thickness of which gradually increases from zero along its length.

HOW A WEDGE IS FORMED

The wedge can be formed from two microscope slides clamped at one end and separated by a thin piece of paper at the other end so that the wedge angle is very small.



Monochromatic light from an extended source is partially reflected vertically downwards by the glass plate G.

When the microscope is focused on the wedge, bright and dark equally spaced fringes are seen parallel to the edge of contact of the wedge.

Some of the light falling on the wedge is reflected upwards from the bottom surface of the top slide and the rest which is transmitted through the wedge is reflected upwards from the top surface of the bottom slide.

Let "l" be thickness of the air wedge at P.

If a dark fringe is formed at P then:

Path difference between the rays at

P = 2l = $n\lambda$

$$l = \frac{n\lambda}{2} \quad \text{.........................(i)}$$

- Where n = 0, 1, 2, 3, …………………
- First bright fringe, n = 0
- Second bright fringe, n = 1

DETERMINATION OF ANGLE OF THE WEDGE

In the figure above, let θ be the angle between the slides/plates (=wedge angle).

For a dark fringe at P we have:

è $2l = n\lambda$

If θ is in radians then we have:

$$l = S_1\theta$$

è $2S_1\theta = n\lambda$ ---------------------- (3)

- For the (n + K$)^{th}$ dark fringe at Q, we have:

$$2S_1\theta = (n+k)\lambda \text{ ----------------- } (4)$$

→ $eqn\ (4) - eqn(3)$

→ $2S_2\theta - 2S_1\theta = (n+k)\lambda - n\lambda$

→ $2\theta(S_2 - S_1) = n\lambda + k\lambda - n\lambda$

→ $2\theta(S_2 - S_1) = K\lambda$

$$\theta = \frac{k\lambda}{2(S_2 - S_1)}$$

è here $S_2 - S_1$ = Distance moved by the microscope

λ = wavelength of the light used.

Special case

If K = 1, then $S_2 - S_1 = \omega =$ fringe width.

$\theta = \frac{1 \times \gamma}{2\omega}$

❖ $$\theta = \frac{\lambda}{2\omega}$$ ---------------- (

Problem 50

An air wedge is formed between two glass plates which are in contact at one end and separated by a piece of thin metal foil at the other end. Calculate the thickness of the foil if 30 dark fringes are observed between the ends when light of wavelength 6×10^{-7} m incident normally on the wedge.

Problem 51

A wedge air film is formed by placing aluminum foil between the two glass slides at a distance of 75mm from the line of constant of the slides. When the air wedge is illuminated normally by a light of wavelength 5.6×10^{-7} m interference fringe are produce parallel to the line of contact which have a separation of 1.2mm. Calculate the angle of the wedge and the thickness of the foil.

Problem 52

A narrow wedge of air is enclosed between two glass plates. When the wedge is illuminated by a beam of monochromatic light of wavelength 6.4×10^{-7} m parallel fringes are produced which are 0.3mm apart. Find the angle of the wedge.

Problem 53

A wedge shaped film of air is formed between two parallel sided glass plates by means of a straight piece of wire. The two plates are in contact along one edge of the film and the wire is parallel to this wedge. In such experiment, using light of wavelength 589nm, the distance between the 7th and 169th dark fringes was 26.3mm and the distance between the junction of the glass plates and the wire was 35.6mm. Calculate the angle of the wedge and the diameter of the wire.

Problem 54

An air wedge is formed between two flat glass plates of length 45mm by using a spacer at one end. Wedge fringes spaced 0.29mm apart are observed by reflection when light of wavelength 430nm is directed normally at the wedge. Calculate the angle of the wedge and the thickness of the spacer.

APPLICATION OF INTERFERENCE

(1 (1) QUALITY TESTING OF OPTICAL SURFACES

Fringes of equal thickness are useful for testing the quality of optical components.

Example

In making of optical flats the plate under test is made to form an air wedge with a standard plane glass surface.

Any uneven part of the surface which require more grinding will show up irregularities. The grinding of a lens surface may be checked if it is placed on an optical flat and Newton's rings observed in monochromatic light.

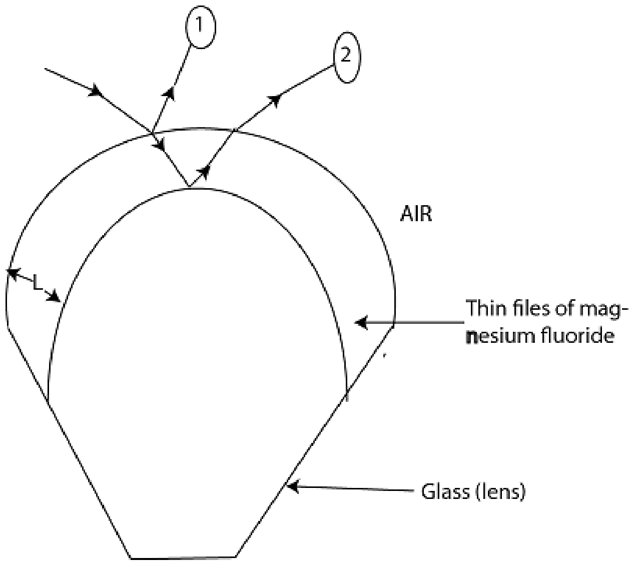
(2) MAKING NON-REFLECTING GLASS / BLOOMING LENSES

In optical instruments containing lenses or prisms light is lost by reflection at each refracting surface which results in reduced brightness of the final image.

The amount of light reflected at the surface can be appreciably reduced by coating it with a film of transparent material example magnesium fluoride. The process is called Blooming**.**

Blooming is the process of depositing a transparent film of a substance such as magnesium fluoride on a lens to reduce (or eliminate) the reflection of the light at the surface.

è The film is about one-quarter of a wavelength thick and has a lower refractive index than the lens.



Light reflected from the top (ray 1) and from the bottom (ray 2) surfaces of the film interfere destructively.

Condition

Destructive interference occurs when:

Path difference = $2l = \frac{1\lambda_2}{2}$

This is because the phase change which occur by reflection is 180° which is equivalent to $\frac{1\lambda_2}{2}$

v

❖ $l = \frac{1\lambda_2}{2}$ ---------------------- (1)

- Where l = thickness of the film

When light passes from one medium to another its speed and wavelength change but frequency (f) remain unchanged.

If V_1 and V_2 are the speeds of light in air and in the film respectively then:

$$V_1 = \lambda_1 f \quad and \quad V_2 = \lambda_2 f$$

- Let n be refraction index of the material of the film.

è $n = \frac{V_1}{V_2} = \frac{\lambda_1 f}{\lambda_2}$

è $n = \frac{\lambda_1}{\lambda_2}$

$$\lambda_2 = \frac{\lambda_1}{n} \text{ - - - - - - - - - - (2)}$$

- Substitute equation (2) in equation (1)

$$l = \frac{1}{4} . \frac{\lambda_1}{n}$$

v

$$l = \frac{\lambda_1}{4n} \text{ ---------------- (3)}$$

Problem 56

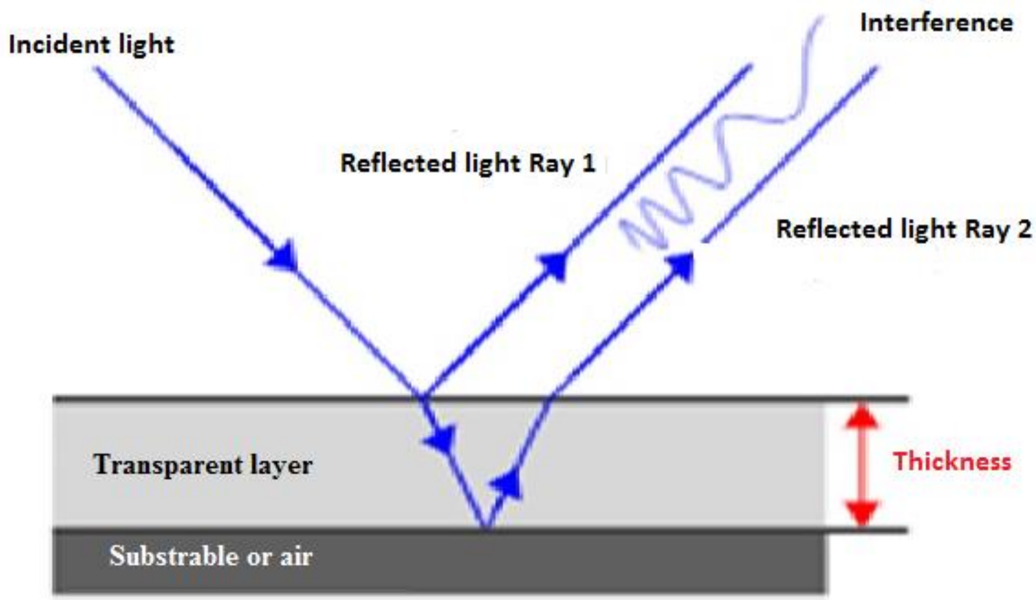
Expensive cameras have non-reflecting coated lens. Normally the lens surface is covered with a thin transparent film of fluoride of refractive index 1.43. suppose that no light of wave length 6000 $\overset{0}{A}$ is reflected when the incidence is normally. Calculate the thickness of the film.

COLOUR IN THIN FILMS OF OIL / SOAP BUBBLE

When a drop of oil spreads on water a thin film of oil is formed on its surface.

In broad day light, the film appears to be made up of beautiful rainbow colors.

The formation of these colors in a thin film is due to interference phenomenon.



The interference phenomenon is the effect which happens when light wave gate reflected from the two opposite surfaces of a thin film.

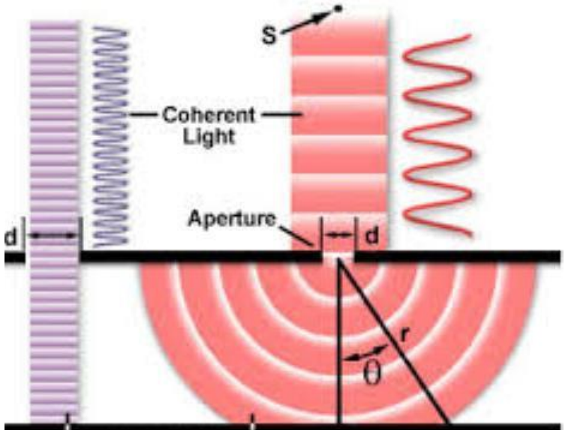
When the path difference gives constructive interference for light of wavelength, the corresponding color is seen in the film.

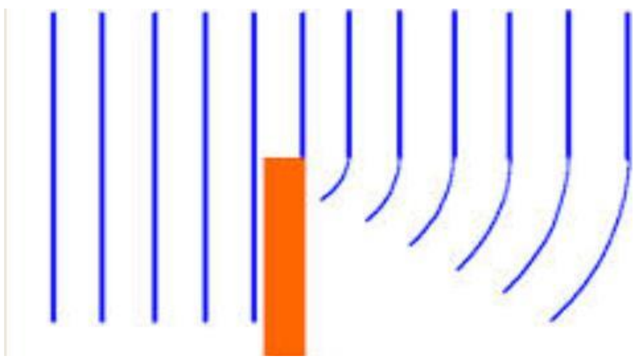
The path difference varies with the thickness of the film and the angle of viewing both of which affect the color produced at any one part.

The colours seen in soup bubbles are also produced in this way.

DIFFRACTION OF LIGHT WAVES

Diffraction of light is the phenomenon of spreading or bending of light waves as they pass through a narrow opening (aperture) or round the edge of a barrier.





The diffracted waves subsequently interfere with each other producing regions of reinforcement and weakening

- Behind the obstacles / apertures at which diffraction occurs, a diffraction pattern of dark and bright fringes is formed.

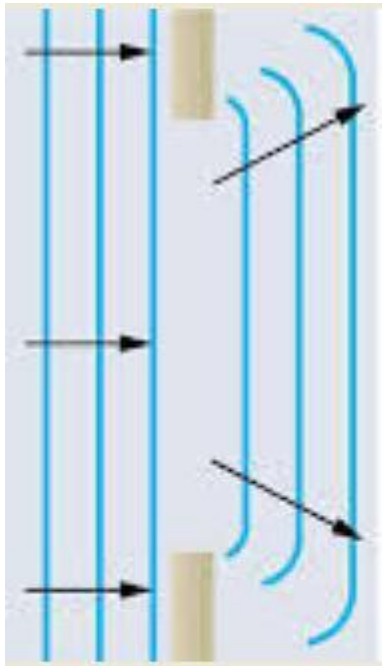
- Diffraction is regarded as being due to the superposition of secondary wavelets from coherent sources on the unrestricted part of a wave front that has been obstructed by an obstacle aperture.

DIFFRACTION EFFECTS

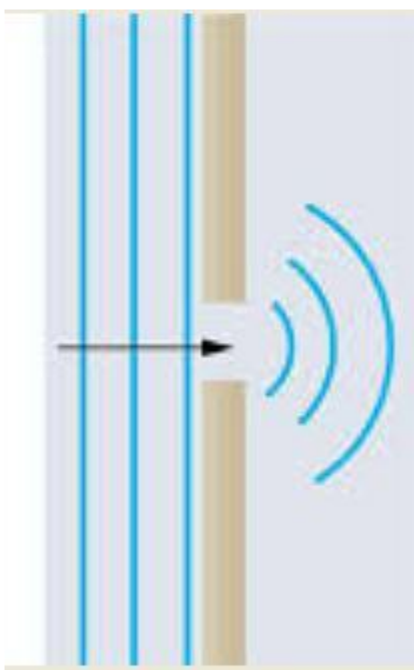
- Diffraction of light is quite pronounced when the width of the opening is comparable with the wavelength of the light.

- This can be illustrated below

(i)

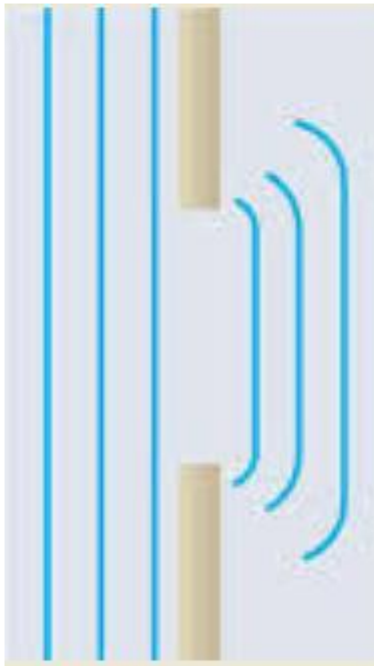


(ii)

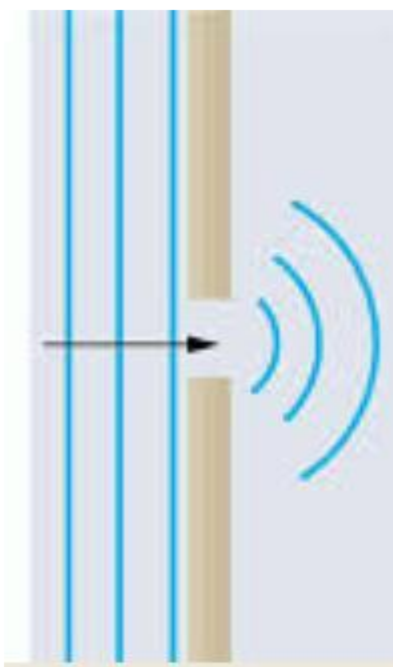


- In figure (i) and (ii) above the apertures are of the same size but the wave length of the incident waves are different.

- The waves are diffracted more when the wavelength is longer



(iii)



(iv)

In figure (iii) and (iv) the two waves have the same wavelength but the sizes of the apertures are different.

The diffraction is more in case of narrow slit.

TYPES OF DIFFRACTION

Diffraction of light is of two types

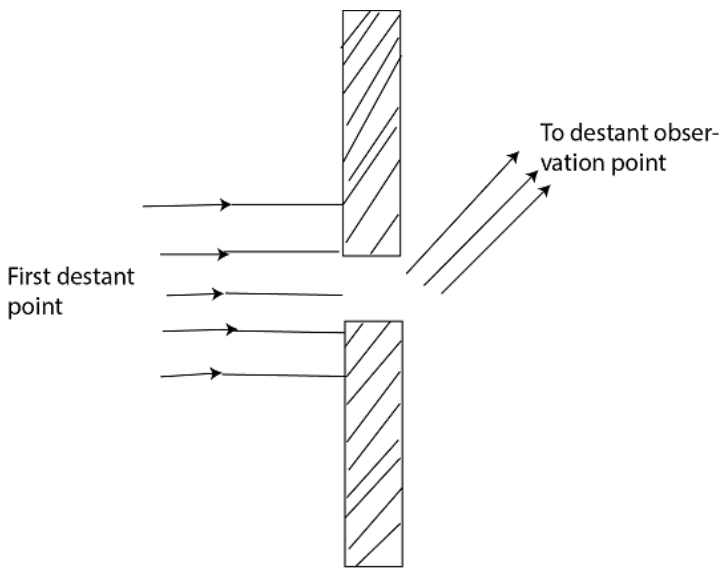
(i) Fraunhofer diffraction

(ii) Fresnel diffraction

FRAUNHOFER DIFFRACTION

This is the type of diffraction which takes place at a narrow slit when parallel rays of light (plane wave front) are incident on it.

Both the light source and the receiving screen should be at infinite distance from the narrow slit.

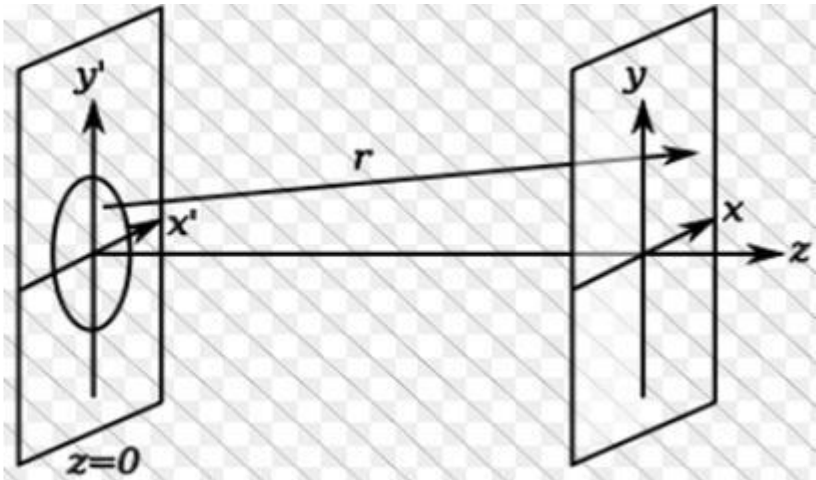


Here a lens is used to obtain the diffraction pattern on a screen placed at a distance equals focal length (f) of the lens.

FRENSEL DIFFRACTION

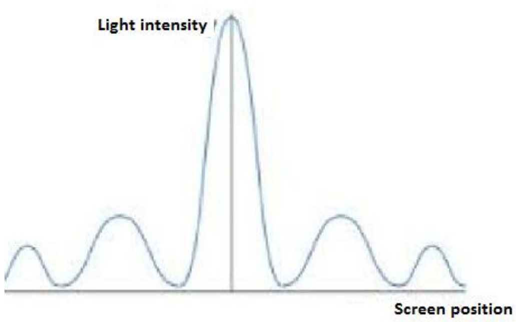
This is a type of diffraction which takes place at a narrow slit when non-parallel rays of light are incident on it.

In this type of diffraction either the light source or the receiving screen or both are at finite distances from the narrow slit.



DIFFRACTION OF LIGHT AT A SINGLE SLIT

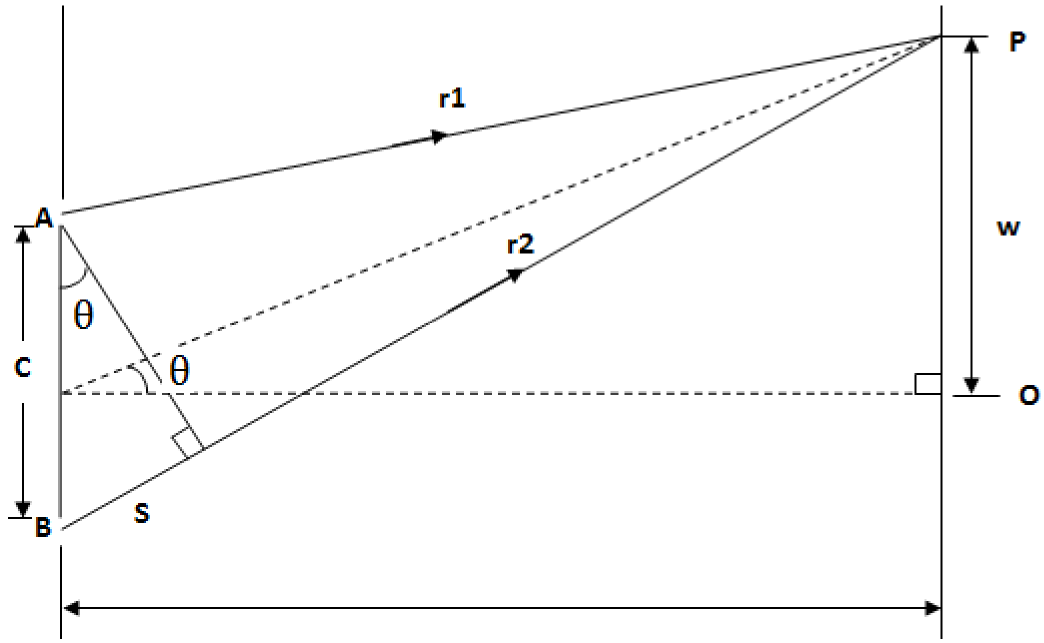
If a plane wave front is incident on a slit AB, then there will be the formation of a bright band on a screen at the centre followed by dark and bright bands of decreasing intensity on either sides of the central bright band.



