CALCULATING DEVICES

SCIENTIFIC CALCULATORS

EXAMPLES

Use a non programmable calculator to find the value of

1. $\frac{2.33\sqrt{9.54 \text{ x } 1.74-1}}{2.56}$

2. $\left(\frac{34.23 \text{ x} 213.56}{156 \text{ x} e^{\sqrt{\pi}} \ln 435}\right)^3$

3. $\frac{\sqrt[5]{\cos\frac{\pi}{4} \log e^1}}{16.9^{\log 3} \text{ x } \tan 30°41}$

Solutions

1. $\frac{2.33\sqrt{9.54 \text{ x } 1.74-1}}{2.56}$

$\frac{2.33 \text{ x} \sqrt{15.5996}}{2.56}$

$\frac{2.33 \text{ x } 3.9496}{2.56} = \frac{9.2026}{2.56}$

$= 3.5948$

2. $\left(\frac{34.23 \ x \ 213.56}{156 \ x \ e^{\sqrt{\pi}} In 435}\right)^3$

$\left(\frac{7310.1588}{156xe^{\sqrt{3.142}} \ln 435}\right)^3$

$\left(\frac{7310.1588}{156 \text{ x } 5.8826 \text{ x } 6.0753}\right)^3$

$$\left(\frac{7310.1588}{5575.2153}\right)^3$$

$$(1.31118)^3$$

$$\underline{= 2.254}$$

3. $\frac{\sqrt[5]{\cos\frac{\pi}{4}\log e}}{16.9^{\log 3} x \tan 30°41}$

Since $2\frac{\pi}{2} = \frac{360°}{2} = 180°$

$$\frac{0.933 \times 0.4343}{16.9^{\log 3} x \tan 30°41'}$$

$$\frac{0.405}{16.9\ (0.477) x \tan 30°41'}$$

$$= \frac{0.405}{2.2842}$$

= 0.1773.

Exercise

Use non - programmable calculator to calculate

1 $\frac{\log e^{\sqrt{\sin\frac{\pi}{2}}}}{1 - \sin 70°}$

2. $\frac{\sqrt{23}[\sqrt[3]{68.2}}{\sqrt{142 \ln 28.7}} + \tan\frac{\pi}{4}]$

3. $\frac{(\sin 42°47')(\cos 42°47')}{\tan 58°13' - 0.037}$

4. $\frac{e^{\sqrt{3}} \log e^{\frac{\pi}{6}}}{\sqrt{9 - \ln 432}}$

Solutions

1. $\frac{0.4343}{0.0603}$

$= 7.2023$

2. $\frac{4.7958[4.0857+1]}{40.002}$

$\frac{4.7958[5.0857]}{40.002} = \frac{24.39}{40.002}$

$= 0.6097$

3. $\frac{(\sin 42°47)(\cos 42°47)}{\tan 58°13 - 0.037}$

$47' = 47 \div 60$ min

$= 0.7833$

$13' = 13 \div 60$ min

$= 0.2167$

Sin $42^o47'$ = Sin(0.78333 + 42)

= sin 42.7833

Cos $42^o47'$= Cos (0.7833+42) =Cos 42.7833

∴Cos $42^o47'$=0.7339

Tan 58°13'=Tan 58+0.2167

Tan 58.2167=1.6139

$= \frac{0.6792 \times 0.7339}{0.6139 - 0.037}$

$= \frac{0.4985}{1.5763}$

$= 0.3161$

4. $\frac{e^{\sqrt{3}}\log e^{\sin\frac{\pi}{6}}}{\sqrt{9-\ln 432}}$

$\frac{5.6522(0.2171)}{-3.0684}$

$= \frac{1.2271}{-3.0684}$

$= -0.3999$

FUNCTIONS

BASIC CONCEPTS

RELATION

Is a set of ordered pairs.