A graph of fringe brightness against position is as shown in the figure above.

-

- Consider a plane wave front of monochromatic light of wavelength to be incident on a slit AB



- By Huygen's principle, every point on the portion AB of the wavefront acts as a source of secondary wavelets spreading out in all directions.

- Those secondary wavelets which go straight across the slits arrive on the screen at point O in the same phase and hence the intensity at point O is maximum.

- Consider the 1st dark band (1st minimum) to be formed at any angle θ to the direction of the incident beam.

- 1st dark band (1st minimum) is obtained on the screen at P where:

Path difference, $\overline{BN} = \frac{1\lambda}{2}$

From triangle ABN

Sin $\theta = \frac{\overline{BN}}{BC}$

But, BN = $\frac{1\lambda}{2}$ $\quad and \quad BC = \frac{\omega}{2}$

Sin $\theta = \frac{\frac{1\lambda}{2}}{\frac{\omega}{2}}$

❖ $$\sin\theta = \frac{\lambda}{\omega}$$ -------------------- (2)

For 1st dark band (1st minimum)

- Therefore, the general condition for a minimum for a single slit diffraction is

$$\sin\theta = \frac{n\lambda}{\omega}$$ -------------------- (2)

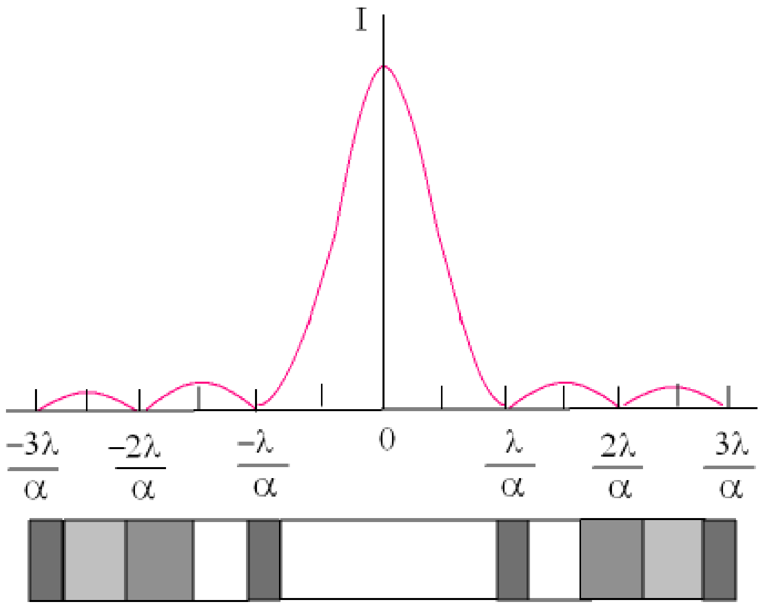
OR

$$\omega = \frac{n\lambda}{\sin\theta}$$ ------------------- (3)

Where n = $\pm$1, $\pm$2, $\pm$3, …………

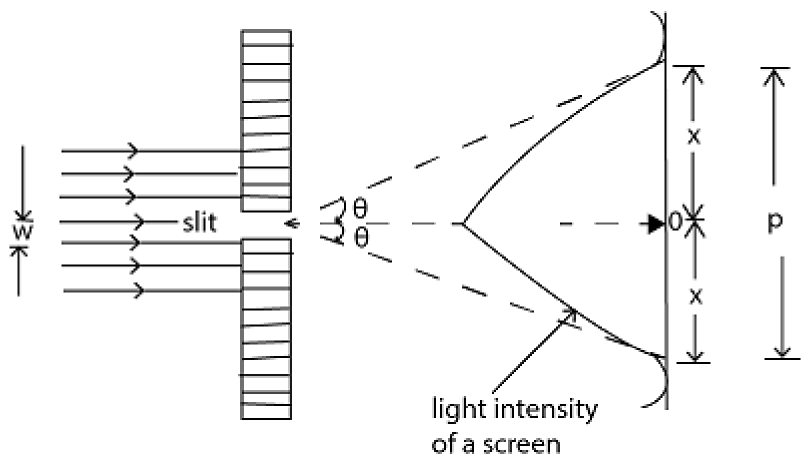
The $\pm$ sign indicate that there are two nth order minima, one on each side of the original direction beam.

The variation of intensity with angle of direction for a single slit diffraction pattern is a shown below.



Single-slit (Fraunhofer) Diffraction

EXPRESSION OF WIDTH OF CENTRAL MAXIMUM



- The width of the central maximum is the distance between the first minimum on the either side of O as shown in the figure above.

- Thus, the width of the central maximum is

$$2x = P$$

- The 1st minimum occurs at

Sin $\theta = \frac{\gamma}{\omega}$

- If θ is small then sin $\theta \approx 0$ in radian

$\theta = \frac{\gamma}{\omega}$ ----------------- (1)

- From the figure above:

Tan $\theta = \frac{X}{D}$

$\theta = \frac{X}{D}$ --------------------- (2)

è equation (1) = equation (2)

$\frac{X}{\omega} = \frac{X}{D}$

v $X = \frac{\lambda D}{\omega}$ --------------------- (3)

- Now P = 2K

$P = \frac{2\lambda D}{\omega}$ ------------ (4)

Problem 57

Monochromatic light of wavelength 6.0 $\times 10^{-7}$ m is incident at a single slit of width 5.0 $\times 10^{-6}$ m. How many orders of diffraction minimum are visible?

Problem 58

A parallel beam of light of wavelength 650nm is directed normally at a single slit of width 0,14mm in a darkened room. A screen is placed 1.50m from the single slit.

(a) Sketch the graph to show how the intensity of light falling on the screen varies with position across the screen.

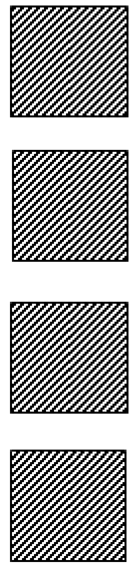
(b) Calculate the width of the central fringe.

DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION FRINGES

INTERFERENCE	DIFFRACTION
(1)Interference fringe are obtained due to the superposition of light coming from two different wave fronts originating from two coherent source	(1)Diffraction fringe are obtained due to the superposition of light coming from different parts of the same wave front.
(2)The width of interference fringe is generally the same.	(2)The width of diffraction fringes is not the same.
(3)The intensity of all bright fringes is the same.	(3)The intensity of all bright fringes is not the same. It is not the same. It is maximum for central fringes and decreases sharply for first, second etc. bright fringes.

PLANE TRANSMISSION DIFFRACTION GRATING

- -This is a device consisting of a large number of equidistant closely spaced parallel lines of equal width rule on glass.
- -The ruled widths are opaque to light while the space between any two successive lines is transparent and act as parallel slits.



Application

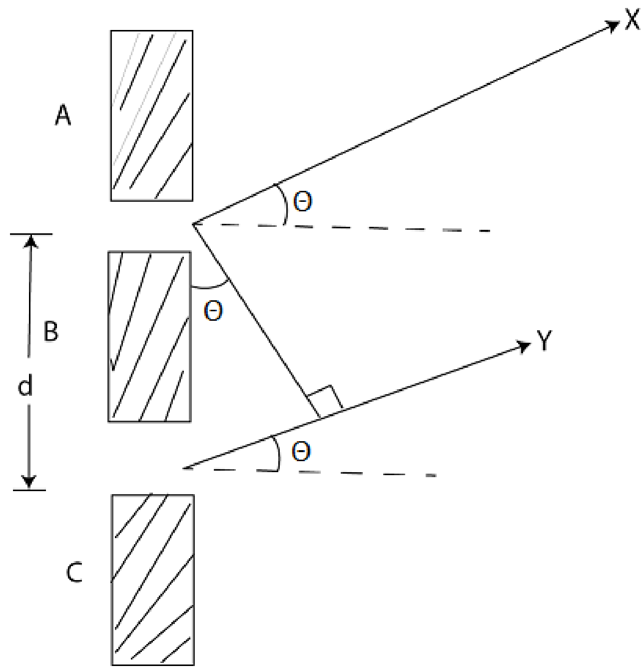
Diffraction gratings are used for producing spectra and for measuring wavelengths accurately.

THEORY

Suppose plane waves of monochromatic light of wave length fall on a transmission grating in which the slit separation (called grating spacing) is d.

Let N be number of lines per meter of the grating (= Grating Constant)

$$\therefore Grating\ spacing = {}^{1}/_{N}$$



- Consider wavelets coming from corresponding point A and B on two successive slits and travelling at an angle to the direction of the incident beam.

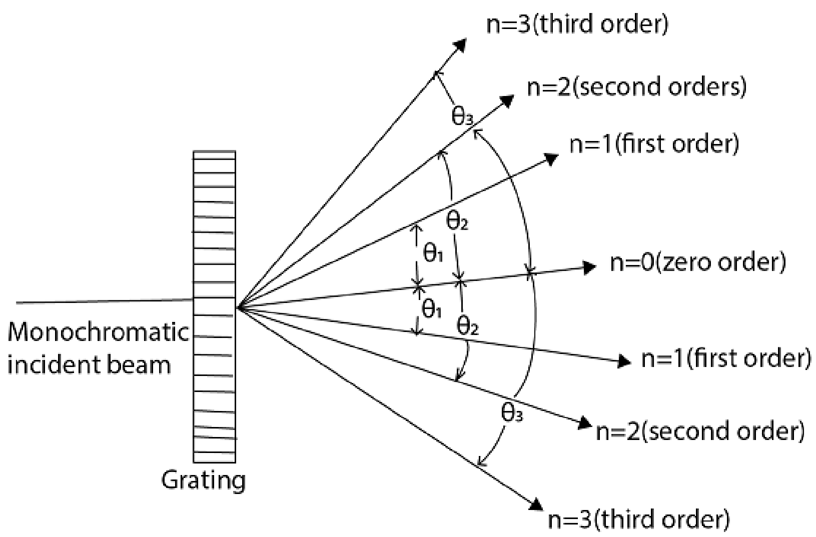
è Path difference, BC = d $\sin\theta = n\lambda$

v $$d\sin\theta = n\lambda$$

Where "n" is an integer giving the order of the spectrum.

Here reinforcement of the diffracted wavelets occurs in the direction θ and a maximum (bright band) will be obtained.

The angular distribution of diffracted beams is as shown below:



NOTE

The actual number of orders possible for a given grating will depend on the width of the grating spaced and the wavelength of the light used.

Since $sin\theta$ cannot be greater than (90° diffraction), then the maximum number of orders possible for any given wavelength cannot be greater than the whole number value of n given by

$$n\lambda = d\sin\theta \;=\; 1$$

"n" is maximum when sin θ = 1

è $$n\lambda \;= d$$

$n = \frac{d}{\lambda} \;=$ Maximum number of orders possible

Problem 59

Monochromatic light of wavelength 600nm is incident normally on an optical transmission grating of spacing 2.00$\mu m.$ Calculate:

(a) The angular position of the maxima

(b) The number of diffracted beams which can be observed

(c) The maximum order possible

Problem 60

The limits of the visible spectrum are approximately 400nm to 700nm. Find the angular spread of the first -order visible spectrum produced by a plane grating having 6000 lines per centimeter when light is incident normally on the grating.

Problem 61

A grating has 500 lines per millimeter and is illuminated normally with monochromatic light of wavelength 5.89×10^{-7} m.

(a) How many diffraction maxima may be observed?

(b) Calculate the angular separation.

Problem 62

What is the angular width of the first order spectrum produced by a diffraction grating of 5000 per cm when a parallel beam of while light is incident normally on it?

Problem 63

A monochromatic light of wavelength $2 \times 10^{-7} m$ falls normally on a grating which has 4×10^{3} lines per cm.

(a) What is largest order of spectrum that can be visible?

(b) Find the angular separation between the third and fourth order image.

Problem 64

A diffraction grating with grating of 2×10^{-6} m is used to examine the light from the glaring gas. It is found that the first order of violet light emerges at an angle of 11.8° and the first order of red light emerges at angle of 15.8°. Calculate:

(a) The wavelength of the two lights.

(b)The angle at which nth order of red light will coincide with the $(n+1)^{th}$ order of violet light.

Problem 65

A diffraction grating has 500 lines per mm when used with monochromatic light of $= 6 \times 10^{-7}$ m at normal incidence. At what angles will bright diffraction images be observed?

Problem 66

A slit 0.1mm wide is illuminated with a monochromatic light of wavelength 5000Å. How wide is the central maximum on a screen 1m from the slits?

Problem 67

A single narrow slits is illuminated with red light of wavelength 6328Å. A screen placed 1.60m from the slits shows a typical single –slit diffraction pattern. The separation between the first two minima is 4.0mm. what is the width of the slits?

Problem 68

Light of wavelength 7500Å passes through a slit of 1.0 $\times 10^{-3}$ mm wide. How wide is the central maximum

(i) In degrees and

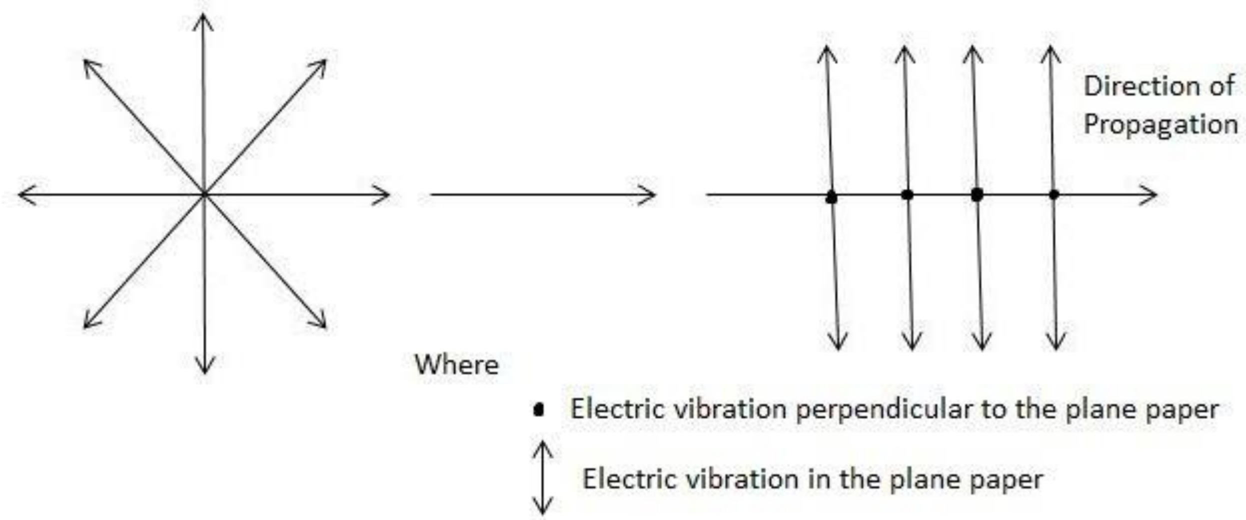
(ii) In cm on a screen 20cm away?

POLARIZATION OF LIGHT WAVES

Light is an electromagnetic wave whose electric and magnetic vibrations are perpendicular to each other and to the direction of propagation.

Polarization of light is the process of confining the vibrations of the electric vector of light waves to one direction.

- In unpolarized light the electric field vibrates in all directions perpendicular to the direction of a wave .
- The commonly used pictorial representation of an unpolarized light wave is as shown below



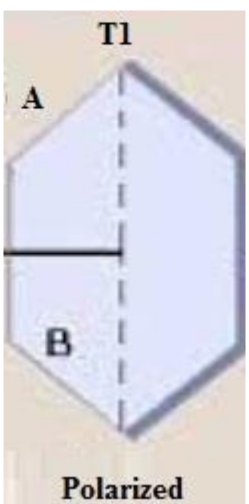
Unpolarized light beam is equivalent to two equally intense beams whose planes of vibration are perpendicular to each other.

After reflection or transmission through certain substances the electric field is confined to the direction and the radiation is said to be plane – polarized light.

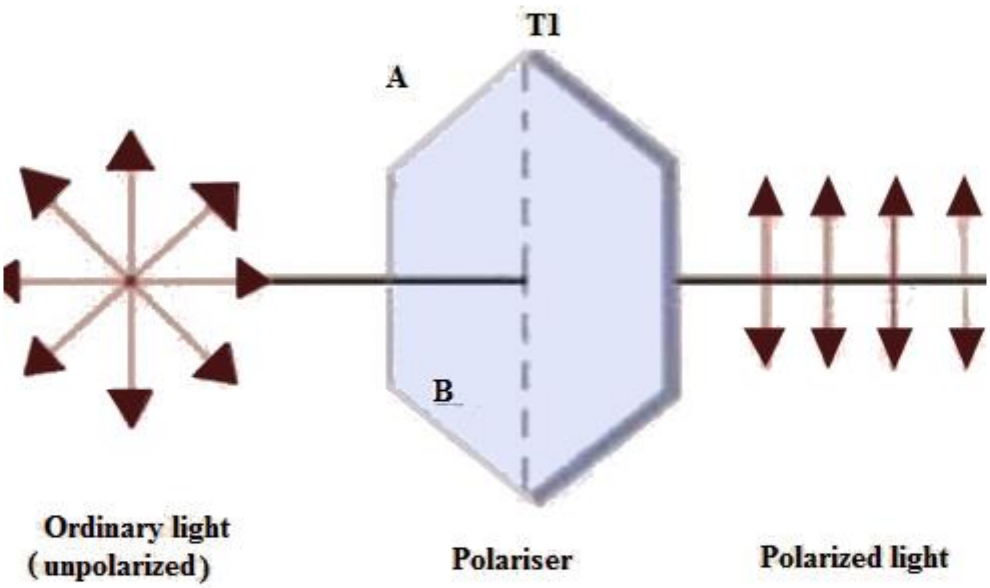
POLAROID

This is a device used to produce plane polarized light.

In a polarizer, there is characteristic direction called transmission axis which is indicated by the dotted line.

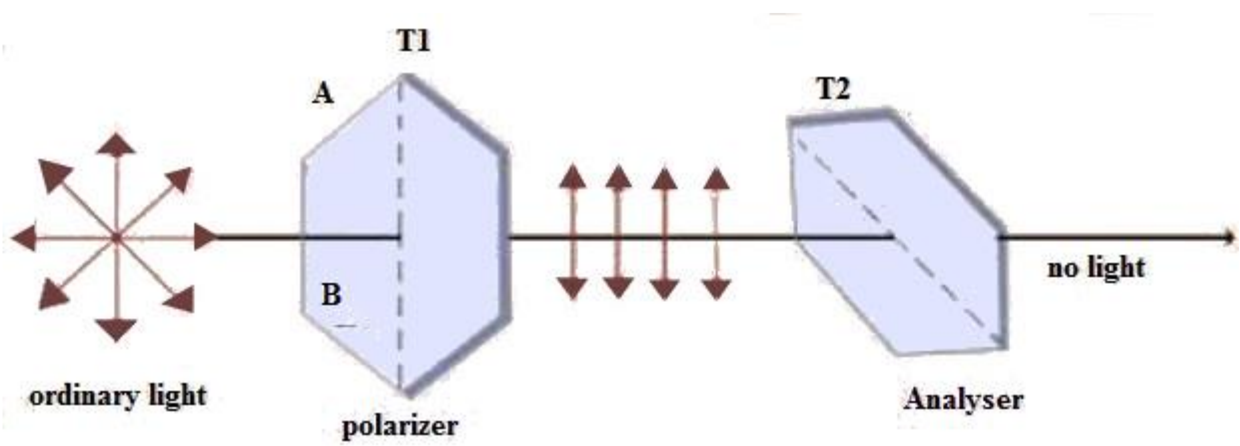


If a polarizer is placed in front of unpolarized light source, then the transmitted light is plane – polarized in specific direction.



Since the human eye is unable to detect polarized light it is necessary to use an analyzer to detect the direction of polarization.

If the plane of polarization of the polarizer and the plane of the analyzer are perpendicular then no light is transmitted when the polarizer and the analyzer are combined.



METHODS / WAYS OF PRODUCTION PLANE POLARIZED LIGHT

(1) By Polaroid

(2)By reflection

(3)By double refraction

(4)By using Nicol prism

POLARIZATION BY POLAROIDS

Polaroid is an artificial crystalline material which can be made in thin sheets. It has the property of allowing light vibrations only of a particular polarization to pass through.

Uses of Polaroid

(i) They are used in sunglasses to reduce the intensity of light and to eliminate glare.

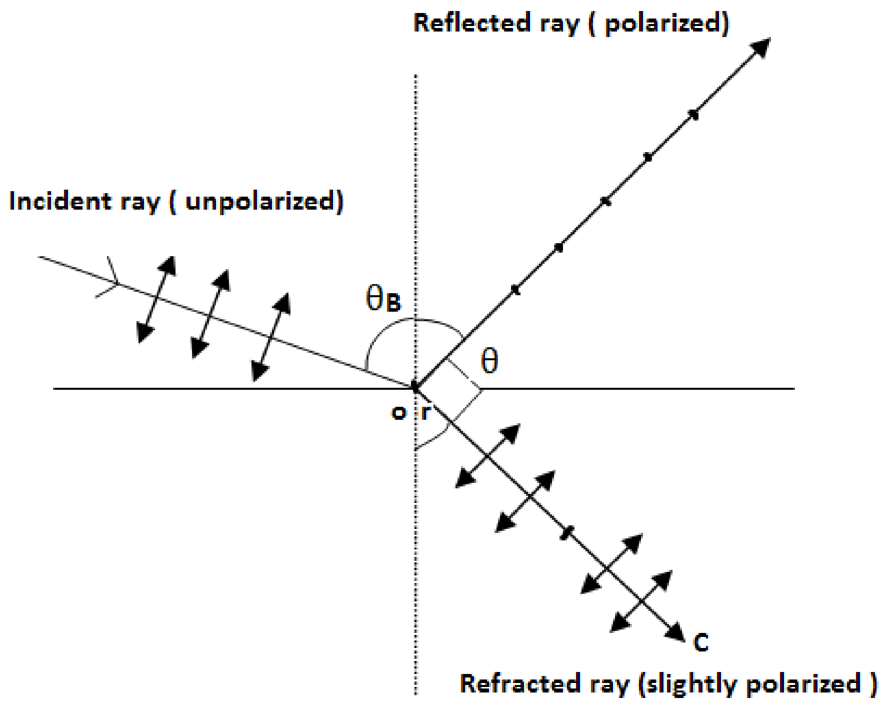
(ii) They are used to control the intensity of light entering trains and aeroplanes.

(iii) They are used in wind shields of automobiles.

POLARIZATION BY REFLECTION

The reflecting surface of a transparent medium can be able to produce plane polarized light.

This happens when unpolarized light is incident to any transparent medium e.g. glass.



Where, AO – Is an incident natural light

OB – Is a strongly plane – polarized reflected ray

OC – Is a partially plane-polarized reflected ray.

BREWSTER'S ANGLE

- Polarization by reflection occurs at a certain special angle of incidence at which maximum polarization occurs.

Example

- For a glass of refractive index 1.5, Brewster's angle is 57°.

BREWSTER'S LAW

- The law states:

The extent of polarization of light reflected from a transparent surface is maximum when the reflected ray is at right angles to the refracted ray.

- By Snell's law of refraction of light.

è $$n = \frac{\sin i}{\sin r}$$

è $$n = \frac{\sin \theta}{\sin r}$$

- When η = refractive index of the transparent medium
- From the figure above, we have:

$$r + 90° + \theta = 180°$$

$$r = 90° - \theta$$

- Equation (1) above becomes:

$$n = \frac{\sin \theta}{\sin(90° - \theta)}$$

$$n = \frac{\sin \theta}{\cos \theta}$$

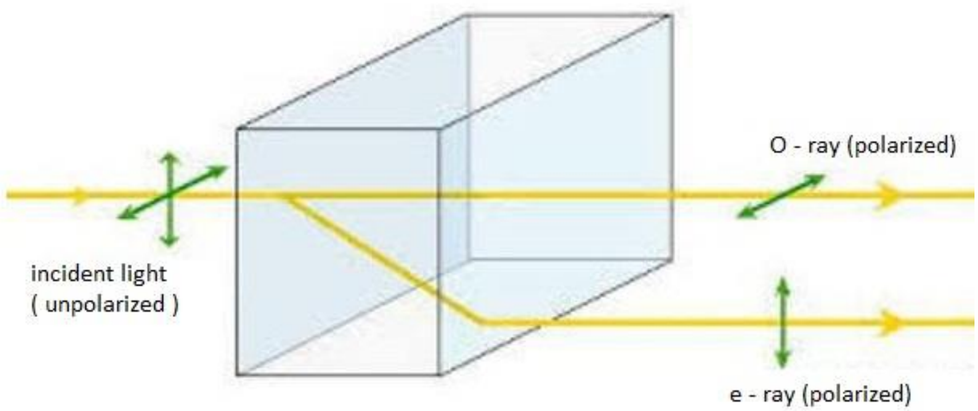
$$n = \tan \theta \text{ --------------- } (2)$$

This equation leads to Brewster's law

- The equation shows that the angle of incident for maximum polarization depends only on the refractive index of the medium.

POLARIZATION BY DOUBLE REFRACTION

Double refraction is the property possessed by certain crystals e.g. calcite, Iceland spar of forming two refracted rays from a single incident ray.



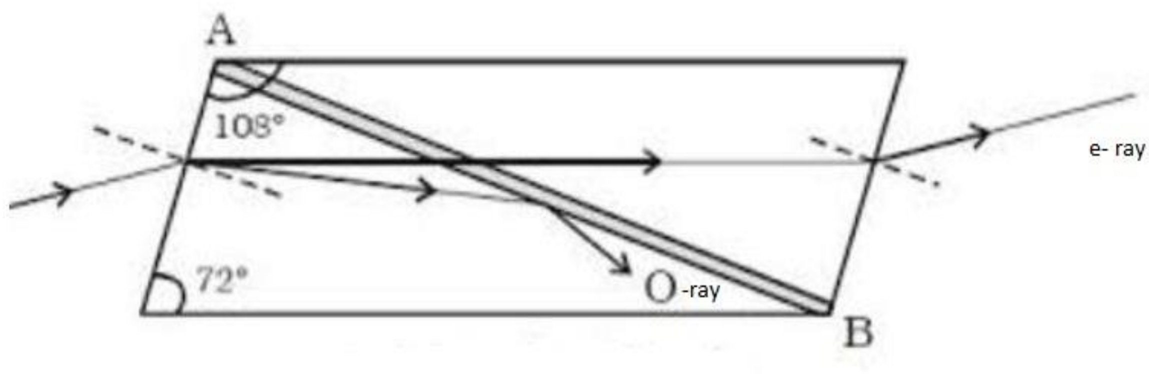
o-ray - ordinary ray
e-ray - extra ordinary

- Where a beam of unpolarized light is incident on one face crystal, its internal molecular structure produces two beams of polarized light E and O whose vibrations are perpendicular to each other.

NICOL PRISM

- Is a device for producing plane-polarized light.

- It consists of two pieces of calcite which are stuck together with Canada balsam material (a transparent material used to join up the two pieces of calcite together)



- The extraordinary ray, E passes through the prism while the ordinary ray, O undergoes total internal reflection at the interface between the two crystals when the angle of incidence exceeds the critical angle value.

NOTE

If the incidence ray to the Nicol prism does not produce double refracted rays it means that the incident ray is a polarized ray of light.

APPLICATION OF POLARIZED LIGHT

(1) Reducing glare

Glare cause by light reflected from a smooth surface can be reduced by using polarizing materials since the reflected light is partially or completely polarized.

Example

In sunglasses.

In photography as filters. Where they are placed in from of the camera lens.

(2) Optical activity

Certain crystals e.g. sugar solutions rotate the plane of vibration of polarized light passing through them and are said to be <u>optically active.</u>

Definition

Optical activity is the ability of certain substances to rotate the place of vibration of plane-polarized light as it passes through them.

For a solution the angle of rotation depends on its concentration which can be measured by the instrument known as Polarimeter.

(3) Stress analysis

When glass, Perspex, polythene and some other plastic materials are under stress (e.g. by bending,or uneven heating) they become doubly refracting.

The effect is called photo elasticity and is used to analyze stresses in plastic models of various structures.

Problem 71

A point P is situated at 20.1cm and 20.28cm from two coherent sources. Find the nature of illumination at the point P if the wavelength of light is 6000$\mathring{A}$.

Problem 72

The path difference between the two identical waves arriving at a point is 85.5λ. Is the point bright or dark? If the path difference is 42.5 micrometer, calculate the wavelength of light.

Problem 73

In young's experiment, the distance between the two slits is 0.8mm and the distance of the screen from the slits is 80cm. If the fringe width is 0.6mm, find the wavelength of light.

Problem 74

In young's experiment, interference bands were produced on a screen placed at 1.5m from two slits 0.15mm part and illuminated by a light of wavelength 6500A.U. find

(i) Fringe width and

(ii) Change in fringe width, if the screen is move away from the slit by 50cm

Problem 75

Two parallel slits 1.2mm apart are illustrated with light of wavelength 5200$\mathring{A}$ from a single slit. A screen is placed at 1.0 m from the slits. Find the distance between the fifth dark band on one side and the seventh bright band on the other side of the central bright band.

Problem 76

In a biprism experiment, the distance between the slit and the eyepiece is 80 cm and the separation between the two virtual images of the slit is 0.25 mm. if the slit is illuminated by a light of wavelength 6000 Å, find the distance of the second bright and from the central bright band.

Problem 77

In a bi prism experiment, with the distance between the slit and the screen as 0.5m and the separation between the two virtual images of the slit as 0.4 cm an interference pattern obtained with a light of wavelength λ = 5500 Å. Find the distance between the 3rd and the 8th bands on the same side of the central band.

Problem 78

In a biprism experiment, the distance of the 10th bright band from the center of the interference pattern is 6 mm. Find the distance of the 15th bright band from the center.

Problem 79

The fringe separation in a biprism experiment is 3.2×10^{-4} m when red light of wavelength 6.4×10^{-7} m is used. By how much will this change if blue light of wavelength 4×10^{-7} m is used with the same setting?

Problem 80

In a biprism experiment, the fringe width is 0.4mm, when the eyepiece is at a distance of 1m from the slit. Find the change in the fringe width, if the eyepiece is moved through a distance of 25cm towards the biprism, without changing any other arrangement.

Problem 81

In a biprism experiment, the distance between the slit and the screen is 1.0m and the distance between the images of the slit is 2.7mm. If the fringe width is 0.2mm, find the wavelength of light used.

Problem 82

In a biprism experiment, the distance between the slit and the screen is 0.8m and the two virtual sources formed by the biprism are 0.4mm apart. The wavelength of light used is 6000Å. Find the band width.

Problem 83

Calculate the distance between the second dark band and the fifth bright band on the same side of the central bright band of an interference pattern produced by coherent sources separated by 1.2mm from each other. The screen is placed at one metre from the coherent sources and the wavelength of light used is 6000Å.

Problem 84

In a biprism experiment, the slit is illuminated by a light of wavelength 5000Å. The distance between the slit and the biprism is 20cm and the distance between the biprism and the eyepiece is 80cm. If the distance between the two virtual source is 0.25cm, calculate the distance between the fifth bright band on one side of the central bright band and the sixth dark band on the other side.

Problem 85

In a biprism experiment, the distance between the two virtual images of the slit is 1.5mm and the distance between the slit and the focal plane of the eyepiece is 1metre. Find the distance between the second and the eighth dark fringe on the same side, if the wavelength of the light used is 5000Å.

Problem 86

In biprism experiment, fringes were obtained with a monochromatic source of light. The eyepiece was kept at a distance of 1.2m from the slit and fringe width was measured. When another monochromatic source of light was used without disturbing slit and biprism, the same fringe width was obtained when the eyepiece was at 0.8m from the slit. Find the ratio of wavelength of light emitted by the two sources.

Problem 87

The distance between two consecutive dark bands in a bi prism experiment is 0.32mm when red light of wavelength 6400Å is used. By how much will this distance change if yellow light of wavelength 5900Å is used with the same setting?

Problem 88

Newton's rings are observed with a plane convex lens in constant with a glass plate. The radius of the first bright ring is 1mm. If the radius of the convex surface is 4metres, what is the wavelength of light used.

Problem 89

The diameter of 10^{th} dark ring in a Newton's ring system viewed normally by reflected light of λ = 5900Å is 0.5cm, calculate the thickness of the air film and radius of curvature of the lens.

Problem 90

Newton's rings formed with sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40th dark ring?

Problem 91

If the diameter of the two consecutive Newton's rings in reflected light of wavelength 5890A.U. are 2.00 and 2.20cm respectively, what is the radius of curvature of the lens surface in contact with the plane glass surface?

Problem 92

Newton's rings formed with sodium light ($\lambda = 5.9 \times 10^{-5}$ cm) between a plane glass plate and convex lens surface. The diameters of two successive dark rings are 2mm and 2.236mm. what is the radius of curvature of the lens surface.

Problem 93

Newton's rings are formed by placing lens on a glass surface. If the 10th bright ring of sodium light by reflection($\lambda = 5893\,\mathring{A}$) be 5mm in diameter. What is the radius curvature of the lens.

Problem 94

In a Newton's ring experiment, the plane convex lens and the glass plate are in optical contact and the thickness of film at that point is zero. Find out the thickness of the air wedge at the fourth bright ring for light of λ = 500Å

Problem 95

In a young's double-slit experiment, sodium light of wavelength 0.59 $\times 10^{-6}$ m was used to illuminate a double slit with separation 0.36mm. If the fringes are observed at a distance of 30cm from the double slits, calculate the fringe separation.

Problem 96

In an experiment using young's slit, fringes were found to occupy 3.0mm when viewed at a distance of 36mm from the double slits. If the wavelength of the light used is 0.59μm, calculate the separation of the double slits.

Problem 97

When red monochromatic light of wavelength 0.70μm is used in a Young's double-slit arrangement, fringes with separation 0.60mm are observed. The slit separation is 0.40mm. Find the fringe spacing if (independently) (a) Yellow light of wavelength 0.60μm is used ;

(b) The slit separation becomes 0.30mm;

(c)The slit separation is 0.30mm and the slits fringe distance is doubled.

Problem 98

Interference fringe are formed in an air wedge using monochromatic light of wavelength 0.60 $\times 10^{-6}$ m. The fringes are formed parallel to the line contact, and a dark fringe is observed along the line of contact. Calculate the thickness of the air wedge at position where:

(a) The twentieth dark fringe and

(b) The thirtieth bring fringe from the line of contact are observed.

Problem 99

When interference fringes are formed using an air wedge, it is found that the twentieth bright fringe is formed at an air thickness of 6.8μm. Calculate

(a)The wavelength of the light used Problem 100

(b) The diameter of the wire

Problem 101

Interference fringes of separation 0.40mm are with yellow of wavelength 0.60μm. Calculate the fringe spacing if the blue light of wave length 0.45μm is used.

Problem 102

Interference fringes of spacing 1.0mm are obtained using the light of wavelength λ incident on an air wedge of angle α. The angle of the wedge is now double and the light replaced by one of the wavelength 1.5λ. Calculate the new fringe separation.

Problem 103

A loudspeaker emits a note which gives a beat frequency of 4Hz when sounded with a standard turning fork of frequency 280Hz. The beat frequency decreases when the fork is loaded by adding a small piece of plastic to its prongs. Calculate the frequency of the note emitted by the loudspeaker.

Problem 104

A note from a loudspeaker gives a beat frequency of 10Hz when sounded with a turning fork of frequency 440Hz. Calculate;

(a) The beat period.

b)Two possible values for the frequency of the note emitted by the loudspeaker

Problem 105

A vibrating sonometer wire emits a note which gives a beat frequency of 6.0Hz when sounded in unison with a standard tuning fork of frequency 256 Hz. When the fork is loaded the beat frequency increases. What is the frequency of the note emitted by the sonometer?

Problem 106

A beam of microwaves of wavelength 3.1cm is directed normally through a double slit in a metal screen and interference effect are detected in a plane parallel to the slit and at a distance of 40cm from them. It is found that the distance between the centers of the first maximum in the interference pattern is 70cm. Calculate an approximate value for the slit separation.

Problem 107

What is the wavelength of light which gives a first order maximum at an angle of 22° when incident normally on a grating with 600 lines mm^{-1}.

Problem 108

Light of wavelength 600nm is incident normally on a diffraction grating of width 20.0mm on which 10.0 $\times 10^{3}$ lines have been ruled. Calculate the angular positions of the various orders.

Problem 109

A source emits special lines of wavelength 589nm and 615nm. This light is incident normally on a diffraction grating having 600 lines per nm. Calculate the angular separation between the first-order diffracted waves. Find the maximum order for each of the wavelengths

Problem 110

When a certain grating is illuminated normally by monochromatic light of wavelength 600 the first-order maximum is observed at an angle of 21.1°. If the same grating is now illuminated with light with wavelength from 500nm to 700nm, Find the angular spread of the first-order spectrum.

Problem 111

When monochromatic light of wavelength 5.0×10^{-7} m is incident normally on a plane diffraction lines are formed at angle of 30°. What is the number of lines per millimeter of the grating?

Problem 112

A spectral line of known wavelength (5.792×10^{-7} m) emitted from the mercury vapour lamp is used to determine the spacing between the lines ruled on a plane diffraction grating. When the light is incident normally in the grating, the third-order spectrum, measured using a spectrometer, occurs at an angle of 60° 19' to normal. Calculate the grating spacing.

Problem 113

Light from a cadmium discharge lamp can be used to determine the spacing of the lines on a plane diffraction grating. This is done by measuring the angle $\emptyset$ between the diffracted beams either side of the normal in the first order spectrum for light incident normally on the grating.

(a) If the measured value of $\emptyset$ is 46° 43' and the red line used in the cadmium spectrum is of wavelength 644nm, calculate the number lines per metre on the grating.

(b) Make a suitable calculation to the whether the second order spectrum of this line will be visible.

Problem 114

A light source emits two distinct wavelengths, one of which is 450nm. When light from the source is incident normally on a diffraction grating, it is observed that the fourth order image formed by the same angle of diffraction as the third order image for the other wavelength. If the angle of diffraction for each image is 46°, calculate;

(a)The second wavelength emitted by the source,

(b)The number of lines per meter of the grating

Problem 115

A horizontal string is stretched between two point points a distance 0.80m apart. The tension in the string is 90N and its mass is 4.5g. Calculate;

(a)The speed of transverse waves along the string and

(b)The wavelength and frequencies of the three lowest frequency modes of vibration of the string.

Problem 116

The fundamental frequency of vibration of a stretched wire is 120Hz. Calculate the new fundamental frequency if

(a)The tension in the wire is doubled, the length remaining constant

(b)The length of the wire is doubled the tension remaining constant

(c)The tension is doubled and the length of the wire is doubled.

(d)The wavelength of waves with frequency 120Hz.

(e)The length of wire which when fixed at its end, gives a fundamental frequency of 120Hz.

Problem 117

The fundamental frequency of vibration of a stretched wire is 150Hz. Calculate the new fundamental frequency if;

(a)The tension in the wire is tripled, the length remaining constant.

(b)The length of wire is halved, the tension remaining constant.

(c)The tension is tripled and the length of wire is halved.

Problem 118

A wire having a diameter of 0.80mm is fixed in a sonometer and has a fundamental frequency of 256Hz alongside it's wire is made of the same material but of diameter 0.60mm. Both wires are stretched over the same bridges on the sonometer but the thinner wire is subjected to only half the tension of the thicker wire. Calculate the fundamental frequency of vibration of the thinner wire.

Problem 119

A closed organ pipe is of length of 0.60 m. Calculate the wavelengths and frequencies of the tree lowest frequency modes of vibration. Take the speed of sound to be 345 ms^{-1} and neglect any end correction of the pipe.

Problem 120

Two open organ pipes are sounding together and produce a beat frequency of 12.0Hz. If the longer pipe has length of 0.400 m. Calculate the length of the pipe. Take the speed of sound as 340 ms^{-1} and ignore end corrections.

Problem 121

A piece of glass tubing is closed at one end by covering it with a sheet of metal. The fundamental frequency is found to be 280 Hz. If the metal sheet is now removed, calculate;

(a)What length the tube is

(b)The wavelength and frequencies of the fundamental and the first overtone 280 Hz. Ignore end corrections.

Problem 122

A tall vertical cylinder is filled with water and a tuning fork of frequency 512 Hz is held over its open end. The water is slowly run out and the first resonance of the air column is heard when the water level is 15.6 cm below the open end. Calculate;

a) The end correction of the tube.

b) The position of the water level when the second resonance is heard.

Problem 129

An open tube of length 30.0 cm has an end correction of 0.60 cm. calculate its fundamental frequency.

Problem 130

Two open pipes of length 0.700 m and 0.750 m are sounded together and vibrate in their fundamental frequencies. Find the beat frequency, assuming that end corrections can be ignored.

Problem 131

Two identical closed pipes of length 0.322 m are each vibrating with their fundamental frequency. If one pipe is held at 00C and the other at 17oC, calculate the beat frequency which is observed. Take the speed of sound at 0oC to be 331 m-1 and ignore end corrections.

Problem 132

A closed pipe is of length 0.300 m. calculate:

a) Its fundamental frequency at 0^oC, given that the speed of sound at 0^oC is 331 m s^{-1},

b) The temperature, in ^oC, at which it will be in unison with a tuning fork of frequency 288 Hz.

Problem 133

Two open pipes of length 0.500 m and 0.550 m are sounded together and vibrate in their fundamental frequencies at 7 ^oC. Calculate:

a) The beat frequency, given that the speed or sound at 7 ^oC is 335 ms^{-1}

If the temperature of the longer pipe is now allowed to change whilst the shorter pipe stays at 7 ^oC, calculate.

b) The value of the temperature of the air in the longer pipe at which the two pipes will be in unison.

Problem 134

Stationery waves are set up in the space between a microwave transmitter and plane reflector. Successive minima are spaced 15mm apart. What is the frequency of the microwave oscillator? Take the speed electromagnetic waves as 3.0 x 108 m s^{-1}.

Problem 135

A system of stationary waves in which the nodes are 2m apart are produced from progressive waves of frequency 200 Hz. Calculate the speed of the progressive waves.

Problem 136

A stretched wire of length 0.7 m vibrates in its fundamental mode with a frequency of 320 Hz. Calculate the velocity of waves along the wire. Why does such a vibration not continue indefinitely?

Problem 137

A wire of mass per unit length 5.0 gm^{-1} is stretched between two points 30 cm apart. The tension in the wire is 70 N. calculate the frequency of the sound emitted by the wire when it oscillates in its fundamental mode.

Problem 138

a) A string of unstretched length 2.0 m and mass 0.15 kg has a force constant of 25 Nm^{-1}.for the experiment, the string is stretched to a total length of 3.0 m. calculate the velocity of propagation of transverse waves along the string.

b) The same string in (a) above is clamped between two rigid supports 3.0 m apart, and set in vibration. Calculate the wavelengths and frequencies of the five lowest frequency modes of vibration which can be excited on the string. When the vibrating string is held lightly at the centre, in which of these modes does the string continue to vibrate? Explain your reasoning.

Problem 139

A vertical steel wire is kept in tension by a piece of iron attached to one end. The wire is set in transverse vibration and emits a note of frequency 200 Hz. The iron is now completely immersed in water and the frequency of the note changes to 187 Hz. If the density of the water is 1000 kg m^{-3} calculate the density of the iron.

Problem 140

A resonance tube is held vertically in water and can be raised or lowered. A tuning fork of frequency 384 Hz is struck and held above open end of the tube on a day when the speed of sound in air is 344 ms^{-1}. The shortest tube length at which resonance occurs is 21.6 cm and the corresponding length when the tube is filled with carbon dioxide is 16.7 cm. calculate:

a) The end correction for the tube,

b) The speed of sound in carbon dioxide.

Problem 141

A turning fork is sounded at the open end of a tube, containing air, which is closed at the other end. Two successive positions of resonance are obtained when the length 49.0 cm and 82.0 cm. calculate:

a) The wavelength of the sound waves in the tube,

b) The end correction of the tube.

Problem 142

If the speed of sound in air is 340 m s^{-1} at a given temperature, calculate the length of an open pipe having a fundamental frequency of 192 Hz. If this pipe were sounded together with another open pipe of length 0.850 m at the same temperature, calculate the beat frequency. Ignore any end corrections.

Problem 143

An open ended pipe, of length 0.50 m, is sounded at 20^oC together with a tuning fork of slightly lower frequency. Five beats per second are heard. Calculate the change in temperature required to bring the pipe and fork back into unison. Neglect end corrections and assume that changes in temperature affect only the speed of sound in air, which is 340 ms^{-1} at 20^oC.

Problem 144

A source of sound, when stationary, generates waves of frequency 500 Hz. The speed of sound is 340 ms^{-1}. Find from first principles the wavelength of the waves detected by the observer and the frequency observed when:

a) The source is stationary and the observer moves towards it with speed 20.0 ms^{-1},

b) The source moves with away from stationary observer with speed 30.0 ms^{-1}.

c) The sources moves with speed 30.0 ms^{-1} in a direction away from, the observed and the observer moves with speed 20.0 ms^{-1}

Problem 145

A train sounding its whistle (frequency 580 Hz) is traveling at 40.0ms^{-1} a long straight section of track, and passes an observer standing closed to the track. Calculate the maximum change in frequency which the observer will hear. Take the speed of sound in air as 340 ms^{-1}.

Problem 146

A motorist approaches a road junction at a constant speed of 15 m s^{-1}. A policeman standing at the junction blows on a whistle with frequency 680Hz.

a) Find from first principles the frequency observed by the motorist. The motorist now reduces his speed at a rate of 5.0 m s^{-1}.

b) Calculate the frequency he observers at subsequent 1 intervals until he stops.

Problem 147

A train sounding its whistle moves at a constant speed of 20 ms^{-1} a long straight section of track. The train passes under a low bridge on which stands an observer. If the observer records a maximum frequency of 638Hz. Calculate:

a) The frequency of the whistle,

b) The minimum frequency the observer hears.

Problem 148

A source of sound of frequency 400 Hz moves at steady speed of 15 ms^{-1} towards an observer. If the observer moves a steady speed of 25 ms^{-1} towards the source, calculate the frequency he observes.

Problem 149

A loudspeaker which emits a note of frequency 250 Hz is attached to a wire and whirled in a vertical circle of radius 1.00 m at a steady rate of 20.0 revolutions per minute. Calculate:

a) The speed of rotation of the loudspeaker in m s^{-1}.

b) The maximum and minimum frequency detected by a stationary observer.

Problem 150

A train sounding its whistle travels at constant speed on a long straight section of track. An observer standing closed to the track records a range of frequencies between 551 Hz and 658 Hz. Calculate:

(a) The speed of the train.

(b) The frequency of its whistle.

Problem 151

A hooter of frequency 360 Hz is sounded on a train approaching a tunnel in a cliff-face at 25 ms^{-1}, normal to the cliff. Calculate the observed frequency of the echo from the cliff-face, as heard by the train driver. Assume that the speed of sound in air is 330 ms^{-1}.

Problem 152

A car travel at a constant speed of 30 ms^{-1} towards a tunnel and sounds its horn, which has a frequency of 200 Hz.the sound is reflected from the tunnel entrance. Calculate the frequency of the echo observed by;

(a) The driver of the car

(b) A stationary observer standing close to the road

(c) The driver of the car travelling at 20 ms^{-1} which is following the first car.

Problem 153

A train emerges from a tunnel at a speed of 20 ms^{-1} and sound its whistle, which has a frequency of 450Hz, Calculate the frequency of the echo from the tunnel entrance as observed by the train driver,

Problem 154

A source of sound which is stationary with respect to air emits a note of frequency 340 Hz. An observer receding at uniform speed, from the source hears an apparent frequency of 300 Hz observer is moving, if the speed of sound in air is 340 ms^{-1}.

Problem 155

(a) Show from first principles that the frequency f of sound in still air, heard by a stationary observer as a source of sound of frequency f_s approaches the observer with a velocity v_s, is given by

$$f_0 = f_s\left[{}^{1}\!/_{1-\frac{v_s}{C}}\right]$$

Where C is the velocity of sound in still air,

(c) When f_s=1.0×10^3 Hz and c = 300 ms^{-1}, what is the percentage change in the frequency heard by the stationary observer when the source velocity changes from 30 ms^{-1} to 35 ms^{-1}?

Problem 156

A model aircraft with an engine producing a note of constant frequency of value 400Hz flies at constant speed in a horizontal circle of radius 12m and completes one revolution in 3.0s. An observer, situated in the plane of the circle and 30m from its centre, monitors the frequency of the sound from the engine.

(a) Explain why the observed frequency shows periodic variations

(b) Derive a relation for the minimum observed frequency in terms of f, the true frequency of the engine, V, the speed of the aircraft and C, the speed of sound in air. Write down the corresponding relation for the maximum observed frequency.

(c) Taking to C be 340 ms^{-1}, calculate the maximum and minimum observed frequencies. Determine the time interval between the occurrence of a maximum frequency and the next minimum frequency.

Problem 157

A ship travelling at 3 ms^{-1} towards a cliff in still air and is sounding its siren at 1 KHz. Find from first principles the frequency of the echo as measured by an observer on the ship. Give sufficient detail for reasoning to be followed. The speed of sound in air is 330 ms^{-1}.

Problem 158

A railway engine traveling at constant speed emits a whistle of constant frequency. When the engine passes a stationary observer closed to the track, the frequency of the sound heard by the observer changes from 600 Hz whilst approaching to 500 Hz whilst receding. Assuming the speed of sound is 340 ms^{-1}; calculate the speed of the engine.

Calculate the frequencies heard if the same engine passes an observer who is travelling at 2 ms^{-1} in the same direction as the engine and close to the track.

STATIC ELECTRICITY

It can be shown that there are two kinds of charges by rubbing a glass rod with silk and hanging it from a long silk thread. If a second glass is rubbed with silk and held near the rubbed end of the first, the rod will repel each other.

On other hand, a hard rubber is rubbed with far, will attract the glass rod rubbed with silk. The modern view of bulk matter is that in its normal i.e neutral, it contains equal amount of positive and negative charges.

If two bodies like glass and silk are rubbed together, a small amount of charge is transferred from…… to the other upsetting the electric neutrality of each. In this case the glass would become positive and silk negative.

FORCE BETWEEN TWO CHARGES OR ANY TWO CHARGED BODIES

Columb found that a for force exist between two electrically charged bodies and that this force and the distance between the charged bodies obey the inccerse square law ie If is the distance between the charged bodies and F is the force of attraction between these charged bodies the

$F\alpha\frac{1}{r^2}$……………………………….(i)

Equation (i) above is known as the inverse square law.

It has been found that if Q_1 and Q_2 are how charged then the force of attraction between the is given by

FαQ1Q2……………………………….(ii)

Combining eq (i) and (ii)

$$\therefore F = \frac{Q1Q2}{4\pi\varepsilon r^2}$$

The permittimity of air at normal pressure is only about 1.005 times that of uacumum (Æ• $_0$) . For most purpose therefore we may assume the value of Æ• $_0$ for the permittinity of air.

a) Calculate the value of two charges if they are one another with a force of 0.1 when situated 50cm apart in a vacuum

b) What would be the size of the charges if they were situated in an insulating liquid whose permitting was 10 times that of vaccum.

Solution

F=0.1

R= 50cm=0.5m

Æ• â—¦=8.854x10-12

Q1=Q2 They repel=Q

Formula

$$F = \frac{Q1Q2}{4\pi\varepsilon r^2}$$

Q^2=Fxð• ›‘Æ• â—¦r^2

$Q=[0.1x4x3.16x8.8854x10^{-12}x(0.5)^2]^{1/2}$

Q=1.67x10-6C

(b) Given

F=0.1

Q1=Q2=Q

Æ• â—¦= 10

r=50c m=0.5m

Formula

$$F = \frac{Q1Q^2}{4\pi\varepsilon r^2}$$

Q2=Fx4xÆ• r^2

Q= (0.1x4x3.14x10x8.854x (E-12)x(0.5x0.5)

Q=5.27x E-6 C

1998P$_2$ Qns

The distance between the electrical proton in the hydrogen atom is about frictional force between those particles.

Me=9.11E-3 kg

Mp=1.67E-27

e=1.6E-19c

Æ• =6.67E-11Nm^{2}kg^{-1}

r=5.3E-11m

Fe=7.25E-29N, Fg=3.61E-19

(i) Gravitational force between particles

Fg α $x = \frac{\varepsilon MeMp}{r^2}$

=9.11E-31kgx1.67/(5.3E-11)^{2}M^2

Fg=3.7E-47 N

(ii) Fe $= \frac{(e)^2}{4\pi \mathcal{E} r^2}$

$$= \frac{(1.6E-19)^2}{4x3.14x8.854E-12(5.31E-11)^2}$$

Fe=8.1E-8 N or 8.1x10^{-8}

ELECTRIC FIELD INTENSITY (E)

Electric field intensity is defined as the region which in electric force is experienced. So an electric field intensity E if an electrostatic field at any point is defined as the force per unit charges which it existence once positive charges.

- If Q is a small test charge placed on a point them

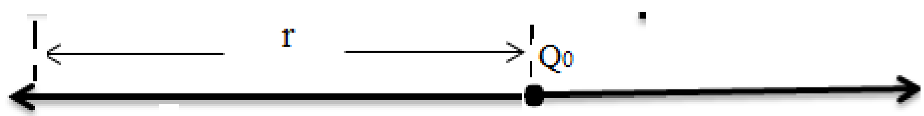
$$E = \frac{F}{Q}$$

The SI unit of E is NC^{-1}

QUESTION

Find the magnitude of an electric field strength such that an electron placed on the field would experience an electrical force equal to its weight mass of electron $=9.1\times10^{-31}$ kg, $e=1.6\times10^{-19}$C, $g=8.9m/s^2$

Consider a test charge Q_0 in vacuum which is placed a distance "r" from a point charge (isolated) charge Q_0



By Coulombs Law the magnitude of a force acting on Qâ—¦ is given by

$$F = \frac{Q\,Q_\circ}{4\pi\varepsilon_\circ r^2}$$

By putting eq (i) in eq(ii)

$$E = \frac{Q\,Q_\circ}{4\pi\varepsilon_\circ r^2}.\frac{1}{Q0}$$

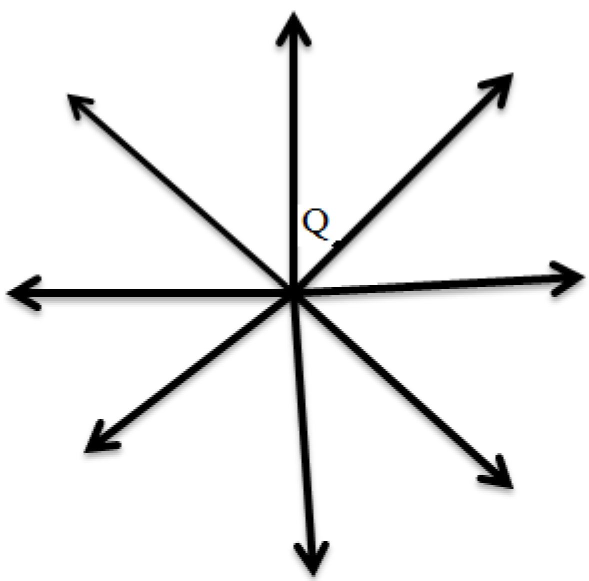
$$E = \frac{Q}{4\pi\varepsilon_\circ r^2}$$

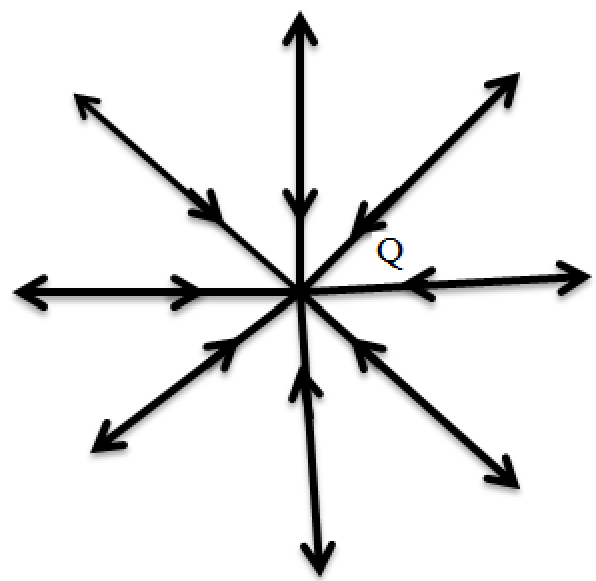
Note:

I. E is the vector quantity

II. From Æ• $= \frac{Q}{4\pi \varepsilon_o r^2}$ E α $\frac{1}{r}$

III. The direction of Æ is radial line from Q point …… out towards. If Q is positive and inside if Q is negative

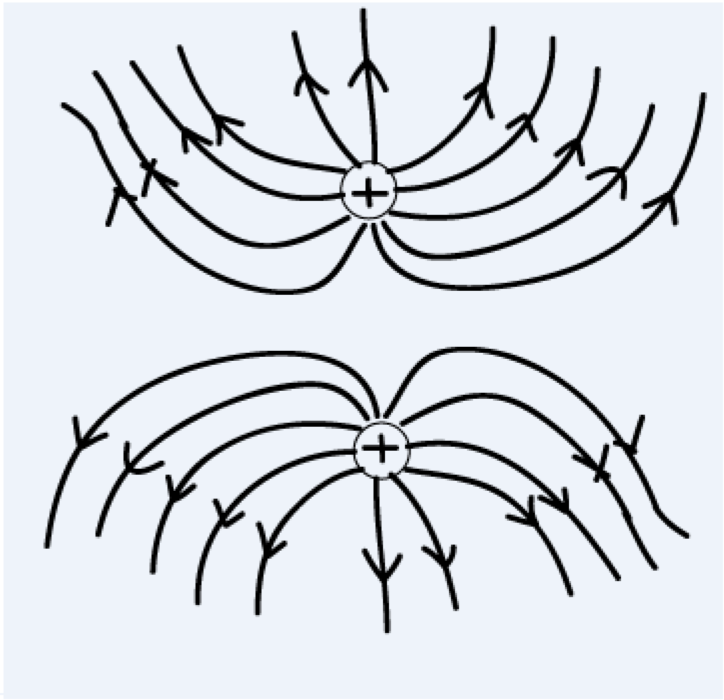




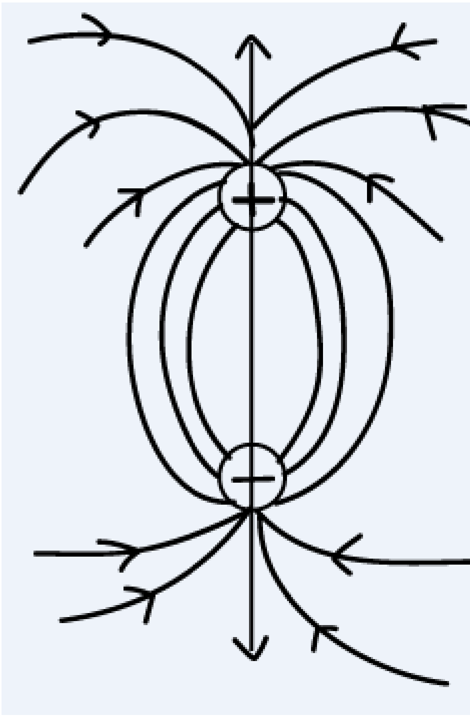
LINES OF FORCE

There (are) is a relationship between imaginary line of force and the electric field strength as follows.
(i)The tangent to a line of the force to any point gives the direction of Æ• at that point

The lines of force are drawn so that the member of force permit crossectional area is proportional to the magnitude of Æ• . Where the lines are close together Æ• is given as large and where they are far apart Æ• is small.



Line of force for the equal positive charges

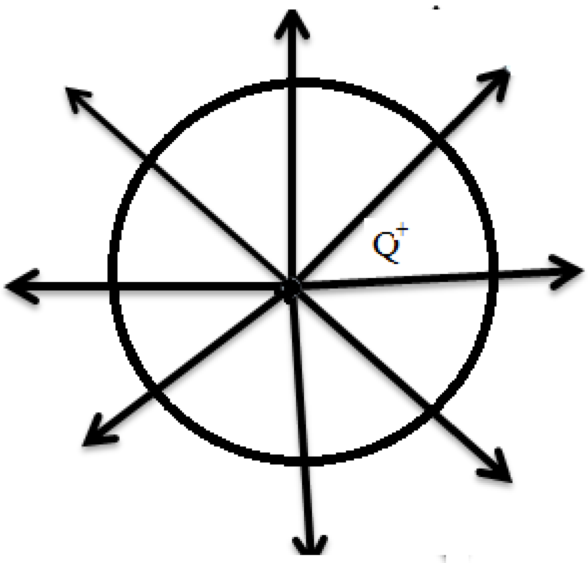


Line of forces for equal but opposite c

ELECTRIC FLUX (Ø$_{Æ}$.)

The electric flux Ø$_{Æ}$. through an area perpendicular total lines of force is the product of. Ex area where E is the electric intensity at that place.

Consider a sphere of radius drawn in a space concentric with a point charge.



Total flux through the sphere is given by

Ð¤Æ• =E x Area of the sphere

=Ex4ð• ›‘r^2

$=\frac{Q}{4\pi\varepsilon_0 r^2}$x 4ð• ›‘$r^2$

i.e

$$\Phi\,\varepsilon = \frac{Q}{\varepsilon_0}$$

If the charge is placed at any other medium apart from air or vacuum then

Ð¤$^{\varepsilon}$=$\frac{Q}{\varepsilon}$

The above equation shows that the total flux crossing any point at drawn sphere concentrically outside the point charge is constant.

1. (OUT SIDE THE CHARGED SPHERE)

ELECTRIC FIELD INTENSITY DUE TO A CHARGED SPHERE

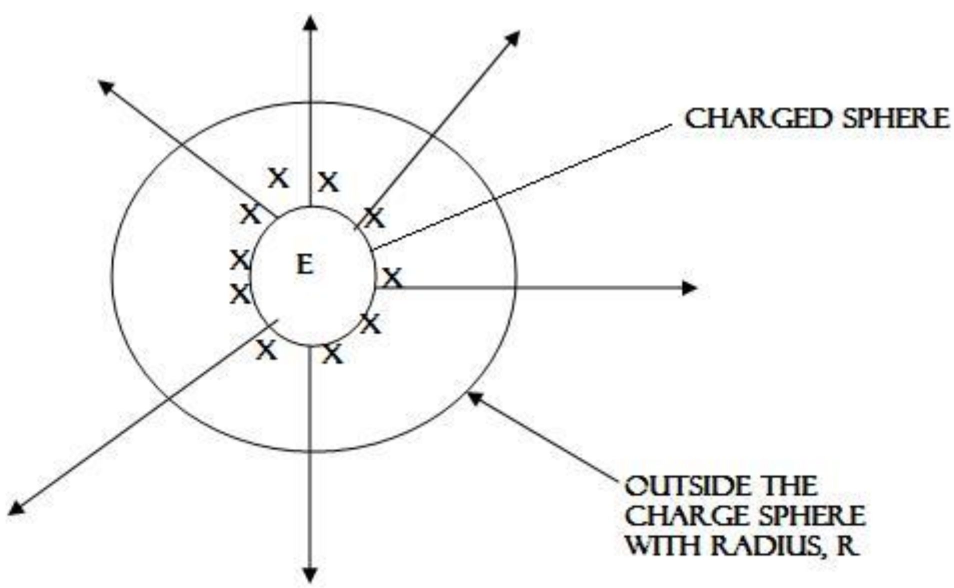
The flux across a spherical surface of radius or concentric with a small sphere carrying charge Q is given by flux= $\frac{Q}{\varepsilon}$

$$E \times 4\Pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

This result shows that the outside of a charged sphere the field behaves as if all charges on the sphere are concentrated at the centre.

2. INSIDE A CHARGED EMPTY SPHERE.

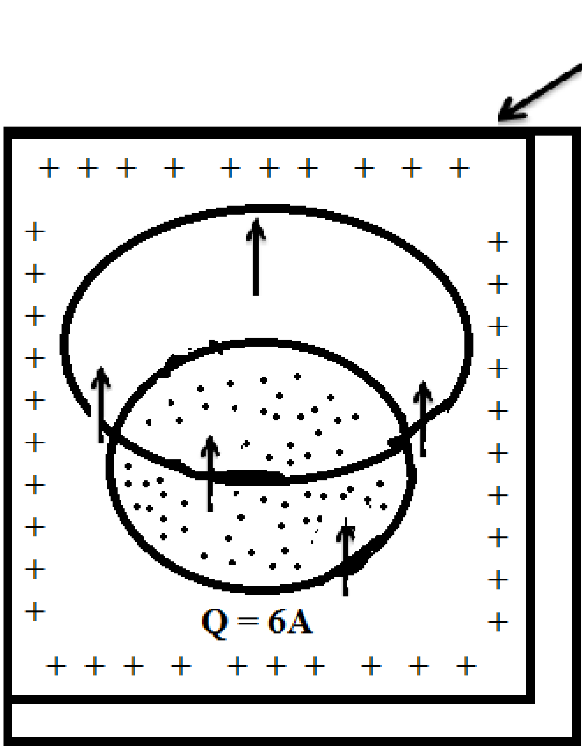


Inside the empty charged sphere there are no charges so the electric field strength E = O therefore;

$$E \times 4\Pi r^2 = \frac{Q}{\varepsilon_0}$$

since Q=0,then the value of ε inside the sphere is also 0

3. ELECTRIC INTENSITY OUTSIDE THE CHARGED PLATES



Consider a charged plane conductor S with a surface change density of 6cm^{-2} let the plane surface 1 as shown above be drawn outside the 5 which is parallel to 5 and has the area Am^2

since $\Phi\varepsilon = \frac{Q}{\varepsilon_0}$ we have

$$x\,A = \frac{QA}{\varepsilon_0}$$

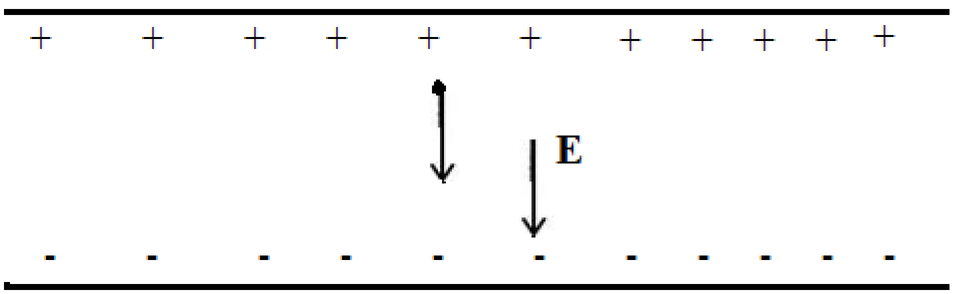
hence

$$E = \frac{Q}{\varepsilon_0}$$

The intensity of in the field must be perpendicular to the surface and the charges will produce this field are those in projection of the area P on the surface S i.e those within the shaded region A.

Question.

A particle of mass M and charge of q is placed at rest in a uniform electric field see the fig below and released. Describe its motion.



The motion reassemble that of the falling body in the earth's gravitational field. The constant to acceleration is given by $a = \frac{F}{m}$

The equation of uniform acceleration to apply therefore with

with $V_o = 0$ we have

$V = at$

put eqn(1) in (2) we have

$$V = \frac{q\varepsilon t}{m} \text{ -----------(iii)}$$

The vertical distance moved by a particle with initial velocity

$$y = \frac{1}{2}at^2 \text{ -----------------(3)}$$

By putting eqn(1) in (3) we get

$$y = \frac{1}{2}\frac{q\varepsilon t^2}{m} \text{ -----------------(4)}$$

From the third equation of motion we have

$$v^2 = 2Ay$$

Putting eqn(1) into eqn(5)

$$v^2 = \frac{2q\varepsilon y}{m} \text{ ------------------ } (6)$$

The kinetic energy attached at the moving a distance y is formed from :-

$$K.e = \frac{1}{2}mv^2 \text{ ------------------ } (7)$$

substitute eqn(6) into (7) we get

$$K.e = \frac{1}{2}m \cdot \frac{2q\varepsilon y}{m}$$

$$\text{i.e } K.e = q\varepsilon y$$

$$\therefore \boxed{K.e = q\varepsilon y}$$

ELECTRIC POTENTIAL (V)

The electric field around a charged and can be described not only by a vector electric field strength E but also by a scalar quantity i.e the electric potential,v.
To find the electric potential difference between two points A and B in an electric field we move a test charge q from A to B and we measure the work

WAB that must be done by agent moving the charge.

Electric potential difference ,v ca be expressed in the form of

$$VB - VA = \frac{WAB}{q^{\circ}} \text{ ....(i)}$$

The unit of the potential difference is obtained for equation (i) that is JC. However volts is also used.

$1JC^1 = 1Volts$

If point A is chosen to be at very far (say at infinity) then the electric potential at infinity distance is arbitrarily taken et zero.

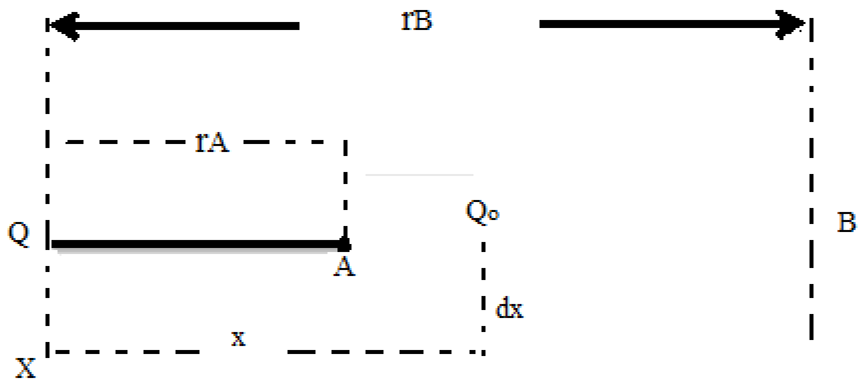
Therefore then putting VA=0 1 in equation (i) and dropping the subscripts we get

$$V = \frac{W}{q} \text{...} \text{(ii)}$$

Definition:

The electric potential at the point is the work done by the force in taking the unit chart from infinity to that point.

CALCULATION OF WORK DONE



Consider a positive charge Q to be at Ra distance as indicate in the figure the work done in taking the charge from A to B is equal to the work done in taking the same distance from B to A. If Q0 is moved by the fo….. from A to B then the force acting on it is

$$F = \frac{Q\ Q0}{4\pi\varepsilon 0 x^2 a}$$

If the charge has moved a distance ∂x the work done is

$$dw = \frac{Q0\ Q}{4\pi \varepsilon r^2}$$

Hence total workdone is taking Qo from A to B is

$$WAB = \int_{rA}^{rB} \frac{Q1Q}{4\pi\varepsilon_o x^2} dx$$

We get

WAB= $\frac{Q\,Q0}{4\pi\varepsilon 0x^2 a}[\frac{1}{rA} \_ \frac{1}{r2}]$..............................(ii)

The potential difference between A and is given by

$$VAB = \frac{WAB}{Q0}$$ ....(iii)

Substitute eq-(ii) into (iii)

We get

$$VAB = \frac{QQo}{4\pi\varepsilon}[\frac{1}{rA} \_ \frac{1}{rB}]\frac{1}{QO}$$

Therefore

$$VAB = \frac{Q}{4\pi\epsilon}[\frac{1}{rA} \_ \frac{1}{rB}]$$

If rB is very large i.e B is at the infinity the potential at A is given by

$$VA = \frac{Q}{4\pi\varepsilon orA}$$

So in general the potential v at a distance r from the point charge Q is given by:

$$V = \frac{Q}{4\pi\varepsilon r}$$

POTENTIAL DUE TO SEVERAL CHARGES

The potential at any point due to a group is found by

a. Calculating the potential Vn due to each charge as if other charges are not present.

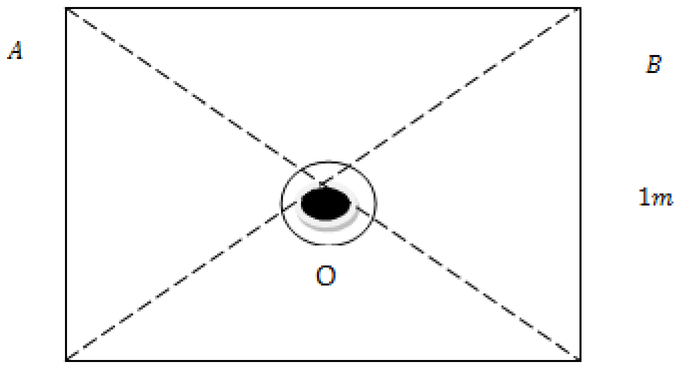
b. Adding the quantity so obtained or

$$V = \sum V_n = \frac{1}{4\pi\varepsilon_o} \sum \frac{q_n}{r_n}$$

1) Where question is the value of the charge and is the distance of this change from the point in question

Question

Calculate the potential at the centre of the squire shown below



From the figure

$(BD)^2 = (BC)^2 + (CB)^2$

$= 1 + 1$

=2

$BD = \sqrt{2}$

But $\overline{DO} = \overline{OB} = \overline{OD} = \overline{OC} = \frac{\sqrt{2}}{2}$.

Solution

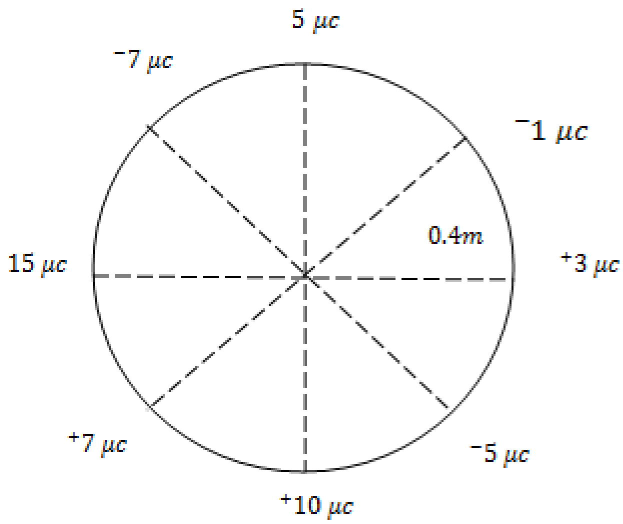
$V = \frac{1}{4\pi\varepsilon o} \sum \frac{qn}{rn}$

$\therefore V = \frac{1}{(4\pi)(8.854x\ 10^{-12})} (1 + (-2) + (2) +) \times 2 \times 10^{-8} \times \sqrt{\frac{1}{2}}$

$\therefore V = 509.2v$

Note. $\frac{1}{4\pi\varepsilon 0} = 9 \times 10^9 Nm^2\tau^{-2}$

2) Eight charges having the values. Shown in the figure below one arranged systematically on the circle of radius 0.4m in air Calculate the potential at the centre 0.



Note. I $MC = 10^{C} C$

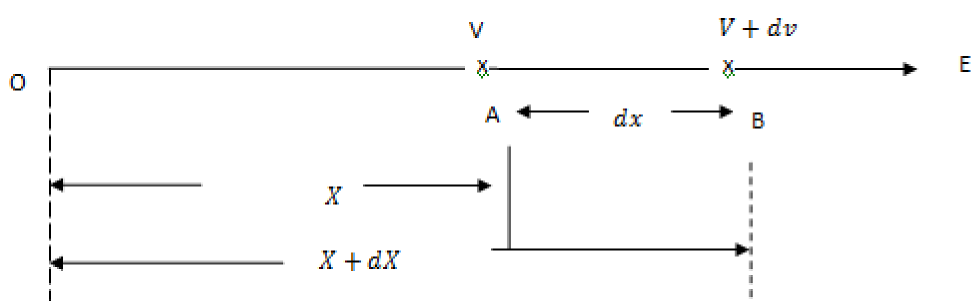
Solution

$V = \frac{1}{4\pi\varepsilon o} \sum \frac{9n}{rn}$ |

$9 \times 10^{9} \frac{(-1+3-5+10+7+15+5)\times 10^{-6})}{0.4}$

$V = 8.075 \times 10^{5} volts$

RELATIONSHIP BETWEEN ELECTRIC INTENSITY (E) AND ELECTRIC POTENTIAL V:



Consider two points A and B at a distance x and x +dx from 0 respectively V and B Vd are respectively potential A and B are very close. So that the electric intensity E is instant. Hence the potential between A and B is $V_{AB} = V_A - V_B$

$$V-(V+DV)\ \ ie$$

$$VAB = -dv - - - - - - - - (I)$$

The work done in taking a Unit charge from B to A

$= force\ x\ distance$

$VAB = Ed \times$ Since Q is unit charge.

From (1) and (2) we have

$Edx = -dv$

$$E = \frac{dv}{dx}$$

The unit of E is Vm^{-1} and the quantity $\frac{dv}{dx}$ is called potential gradient

Qn 1985 p Qn 13

Two points A and B in a uniform electric field are 5mm apart. Where a charge of $1.5 \times 10^{9} C$ is moved between A and B. 1.5×10^{-6} of work is done. Calculate

i) The potential difference between A and B

ii) The electric intensity of the field.

Iii) The electrostatic force exerted on the charge by the field.

Solutions

(i) ; $p.d\ of\ A\ and\ B = VAB = \frac{W.D}{Q} = \frac{1.5 \times 10^{-6} J}{1.5 \times 10^{-9} C}$

$VAB = 10^{3} jC^{-1}$

(ii) $E = \frac{dv}{dx} = \frac{1000V}{5 \times 10^{-3} m} = 2 \times 10^{5} Vm^{-1}$

(iii) $E = \frac{F}{Q}$

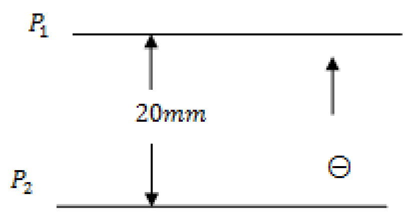
$F = E \times Q = 2 \times 10^{5} \times 1.5 \times 10^{-9} = 3 \times 10^{-4}$

Force $= 3 \times 10^{-4} N$

QN 1994. P, Qn B

An electron is liberated from the lower part two large parallel plates separated by a distance h. The upper plate has a potential of 2400V relative lower. How long does the electron take to reach

Solutions



$E = \frac{dv}{dx} \quad \frac{2400v}{20\times10^{-3}} = 12$

$\therefore E = 1.2 \times 10^{5}vm^{-1}$ But $F = ma$

So $S = ut + {}^{1}/_{2}\, at^{2}$ $a = \frac{F}{m}$

$S = {}^{1}/_{2}\, at^{2} where\ u = 0$

$F = Ee - - - - - - (1)$

Put 1 (in) (2)

$a = \frac{Ee}{m} - - - - (3)$

$f^{2} = \frac{25}{a} — (4)\ put\ in(4)$

$f = \left(\left(\frac{2\times20\times10^{-3}}{1.2\times10^{5}\times1.6\times10^{-19}}\right) \times m\right)$

Where m is the mass of an electron.

EQUIPOITENTIAL SURFACE

It is the evident from the equation V = Q/4ÐŸÆ• that all points which are at the same distance from point charge are at the same potential
Any surface over which the potential is constant is called **equipotential surface**

Equipotential surface has the surface property that along direction lying on the surface there is **No electric** field for there is no potential difference dv/dx =0 since v is constant.

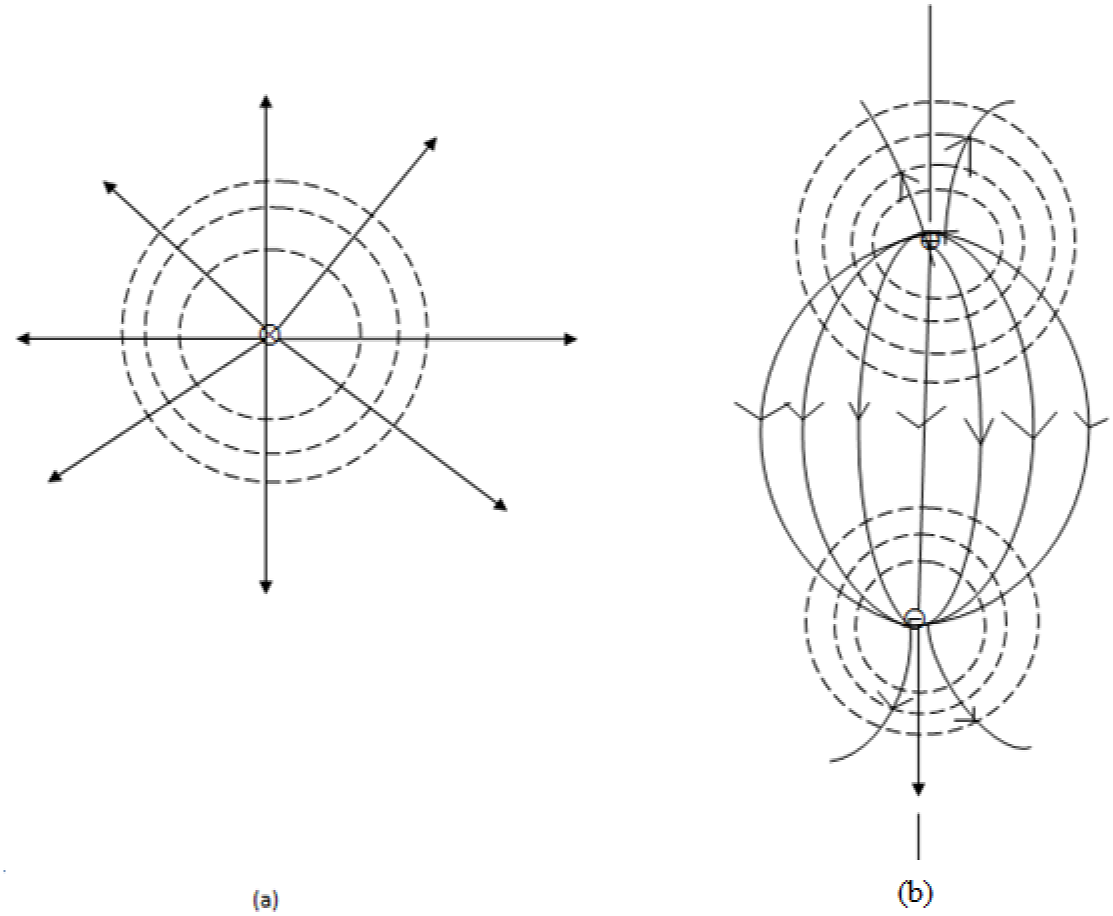
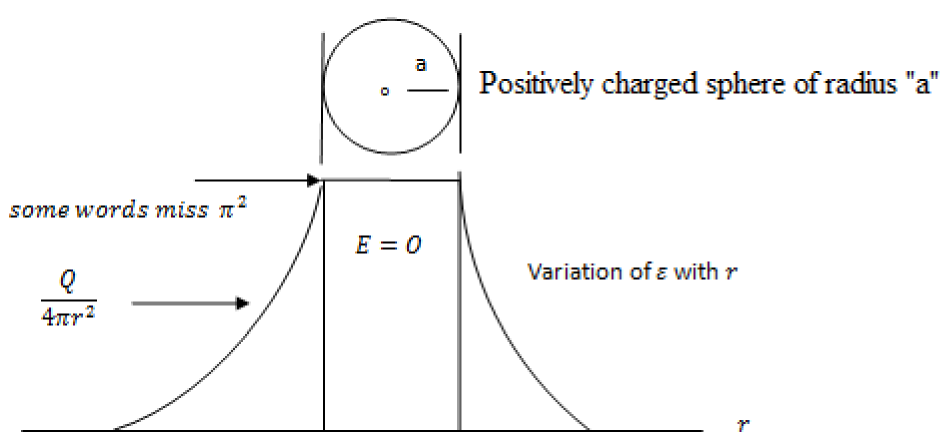
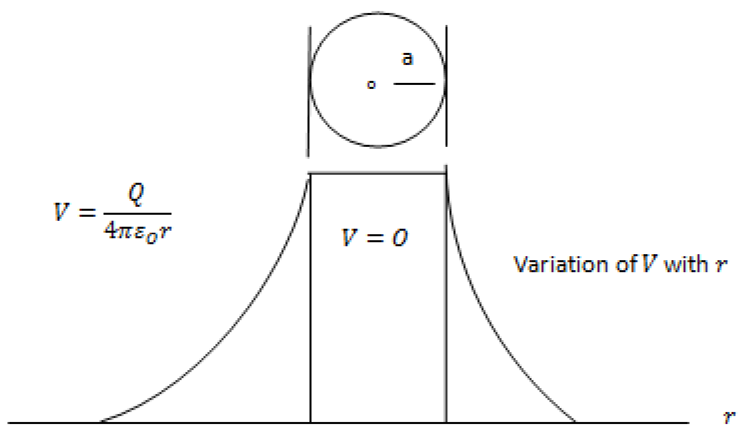


figure (a) And (b) above shows equipotential surface dashes times while solid (continuously) lines duplicating lines force.



Graph- shown below shows the variation of electric field strength within an isolated sphere and that at external part of it.

Graph below shows variation of electric potential at an isolated sphere and at the external part of it.

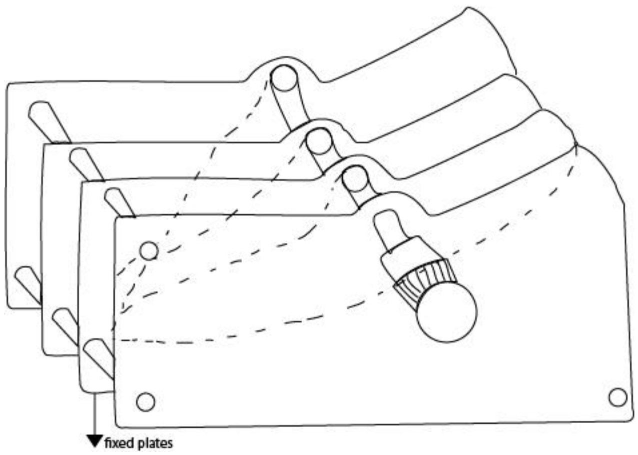


Formula.

$$C = \frac{A\varepsilon r\varepsilon o}{d} = \frac{3.5\times10^{-}m \times 2.7\times8.85\times10^{-12}}{2\times10^{-5}} \quad 4.18 \times 10^{-7}F$$

TYPES OF CAPACITOR

(i)VARIABLE AIR CAPACITOR



Variable (capacitance) capacitor is the one in which the effective area of the plates can be adjusted. The capacitance of variable capacitor can be varied as you wish but at a certain limits.

These are widely used in the turning circuits of radio receivers. They are constructed of number affixed parallel metal plates. Plates. Connected together and constituting one plate of the capacitor. The second parts of movable plates also connected together and form the other plates.

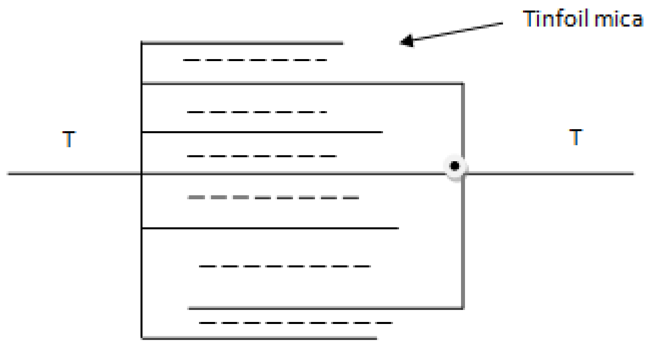
By rotating plates on which the movable plates are mounted the second set may be caused to interleave the first to a lesser or greater extent.

The effective area of the capacitor is that of the interleaved portion of the plates only

The plats of the capacitor may be made of brass or Aluminum. The dielectric may be oil, air or mica.

(ii)A MULTIPLE CAPACITOR MICA DIELECTRIC

Tin foil



The plates of this type of capacitor are made of tin foil. The capacitance of the capacitor is “n” times the capacitance (of the) between the two successive plates. When “n” is the number of dielectric between the plates.

Qn 1999. P2 Qn Sc

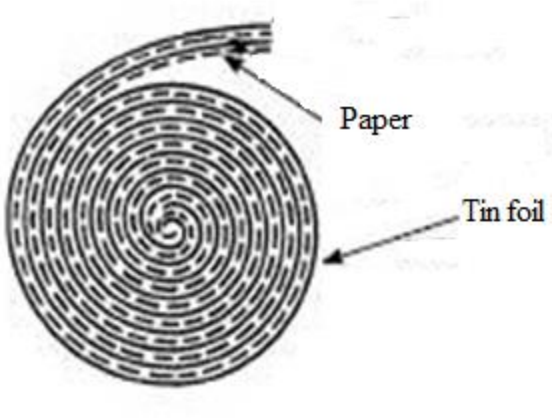
Given that the distance of separation between the plates of a capacitor is 5mm and the plate has an area of $5m^2$ .A potential difference of 10V is supplied the capacitor which is parallel in vacuum.
compute

(i) The capacitance.

(ii) The electric intensity in a space between the plates

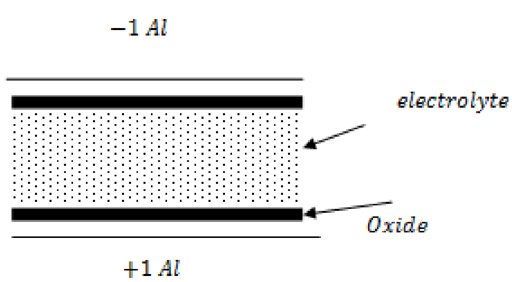
(iii) The change in stored energy, if the separation of the plates is increased from 5m to 5.5mm.
Æ• â—¦= 8.85 pF/m

(iii)**PAPER CAPACITOR**



The paper capacitor has a dielectric of paper impregnated with paraffin wax or oil unlike the mica capacitor the paper can be rolled and scatted into a cylinder relatively small volume. Now a days the paper has been replaced by thin layer of polystyrene.

(iv) **ELECTROLYTIC CAPACITOR**



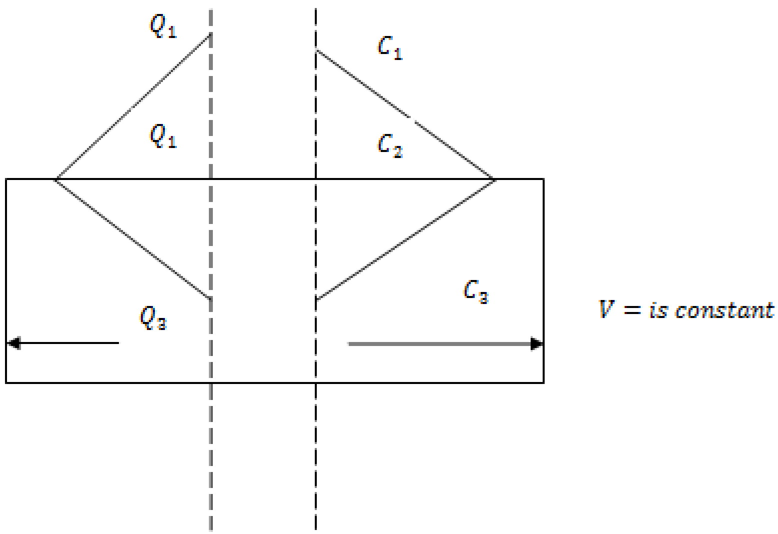
They are produced by passing a direct current between the two sheets of aluminum foil with a suitable electrolyte of lightly liquid conductor between them. A very thin film of aluminum oxide is then formed on the oxide plate which is of positive side of the d.c supply.

This film is an i**nsulator**. It forms dielectric in between two plates the electrolyte being of a good conductor.

Since the dielectric d is very small and C α 1/d *the capacitance value can be very high*.

ARRANGEMENT OF CAPACITOR

1) **Parallel arrangement of capacitor**



All the left hand plates are connected together and all the right hand plates are connected together and in the case of parallel arrangement of capacitors (see figure above)

When a cell is connected across these capacitor is parallel they have the same potential difference (v)

So

(i) $Q_1 = C_1V$

(ii) $Q_2 = C_2V$

(ii) $Q_3 = C_3V$

Let the total charge be Q then $Q = Q_1 + Q_2 + Q_3$...........(4)

Put

(I) (II) (III) in (IV)

$= C_I V + C_2 u + C_3 v - - - - -$

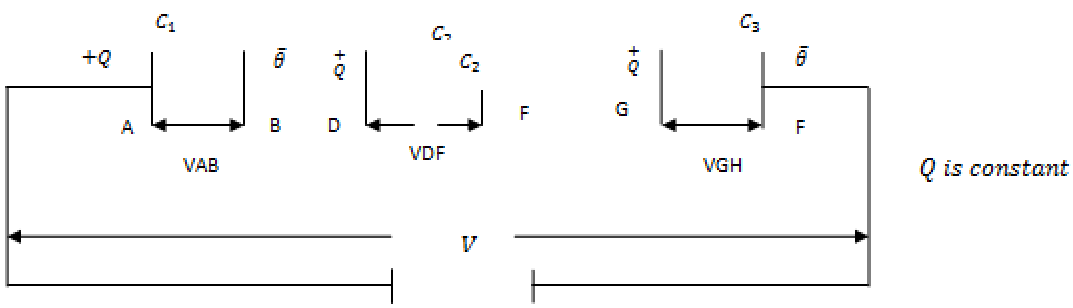
$Q = V(C_1 + C_2 + C_3 - - - - - (V)$

Divide equation (V) by U both side we get

$C = \frac{Q}{V} = C_1 + C_2 + C_3$ Or $C = C_1 + C_2 + C_3$

Where "C" is the equivalent capacitance.

(ii) SERIES ARRANGEMENT OF CAPACITOR.



When the right hand plates of one capacitor is connected to the left hand of the next and so on then these capacitors are said to be connected in series.

When the cell is connected across of the end of the system a charge is transferred from the plates H to A, A charge –Q being left on it. This charge induce a charge +Q on plate Q. This process is repeated with other plate.

Now $V_{AB} = \frac{Q}{C_1}$ $VDF = \frac{Q}{C_2}$ $VCH = \frac{Q}{C_3}$

But. VAB + UDF + VOH

$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

Divide both sides by Q

$\frac{V}{Q} = {}^{1}/_{C_1} + {}^{1}/_{C_2} + {}^{1}/_{C_3}$

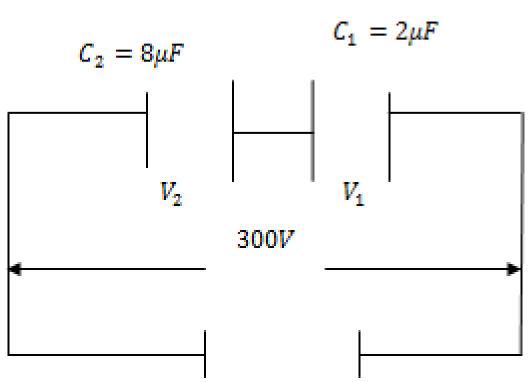
I.e. ${}^{1}/_{C} = {}^{1}/_{C_1} + {}^{1}/_{C_2} + {}^{1}/_{C_3}$

Where C is the equivalent capacitance.

Question

Two capacitors of capacitance C_1=2µF and C_2= 8µF are connected in series and the resulting combination is connected across 300volt. Calculate the charge and potential difference

Solution



(i) $I\mu F = 10^{-6}F$

(ii) $Ip\,F = 10^{-12}F$

(iii) $I\,nF = 10^{-9}F$

$V_1 = \frac{Q}{C_1}, V_2 \frac{Q}{C_2}$ But $V = V_1 + V_2$ where V = 300volts

$\therefore 300 = \frac{Q}{2\times10^{-6}} + \frac{Q}{8\times10^{-6}} = 300 = Q\frac{(4\times10^{6}+10^{6})}{8}$

$\therefore Q\ \frac{300\times8}{5\times10^{6}} = \frac{60\times8}{10^{5}} = \frac{480}{10^{5}} = 4.8\times10^{-4}C$

$\therefore Q = 4.8\times10^{-4}$ C

(b) Potential difference;

$V_1 = \frac{4.8\times10^{-4}C}{2\times10^{-6}} = 2.4\times10^{2} = 240$

$V_2 = \frac{4.8\times10^{-4}}{8\times10^{-6}} = 6\times10^{1} = 60$

$V = V_1 + V_2 = 240 + 60 = 300$ proued

The charge across the capacitor is $4.8\times10^{-4}C$ and the

pd Across the 1^{st} capacitor $= 240v$

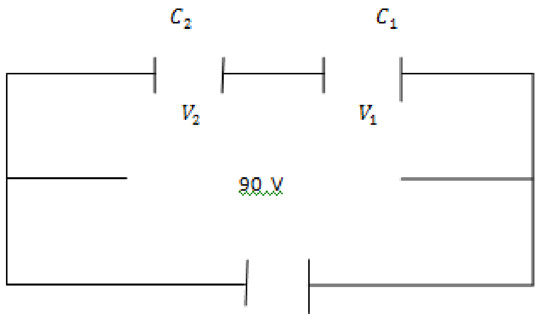
pd Across the 2^{nd} capacitor =60V

Qn. 1998 Qn 13

Capacitor of 5μF and 25μF are connected in series and the combination is connected to the battery of 90 volt. Calculate

a) Charge on each capacitor

b) The p.d across each capacitor



Where $C_1 = 5\mu F$

$C_2 = 25\mu F$

$V_1 = \frac{Q}{C_1} \qquad = \frac{Q}{C_1}$

$\therefore V = V_1 + V_2$ But v = 90 volts

So $V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 90 = Q\left(\frac{1}{5} + \frac{1}{25}\right) \times 10^6 F$

$90 = Q \times 10^6 F\left(\frac{5+1}{25}\right)$

$\therefore Q = \frac{90 \times 25}{6 \times 10^6 F} = 15 \times 25 \times 10^{-6} C$

$Q = 3.57 \times 10^4 C$

From above formular V=

$V_1 \ \frac{3.75 \times 10^{-4}}{5 \times 10^{-6}} = 705 \ \times 10 = 75$ Volt

$V_2 = \frac{3.75}{25} \times 10^{-4+6} = 15$ Volt

$\therefore$ The charges across the capacitor is $3.75 \times 10^{-4} C$ and the p.d of the 1st capacitor is 75 volts and the second capacitor = 15 volts.

ENERGY STORED IN A CAPACITOR

Consider a capacitor of capacitance C to have been charged to a potential difference V and let a small charge dQ be transferred from the negative plate to positive plate. Then the work don't in moving a charge dQ will be dw = VdQ but V=Q/C

Hence $dw = \frac{Q}{C} dQ$

Suppose a capacitor is at first discharge d and then charged until the final charge on the plate is Q The work done in charging it is given by

$$W = \int_0^{Q1} dw = \frac{1}{C}\int_0^{Q1} QdQ$$

$$\frac{1}{C}\int_0^{Q1} Q\partial Q = \frac{1}{C}\left[\frac{Q2}{2}\right]_0^{Q1}$$

$$\frac{1}{2}\frac{Q1^2}{C}$$

$\therefore W = \frac{1}{2}\frac{Q^2}{C}$

In general if C is capacitance of a capacitor carrying charge Q at potential difference then,

$W = \frac{1}{2}\frac{Q^2}{C}$ …………………(i)

But Q is equal to CV then (i) becomes

$W = \frac{1}{2}CV^2$……………………(ii)

Also $C = \frac{a}{v}$…………………………(iii) put into (i) we get

$W = \frac{1}{2}QV$………………………….(iv)

Equation (i), (ii) and (iv) give the energy stored in a capacitor.

Question

1998 P2B Qn

A capacitor of a capacitance 3ð• œ‡F is charged until a potential difference of 200v is developed across. Its plat…..another capacitor of capacitance 2ð• œ‡F developed apd of 100v across its plates on being charged.

i. What is the energy stored on each capacitor?

ii. The capacitors…them connected by a wire of negligible resistance so that the plates carrying like charges are connected together. What is the total energy stored in the combined capacitors?

Solution

C1=3x10^{-6}F

V1= 200v

C2=2x10^{-6}F

V2=100v

Formula

E1=$\frac{1}{2}$C1V²

=1/2x3x10^{-6}x(200)

=6x10-2 Joules

E1=0.06 Joules

E2=$\frac{1}{2}$C2V2

=1/2x2x10^{-6}x10^{4}

=10-2Joules

(ii) $\frac{1}{C}=\frac{1}{C1}+\frac{1}{C2}$

C=$\frac{C1C2}{C1+C2}$

$=\frac{2x3}{2+3}$

=6/5

=1.2

$C=1.2x10^{-6}F$ V=V1+V2, 100+200=300V

$E=\frac{1}{2}CV^2$

$=1/2x1.2x10^{-6}x(300)^2$

$=0.6x10^{-6}x\ 9x10^4$

$=5.4x10^{-2}$ Joules.

The total energy stored in the combined capacitor $5.4x10^{-2}$

Question

The capacitance of a parallel plate capacitor is 400 pic…Farad and its plates are separated by 2m of air

i. What will be the energy when it is charged to 500 volts.

Æ• â—¦=$8.854x10^{-12}$C2NM

How much work (energy) is needed to double the distance between the plates?

Solution

$C=400x10^{-12}$ F

$D=2x10^{-3}m$

Æ• â—¦=$8.854x10\text{-}12C^2N^{-1}M^{-2}$

Energy stored=$\frac{1}{2}C\ V^2$ or $\frac{1}{2}QV$ or QV

$\frac{1}{2}C\ V^2=\frac{1}{2}x\ 4.x10E-10X25X10$ $=5X10^{-5}$

$\frac{1}{2}C\,V^2 = \frac{1}{2}x\ 2x10EX - 10X2.5X105$ =2.5X10^{-5}

Energy needed to separate /double the space twice is 2.5x10^{-5} J

DISCHARGE IN C-R CIRCUIT

Consider a capacitor initially charged to a p.d V0 so that its charge is that Q=CV0

At a time t, after the discharge through R has begun the current, I flowing=V/R……………..(i) where V is the potential difference across C

Now V=$\frac{Q}{C}$…………………(ii) and I=$\frac{dQ}{dt}$ ……………………………(iii)

The minus sign shows that Q decreases with increasing time from equation (i), (ii) and (iii) we have

$\frac{-dQ}{dt} = \frac{1}{CR}Q$ ……………………………(iv)

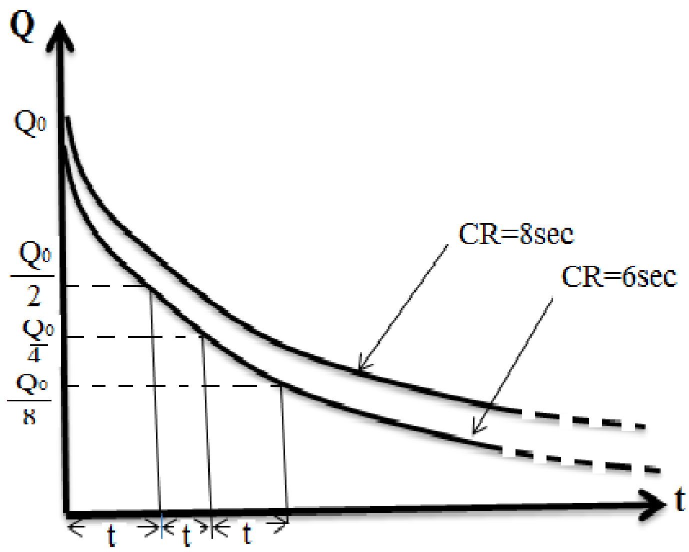
$\int_{Q0}^{Q}\frac{dQ}{Q} = \frac{1}{CR}\int_{0}^{t}dt = [\ln Q]_{Q0}^{Q} = -\frac{1}{CR}[t]_{0}^{t}$

=mQ-mQ0= $-\frac{1}{CR}t$

=ln$\left(\frac{Q}{Q0}\right) = -\frac{t}{C\ R}$

$= \frac{Q}{Q0} = e^{-t/cR}$ =Q=Q0$e^{-t/cR}$ ……………………(v)

From equation (v) Q decreases experimentally with time



Since the P.d, v across C is proportional to Q then $V=V0e^{-t/C}$ also since is the circuit is proportional to v then, $I=I0e^{-t/CR}$ where I0 is the initial current value $\frac{V0}{R}$ from the equation(i) Q decreases from Q0 to half of its value

$\frac{Q0}{2}$ in time t given by

$e^{-t/C}=1/2=2^{-1}$

Taking logarithm to base e both sides we get

$\frac{-t}{CR}=-\ln 2$

Therefore the time for a charge to diminish to half its initial value no matter what the initial value may be is always the same.

TIME CONSTANT

The time constant (T) of the discharge circuit is defined as CR seconds where C is the in Farad and R is in Ohms.

A resistor of the resistance R=10μâ„¦ is connected in series with a capacitor of capacitance 1μF. Find the time constant and half life.

Solution

$C=1x10^{-6}F$

$R=10x10^{6}$â„¦

Time constant $CR=10^{-6}x10^{7}$

=10 Seconds

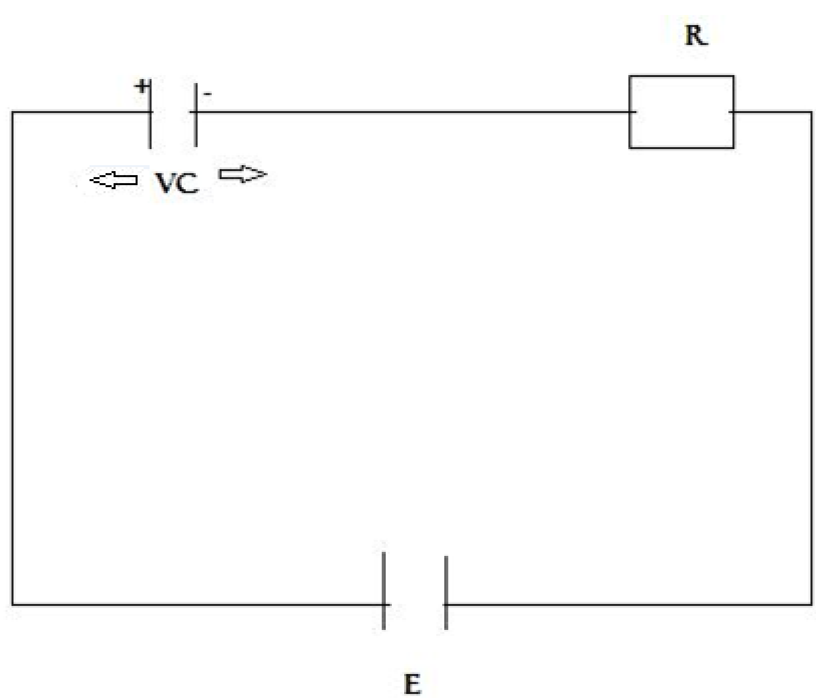
T1/2=10……..=10x0.693

The time constant =10 second

Half life=0.693

CHARGING OF CAPACITOR THROUGH A RESISTOR

Consider a charging of a capacitor with capacitance C through a resistor R in series.



Suppose the supplied battery has an e.m.f E and negligible internal resistance.

Initially there is no charge on care so no p.d across after connecting the battery the P.d across R=E the applied circuit P.d the initial current flow I0=$\frac{E}{R}$. Suppose I is the current flowing after a time t, the if VC is the P.d across C

$$I = \frac{E - V_c}{R} \quad \text{..................} \quad (1)$$

Since $E = IR + V_c$

But $I = \frac{dQ}{dt}$ ....(2) and

$$V_c = \frac{Q}{C} \quad \text{.......................}(3)$$

Substituting eqn(2) and (3) in eqn (1) we get

$$\frac{d\theta}{dt} = \frac{E-Q/C}{R} = \frac{CE-Q}{CR}$$

$CR\frac{d\theta}{dt}$=CE-Q................................(iv)

CE is the final charge on C when no further current flows through Q0.

Therefore equation (iv) becomes

$CR\frac{d\theta}{dt}$=CE-Q

Integrating the above expression gives

$$\frac{1}{CR}\int_0^t dt = \int_{Q0}^{Q}\frac{d\theta}{Q0-Q}$$

$$= \frac{t}{CR}$$

Let U=Q0-Q

du=-dθ

-$\int \frac{du}{u} =$ -ln u=-ln(Q0-Q)from 0-Q

-ln[ln(Q0-Q)-L=lnQ]= +ln($\frac{Q-Q0}{Q0}$)=-$\frac{t}{CR}$

ln($\frac{Q-Q0}{Q0}$)= -$\frac{t}{CR}$= $\frac{Q-Q0}{Q0}$ =$e^{-t/CR}$

Q0-Q=Q0$e^{-t/CR}$

$$Q = Q0(1 - e^{-t/CR})$$

∴

$$Q = Q0(1 - e^{-t/CR})$$

To show that CR i.e the time constant takes the unit of time i.e seconds

CR can be expressed as

C(Farad)=$\frac{Q}{V}$$\left(\frac{Coulombs}{Volts}\right)$=$\frac{It(AmperexSeconds)}{volts}$

R=Ohms=$\frac{V}{I}$=$\frac{(Volts)}{Ampere)}$

CR= $\frac{AmpsxSec}{Volts}$ x$\frac{Volts}{Amps}$=Seconds

The plates have the electric flux

Ð¤=E x A………………………….(i)

But also Ð¤=$\frac{Q}{\varepsilon}$…………………………….(ii)

From equation (i) and (ii) we have

EA=$\frac{Q}{\varepsilon}$ or E=$\frac{Q}{A\varepsilon}$……………………………………(iii)

The work required to take a test charge Q0 from the plate to the other is

W=Force x Distance

W=EQ0d……………………………………………(iv)

But also work done

W=Q0V………………………………………(v)

∴ Q0V= Æ• Q0d

V=Æ• d……………………………..(vi)

Put equation(iii) into equation (vi

V=$\frac{Qd}{A\varepsilon 0}$…………………………………(vii)

But capacitance=C=$\frac{Q}{V}$ …………………..(viii)

Substitute equation (vii) into equation (viii) we get

C= $\frac{A\varepsilon 0}{d}$

If the space between plates is filled with air or vacuum, the equation above becomes

C= $\frac{A\varepsilon 0}{d}$

QUESTION

1997P2 Qn 6

What is the capacitance of a parallel plates capacitor if it consist of a mica 0.1mm thick and 1.5cm^2 silvered o both sides the permittivity of mica plate is 6.0 (Æ• â—¦=8.854x10-12C^{2}N^{-1}M^{-2})

QUESTION

What is the electric field strength at the surface of a………….metal sphere of radius 100mm if it carries the charge $2x10^{-7}$ in vacuum Æ• â—¦=8.854x10-12$C^2N^{-1}M^{-2}$)

Solution

$$E = \frac{Q}{4\pi\epsilon r^2}$$

$$E = \frac{2E-7}{4\pi(8.854E-12)x(0.1)^2}$$

$=1.8x10\ NC^{-1}$

CAPACITANCE

It can be shown by sending a positive or negative charge close to a charged body lower s or va….the potential difference between the system of the charged bodies. This shows that the charge is proportional to the potential difference.

If Q is the charge and v is the p.d then

Q αV i.e

Q=CV…………………….(i)

Where the constant C is known as Capacitance.

Definition

The capacitance of a system of bodies is the charge necessary to raise potential by a unit i.e

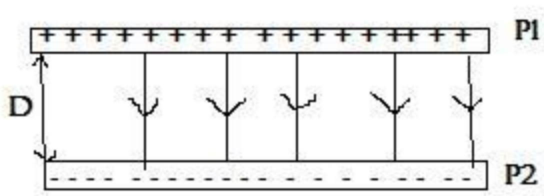
$C=\frac{Q}{V}$

Capacitance is measured in Farad but other units like ð• œ‡F and Hf are also used.

The symbol of capacitor is  The two parallel…………found in the symbol above can also be horizontal i.e depending on the circuit.

THE DERIVATION OF CAPACITANCE IN PARALLEL PLATES CAPACITOR

Consider the two parallel plates capacitor with plates ……………….



Each plate is carrying charge equal to Q. If the electric field strength between the plates

The plates have electric flux Ð¤ = E x A ……………(1)

But also Ð¤ = Q/ Æ□ ………………………………(2)

From equation (1) and (2) we have

EA = Q/ Æ• â—¦ or E = Q/A ………………………..(3)

The work required to take a test charge from one plate to another is

W = Force x distance

W =E Qâ—¦d……………………………(4)

But also work done= w Qâ—¦V…………(5)

Qâ—¦V = Æ• â—¦ Qâ—¦d

V = Æ• d ………………………(6)

Put equation (3) into (6) we have

V = Qd/A Æ• â—¦ …………………..(7)

But capacitance ,C = Q/V …………………………(8)

Substitute eqn(7) into (8) we get

C = A Æ• â—¦ /d

If the distance between plates is field with air or vacuum the equation above becomes

C = A Æ• â—¦ /d

NECTA 1997 P2 Qn 6
What is the capacitance if parallel plates capacitor it is consist of a sheet of mica 0.1mm thick and 1.5cm square silvered on both sides? The permittivity of mica is 6.0 times that of vacuum (Æ• â—¦ = 8.854×10^{-12} $JN^{-1}m^{2}$)

Solution
C=AÆ• /d

But
$A=22.5 \times 10^{-4} m^{2}$

$D=1.0 \times 10^{-4} m$

Æ• =6x8.854x10CN^{-1}m^{-2}

C=1.2x10^{-10}F

FACTORS WHICH DETERMINE CAPACITANCE

i.Distance between the plates

From the equation **V = Cd/AÆ•** It follows that if more far apart the p.d will increase and hence C**=Q/v**

The capacitance decrease. Therefore the capacitance decreases when the separation of the plates increase.

ii.Dielectric

When the dielectric e.g sheet of glass or ebonite between the potential difference between the plates decreases i.e **V=Qd/AÆ•**

Hence from C**=Q/v**

The capacitance has increased. Therefore so when **Æ•** r increases the capacitance will also increase.

iii.Area of plates

From V = Qd/Æ• A

It follows that the p.d decreases area of the plates.Increases, so the capacitance C = Q/V must increase.

DIELECTRIC CONSTANT(Æ• r)

Dielectric constant is also known relative permittivity

The ratio of the capacitance with dielectric to the one without the dielectric between the plates is called dielectric constant or relative permittivity of the material used.

Consider the case of parallel plates capacitor capacitance with dielectric (C)=AÆ• /d and capacitance without dielectric (Câ—¦)=AÆ• â—¦/d

Hence;

Æ• r= $\frac{C}{C0}=\frac{A\mathcal{E}}{d}.\frac{d}{A\mathcal{E}0}=\frac{\mathcal{E}}{\mathcal{E}0}$

∴ Æ• r= $\frac{\mathcal{E}}{\mathcal{E}0}$ or Æ• r.Æ• 0…………………..(i)

From equation (i) it follows that the capacitance of a capacitor ca also be given as

$C=\frac{\mathrm{A}\mathcal{E}\mathrm{r}.\mathcal{E}0}{\mathrm{d}}$ …………………………(ii)

A capacitor which leads to capacitance is a device for storing charges. Essentially all capacitors five metal plates separated by an insulator. The insulator are called dielectric in some capacitors dielectric used are oil, air, polyethylene, etc.

1997P2 Qn 6

Determine the capacitance of a parallel plates capacitor if it consist of a sheet of mica 0.1mm thick and surface area $15cm^2$, silvered on both sides and that mica has dielectric constant of 6.0

Solution

Given

$A=1.5x10^{-4}m^2$

$d= 1x10^{-4}=10^{-4}$

Æ• $0=8.854x10^{-12}$

Æ• r=6

Formula

$$C=\frac{\mathrm{A}\mathcal{E}\mathrm{r}.\,\mathcal{E}0}{\mathrm{d}}$$

$$C=\frac{1.5\mathrm{E}-4\mathrm{x}6\mathrm{x}8.85\mathrm{E}-12}{\mathrm{E}-4}$$

$C=7.96x10^{-11}$ Farad.

Question

A paper capacitor consists of a sheet of paper 35 width, 10mm long and $2.0x10^{2}$mm thick between sheets metal foil. If the relative permittivity of paper is 2.7, what is the capacitance? Æ• $0=8.85x10^{-12}$

Solution

Æ• r=2.7

$Area=35x10x10^{-3}m^{2}=3.5x10^{-1}m^{2}$

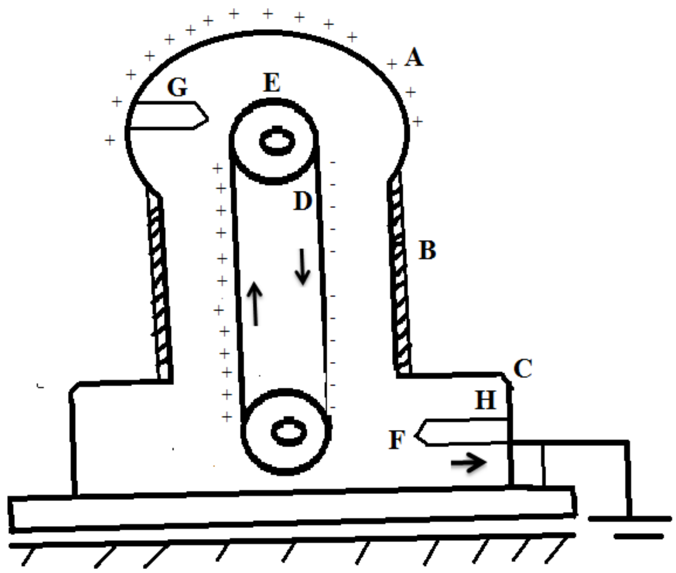
d=2.0 x10-2 x 10-3=2 x 10-5 m

Æ• 0= 8.85 x 10-12

VAN DE GRAAFF GENERATOR

If a positively charged body makes extrexual contact with an uncharged body the first will loose some of its charge and its potential will decrease while the sea will gain charge and its potential will increase, the flow of charge will cease when both bodies are at the same potential but there will be still remain some charges from the first body.

Consider the figure shown below of Van De Graaff generator



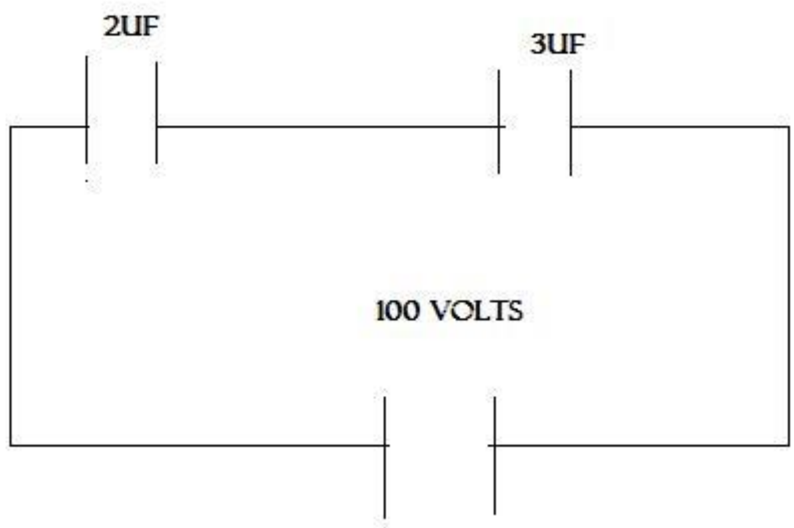
It consists of a hallow metal conductor A approximately spherical is supported and an insulating tube B mounted on a metal base C which is normally grounded. A non conducting ended belt D runs over two non conducting pulley E and F. Pulley F may be driven by hand or by small electric motor pulley and F are covered by different materials chosen so that when the belt D makes contact with F acquires positive charges while on contact with E it acquires negative charges

The charges developed on the belt as it makes contact with the pulleys, stick to it and are carried along by it. As the belt passes through it induces a charge on it , this conductor…..which because of the sharp point result in sufficiently high field intensity to ionize the air between the point and the belt. As the belt leaves the pulley, E it becomes negatively charged and the right hand side of the belt carries negative charge out of the upper terminal. Removal of negative is equivalent to addition of positive charge . So sides of the belt act to increase the net positive charge of the terminal A.

Question

A 100 volts battery is connected across two and 3ð• œ‡F capacitor in series. Calculate the potential difference between each capacitor and the energy/.

Solution



V=100u

From the series capacitor formula

$\frac{1}{C} = \frac{1}{C1} + \frac{1}{C2} = \frac{1}{2E-6} + \frac{1}{3E-6} = \frac{E6}{2} + \frac{E6}{3} +$......................

$\frac{1}{C} = \frac{5E6}{6}$

∴$C = \frac{6}{5E6}$

$C = 1.2 \times 10^{-6}$

∴Potential difference across each capacitor is given by

$V = \frac{Q}{C}$ but Q=θ=CxV=100x1.2×10^{-6}

$Q = 1.2 \times 10^{-4}$

$V1 = \frac{Q}{C} = \frac{1.2XE-4}{2E-6}$ =0.6x 10^2=60 volts

V2= $\frac{1.2XE-4}{3E-4}$ =0.4x 10^2=40 volts

Energy= $\frac{1}{2}CV^2 = 1/2 x 1.2 x 10^{-6} x 10^4 = 6 x 10^{-3} J$

Energy= $6 x 10^{-3} J$

The potential difference at each V1 = 60v and V2 = 40v and the total energy = $6 x 10^{-3} J$

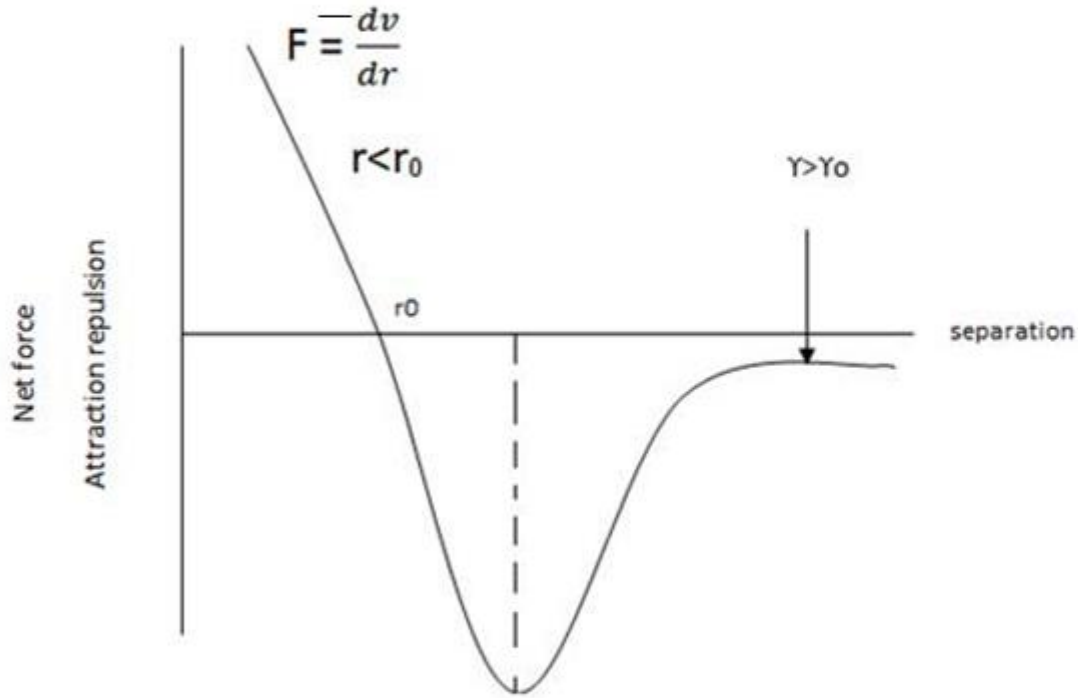
SURFACE TENSION

–

INTERMOLECULAR FORCE

Force which exist between the molecules of solid, liquid and gaseous are known as intermolecular forces. These forces arise from the two main causes (i) The potential energy of the molecules (ii) The thermal energy of the molecules this is K.E of the molecules and it depends on the temperature of the substance concerned

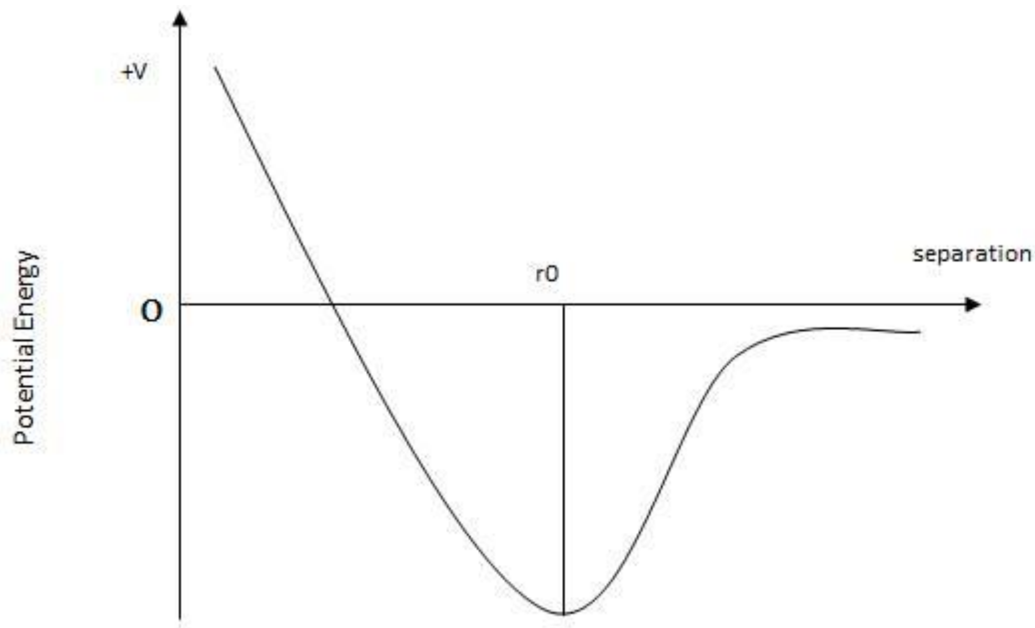
The figure below shown a force separation graph from inter molecular forces between two molecules.



ðš›o is the (separation) Equilibrium separation of the to molecules. The net force here is zero. When the separation of the two molecules is less than ro the net force is repulsive and when the separation is greater than ro the net force is attractive.

N:B the force between two molecules changes as they gradually brought together from the separation greater than ro to one less than ro. At large distance the force is negligible. As the molecules are brought closer there is a net attractive force which increases to a maximum value before diminishing to zero ro. Further bringing the molecules together results cute a net repulsive force.

The figure below shows that the potential energy separation graph for intermolecular forces between two molecules.



From the figure above than be observed that the separation of the molecules for the minimum potential energy is ro and the net force on the molecules at the minimum potential energy position is zero. The potential energy have positive values at small separations but negative values at large separations. Positive value means that the work has to be done molecules.

Molecules in solids are said to be in a condensed phase or state. They have relatively low thermal energy compared with their R.E U and vibrating about which the minimum of the curve

If the thermal Energy is increase by amount corresponding to cc the molecules oscillate between the limits X AND Y the molecules on the left of C experienced as greater force to wards it than when on the right. Thus the molecule return (quickly) e quicker to C^{I} therefore the means position is on the right of C^{I} this corresponds to a mean separation of molecules which is greater than ro . Thus the sold expands when its. Thermal energy is in creased.

SURFACE TENSION

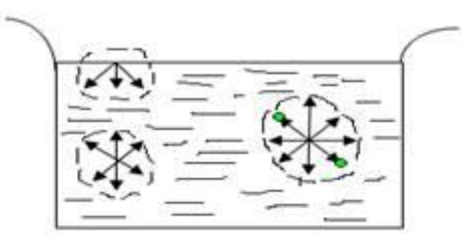
If the clean glass rod is dipped into the water and then removed some water aling to the glass rod. We say that the water wet the glass. The adhesion of molecules of water to glass must therefore be greater than the cohesion to glass.

If you carefully place the razor blade on the surface of water it remains in the surface even though it has more than seven times as dense as water

The water acts as through it has a thin elastic film

These as observations shows that surface of a liquid acts like an elastic skin covering the liquid or in state of tension. All liquids shows surface tension.

In many liquids the surface tension is not as strong as that of water or mercury. The cleaning action as that of water or mercury. The cleaning action of the water or mercury. The cleaning action of the water or mercury. The cleaning action detergents is due to their ability to lower the surface tension of the water making it possible for the water and detergents more readily into the pores of the substances being cleaned.



Molecules at A attracted in all directions by cohesion force of the surrounding molecules. A molecules at B attracted equally on all side but move strongly downwards. The molecules at c is not attracted in upward direction at all. There is unbalanced force tendency to pull such surface molecules towards the enter of the liquid and keep the free surface of the liquid as small as possible.

The effect of this unbalanced contracting force i to make the liquid behave as though it was contained in a stretched elastic skin. The tension in the is skin is the surface tension when the force acts on a liquid surface firm and distort it the cohesive if the liquid molecules exerts an equal and opposite forces to restore the horizontal surface.

Thus the weight of the supported razor blade produce a depression in the water surface firm. The cohesion of water molecules exert balancing upward force on the razor blade by intending to restore the surface of the liquid to its original condition.

Definition of the surface tension its unit and dimension

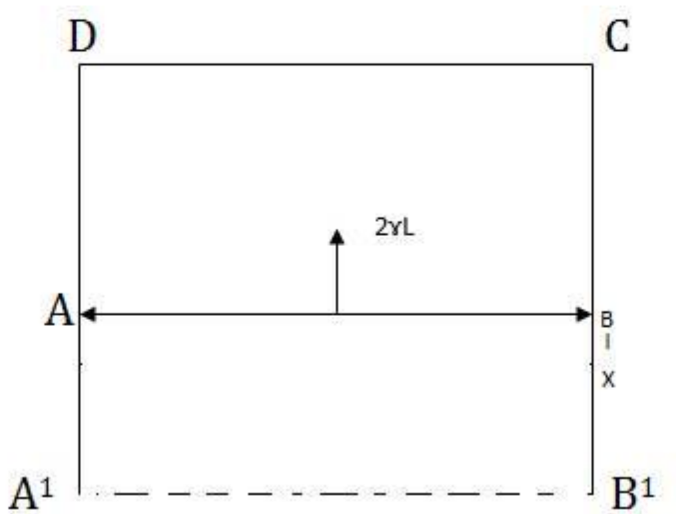
The surface tension γ of a liquid in the coefficient of the surface tension of the liquid as defined as the force per unit length acting in the surface at right angles to one side of the side of the line drawn in the surface. The unit of the É¤ is newton meter (NM the dimension of surface tension is

$$= \frac{dimension\ of\ force}{dimesion\ of\ length} = \frac{MLT^2}{L} = MT^2$$

SURFACE ENERGY

Molecules on the surface of the liquid of constant by pulled inwardly and therefore in order to bring the molecule from within the liquids to surface of the water. Some work has to be done. All molecules in the surface film has higher potential energy than those deep inside the liquid. This shows that work has to be done to create a surface is known as surface energy.

Consider the wire frame A B C D such that AB is moving over it. The surface tension will pull the wire inwards by a force F given by F = 2 É¤L



When É¤ is the surface tension and L is the length. AB the factor of 2 is because of the fact that the soap film has the surface let A 'B' be the film by small distance x. Let A 'B' be the new position of the A B Work done in enlarging surface

Energy = Force x distance

= 2 É¤ L.X

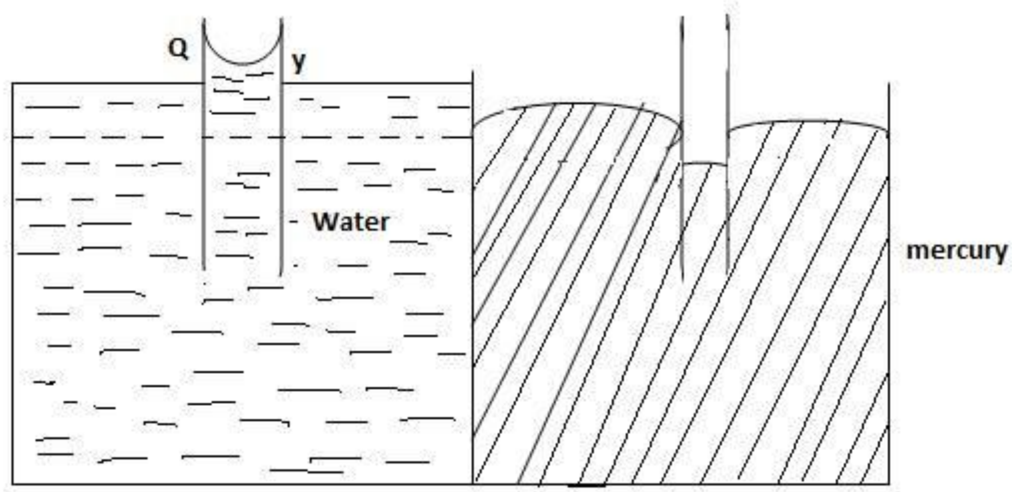
= É¤ 2LX

But the 2LX is the total increase in the surface area of the film.

Work done per unit area in enlarging area É¤. Thus the surface tension É¤ is defined as the work done per unit are a in increasing the surface area of the liquid under Isothermal conditions. Surface tension is therefore also called the free surface energy per unit area.

<u>CAPILLARITY TUBE É¤ ANGLE OF CONTACT</u>

When the capillary tube is immersed in water and one end is on the surface of water the following are observations in case of mercury level in the tube compared to outside level there is depression. All these explain the phenomenon of capillarity. The molecular explanation of this due to adhesive and forces of interaction



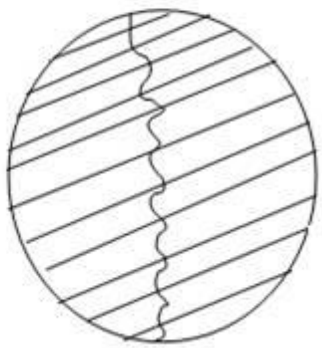
It is assured that the meniscus of water assumes a hemispherical shape and if the glass tube is less. But in uncleaned tube there occurs as angle of contact from the fig above the tangent X,y makes an angle θ with the table surface.

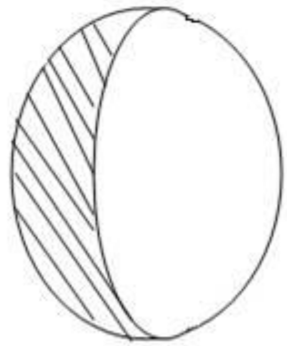
The angle ÆŸ is known as angle of contact and is measured through the liquid and its is an acute angle. In case of mercury the angle of contact is an obtuse.

Phenomenon of soap bubbles and the tension of the thread on the soap bubbles

When the thread is tied cross the ring and dipper the soap of the soap film and the thread is loose.

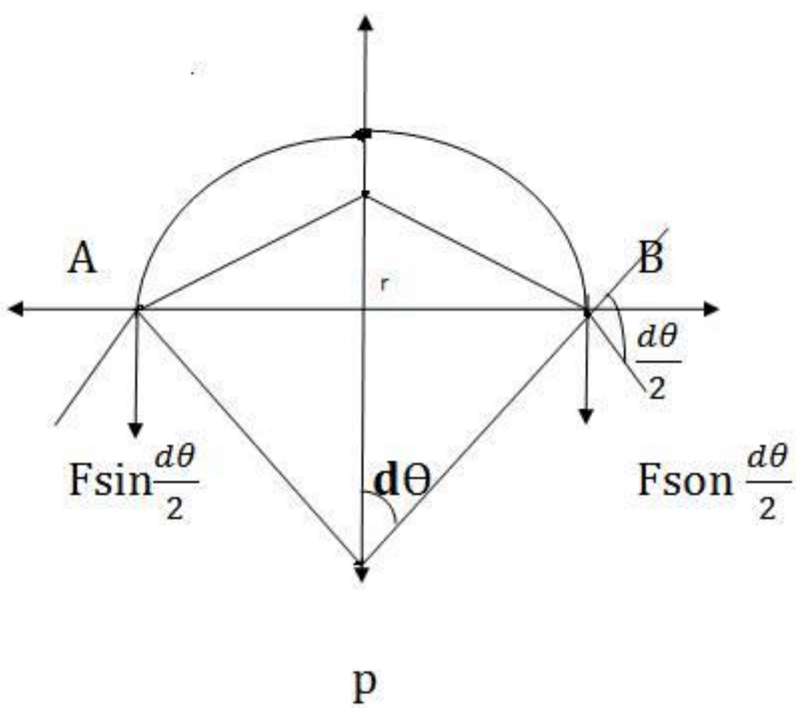
Loose thread with soap film   Tension of an thread after breaking one side





If the however the film on the one side of the thread is broken it is immediately drawn foward the other side of the tension which now exists along the boundary of the film.

A thread subjected to a pull by soap film in one side only.



Consider an element of the thread APB. Let the radius of curvature of P be r and the all which subtends the angle dÆŸ at the centre 0, the length AB = rdθ and the arc is in equilibrium under three forces

i) Forces acting tangentially at A and B.

Force 2 É¤ r d ÆŸ acting normally outward at P

Resolving in the direction of OP and at right angles to it the component of F at right angles to OP cancel and that in the direction of OP we have.

2Æ"rdÆŸ = F sin $\frac{d\theta}{2}$ + F sin $\frac{d\theta}{2}$

29rdÆŸ = 2F sin $\frac{d0}{2}$

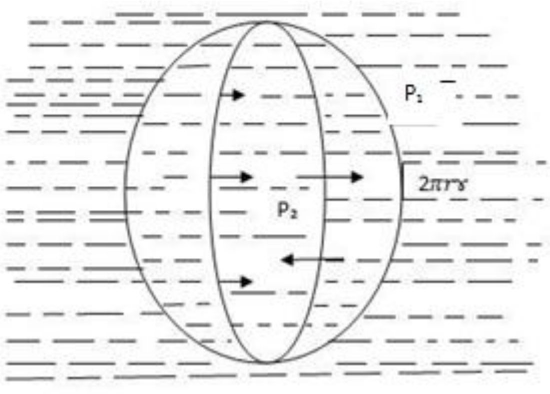
As dÆŸ $\rightarrow 0$ sin $\frac{d\theta}{2}$

Thus 2É¤ r dÆŸ =2F $\frac{d\theta}{2}$

F = 2É¤r

As the surface tension is the same wherever P is taken and the tension F in the string must be the same throughout its length it follows that must be the same wherever P is taken so that the tread sets in an arc of a circle.

Pressure difference in a bubble or curved surface.



Consider a bubble formed inside a liquid. The bubble is in equilibrium under three forces.

- The forces due to external pressure P_1 (FP_1)
- The surface tension force FÉ¤
- The force due to internal pressure P_2 inside the bubble (FP_2)

For equilibrium we have

Force due to external pressure + Force due to surface tension = Force due to internal pressure P_2

FP_1 + FÉ¤= FP_2

$P_1\pi r^2 + 2\pi r\gamma = P_2\pi r^2$

$P_2 - P_1 = \frac{2\gamma}{r}$

Now ($P_2 - P_1$) is the Excess pressure P in the bubbles over the out side p pressure

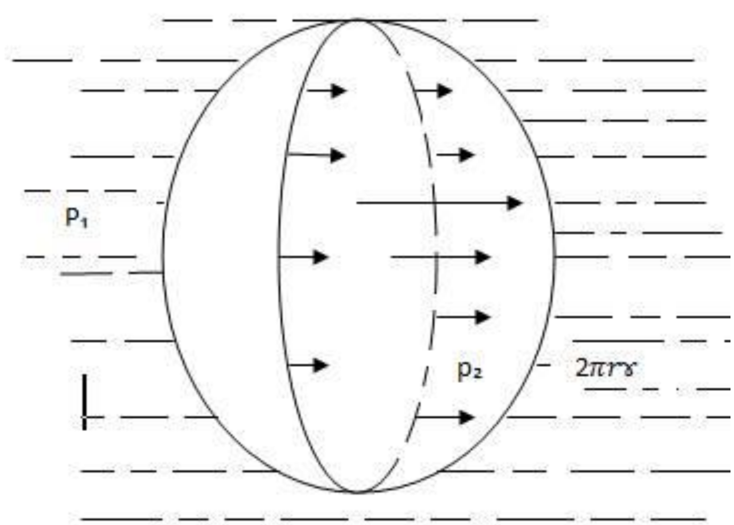
Excess pressure = $P_1 = \frac{2\gamma}{r}$

Although use consider a bubble the same formula for excess pressure holds for any curved surface or meniscus where r is its radius it circumference and r its surface tension provided the angle of contact is zero. If the angle of contact is ÆŸ the formula is modified by replacing É¤ by É¤COSÆŸ.

Thus excess pressure $P_1 = \frac{2\gamma COS\theta}{r}$

EXCESS PRESSURE IN A SOAP BUBBLE

A soap bubble with radius r has two surface so in case of liquid bubbles, the soap bubble is in equilibrium under three forces namely FP_1,Fγ and FP_2



FP_1 + FÉ¤ $=Fp_2$

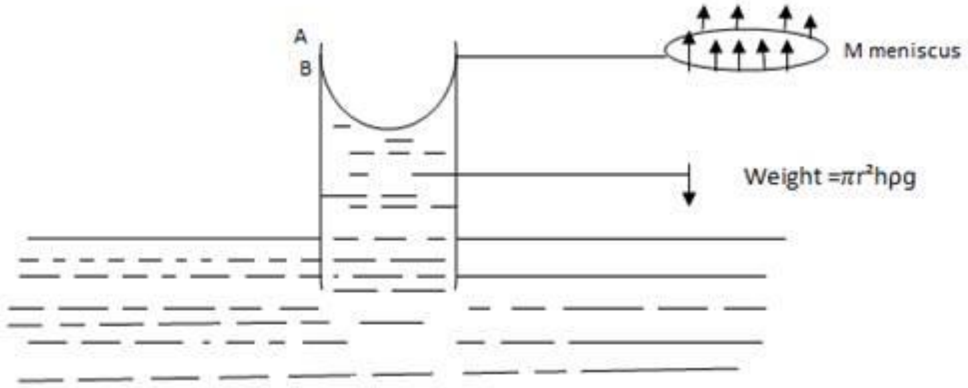
$$P_1\pi r^2 + 4\pi r\gamma = P_2\pi r^2$$

The factor of 2 in FÉ¤ occur because the soap bubbles have two surface.

$\therefore P_2 - P_1 = \frac{4\gamma}{r}$, hence the excess pressure $P = \frac{4\gamma}{r}$

MEASUREMENT OF SURFACE TENSION BY CAPILLARY TUBE.

Suppose a clean glass capillary tube is dipped into water, water level rises and the angle of contact is zero let Æ"be its magnitude of surface tension. The fig below shows a section of the meniscus M at B which is hemisphere.



Glass AB is tangent to the liquid at its meniscus.

The surface tension force acts along the boundaries of the liquid and the air vertically down wards on the glass.

According to reaction and action the glass exerts an upward force on the liquid meniscus if r is the radius of the capillary tube length of water in contact with glass is 2xr.

∴ $2\pi r\gamma$ = Upward force on liquid (I)

This force supports the light of the liquid column of height h above the outside level.

Volume of the liquid = $\pi r^2 h$

Mass of the liquid = $\pi r^2 hP$ where P is density

The weight of liquid = $\pi r^2 hPg$

From i) we have

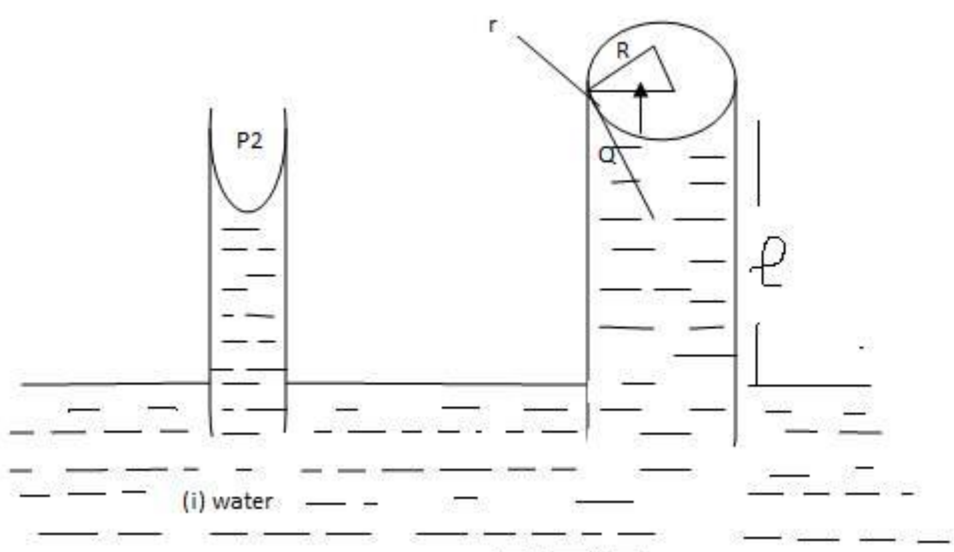
$2\pi r\gamma = \pi r^2 hPg$

$\therefore \gamma = \frac{rh\rho g}{2}$

Therefore, the angle of contact is assumed equal to zero. The weight of small amount of liquid above the meniscus, has been neglected.

CAPILLARY RISE AND FALL BY PRESSURE METHOD

In figure (I) below the liquid rises up the tube to a height (h), so that the pressure P_1 has a less than pressure P_2 and $\frac{2r}{r}$



If the capillary tube is dipped into water the angle of contact is practically zero. Fig (i) This is the atom ospheric pressure and P_1 is the pressure in the liquid we have

$P_2 - P_1 = \frac{2\gamma}{r}$

If **H** is the atmospheric pressure **h** is the height of the liquid the liquid in the table and ρ its density

P_2 =H and P_1 = H – h ρÉi

∴ H –(H - hρÉi) = $\frac{2\gamma}{r}$

∴ hρÉi = $\frac{2\gamma}{r}$

h = $\frac{2\gamma}{\rho g r}$

h increases as r decrease ie the narrow the tube the greater the height to which the water raised. Suppose tube is pulled down until the top of height

if the height l of the tube is above the water than the calculated value of h in the above formula the water surface at the top of the tube now meet at an angle of contact. This angle is an acute one. The radius R of the meniscus is greater than the capillary tube of radius r.

From figure we have COSÆŸ = r/R

From the excess formula for the meniscus

$P_2 - P_1 = \frac{2\gamma}{R}$

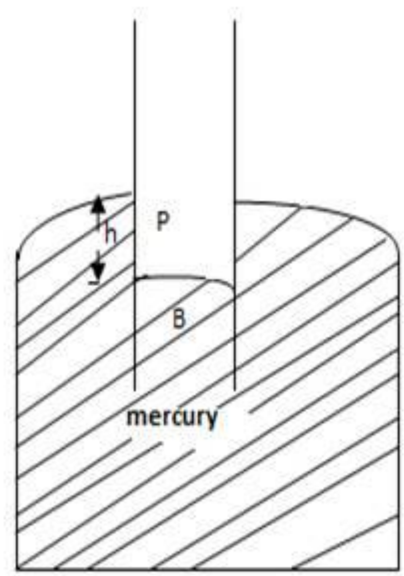
H – (H - hρÉi) = $\frac{2\gamma}{R}$ = Lpg

∴ L= $\frac{2\gamma}{RPg}$

Cos ÆŸ = $r/_R$ = $L/_h$

Let the depression of the mercury inside the tube of radius r be h in figure below, the pressure P_2 below the curved surface of mercury is the atmospheric pressure R

Out side the curved surface



P_2 – P_1 = 2É¤ CosÆŸ

When ÆŸ is the supplement of an obtuse angle of contact of mercury with the glass.

ie ÆŸ is an acute angle and its cosine is positive

But P_1 = H and P_2 = H + hÆ¿Éi

$$\therefore (H + h\rho \acute{E}i) - H = \frac{2\gamma Cos\theta}{r}$$

$$h\rho \acute{E}i = \frac{2\gamma cos\theta}{\gamma}$$

$$\therefore h = \frac{2\gamma cos\theta}{prg}$$

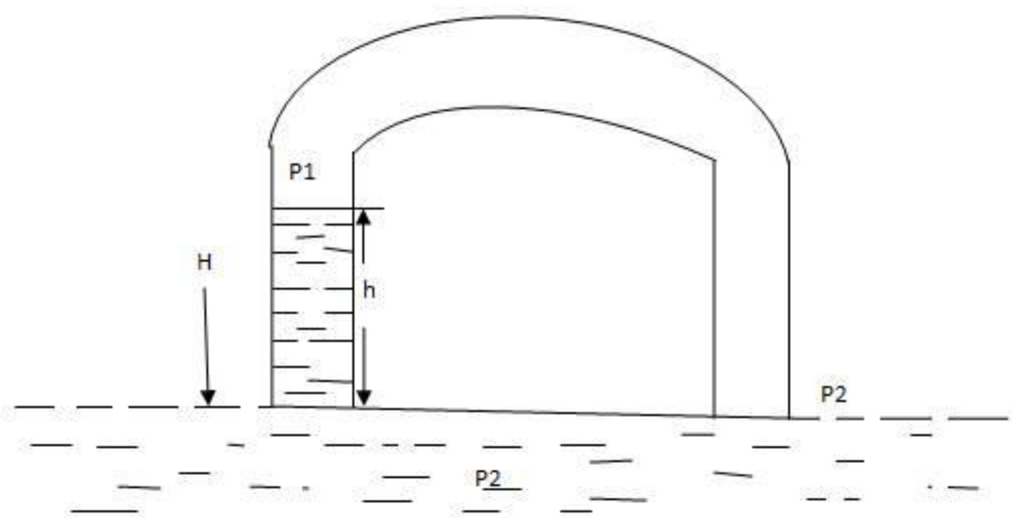
The height of depression h increased as the radius decrease.

Example

A glass U-tube is inverted with an open end of straight limbs of diameters respectively 0.5mm and 1.00min below the surface of water in the beaker.

The air pressure in the upper part is immersed until the maximum in one limb is level with the water outside. Find the level of water in the other limb.

Solution



Given

É¼â‚ = 0.025, É¼â‚‚ = 0.05

H -(H -Æ¿Éih) = $\frac{2\gamma}{r_1}$

$P_1 - p_2 = \frac{2\gamma}{r_1}$

But P_2 = H - ρÉih

P – H + ρÉih = $\frac{2\gamma}{r_1}$

$\frac{2r}{y_2}$ + ρhÉi = $\frac{2\gamma}{\gamma_1}$

ρhÉi = $\frac{2\gamma}{r} - \frac{2\gamma}{r_2}$

h = $\frac{2\gamma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

É¤ = 7.5×10^{-1}

Questions

1. Water rises to a height of 5cm in a certain capillary tube in the same tube. Mercury is depressed by 1.71 cm. Compare the surface tension of water and mercury specific gravity of mercury is 13.6, angle of contact for water is zero and that of mercury is 135°

2. A liquid of surface tension γ is used to form a film between a horizontal rod of length L and another shorter of rod of mass m supported from by the two light mentensible strings of egual length. Joining adjacent end of each rod. The film fells the verticle place within

rods and strings. What is the shape of each string?

Show how that the tension in each is $\frac{mg-2\gamma L}{2Sim\theta}$

Where ÆŸ is the angle which the tangent to each string make with the upper net.

3. A soup bubbles in Vacuum has a radius of 3cm and another soap bubbles in the vaccum has a radius of 6cm. If the two bubble coalesce under isothermal conditions. Calculate the radius of the bubble formed under isothermal conditions É¤ is constant.