R = $\{a,b\}$

Examples

1) Which of the following ordered pairs belong to the relation

R ={ (x,y) : y > x}

(1,2), (2,1), (-3,4), (-3,-5), (2,2), (-8,0), (-8,-3)

Solution

{(1,2), (3,4), (-8,0), (-8,-3)}

2) Give 5 ordered pairs which satisfies the relation

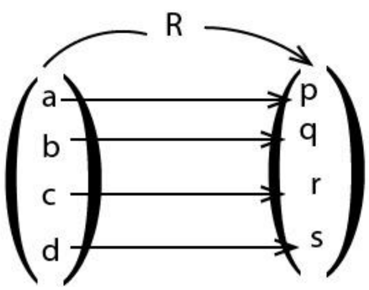
R = { (x,y):y=2x}

Solution

(1,2)(2,4)

Pictorial representation of a relation

If A= $\{a,b,c,d\}$ and B = $\{p,q,r,s\}$then the relation between A and B can be represented as follows.



Note

We say that elements of a set A mapped into set B i.e

$\longrightarrow$A B

Examples

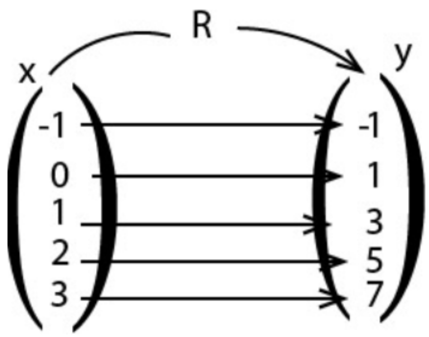
Given that A = $\{-1,0,1,2,3\}$ draw a pictorial representation of the relation

a) $R: \longrightarrow 2x+1$
b) $R: \longrightarrow x^2-2$

Solution

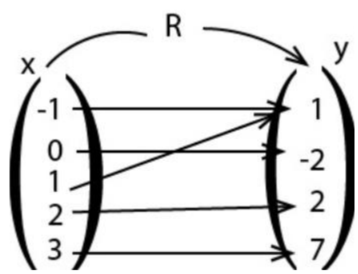
a) $R: x \longrightarrow 2x+1$

x	**-1**	**0**	**1**	**2**	**3**
y= 2x +1	-1	1	3	5	7



b) R ; x → $x^2 - 2$

x	-1	0	1	2	3
$y = x^2 - 2$	1	-2	-1	2	7



Domain and range of a relation

If R is the set of all ordered pairs [x, y] then;

Domain: $\{x: [x, y] \text{ belongs to } R \text{ for some } y\}$

Range : $\{y: [x, y] \text{ belong to } R \text{ for some } x\}$

Examples

1) (1) Let R = $\{[0,3]\ [1,4]\ [2,5]\ [3,6]\}$

What is the domain and range of R?

Solution

Domain = $\{0,1,2,3\}$ all values of x

Range = $\{3,4,5,6\}$ all values of y

2) (2) If R = { (x, y): x are real numbers and y = $x^2 + 1$}

Find all selected pairs which belong to R when the domain is

$\{-1,0,1,2,3\}$

Solution

R = $\{[-1,2][0,1][1,2][2,5][3,10]\}$

3) (3)Find the domain and range of a relation y = $2x^2$-1

Solution

Domain = $\{x: x \in IR\}$

Range of y =$2x^2$-1

$2x^2 = y + 1$

$x^2 = \frac{y+1}{2}$

$x = \frac{\pm\sqrt{y+1}}{2}$

y +1 $\geq$ 0

y $\geq$-1

$\therefore Range = \{y : y \geq -1\}$

Exercise

1) If R = $\{x, y\}$: x and y are real number $y = x^2$ find

a) a) A set of ordered pairs belonging to R where domain is

$\left(1, -2, \frac{-2}{3}, 3\right)$

b) A set of ordered pairs in R where range is $\left\{0, -27, 8, \frac{1}{25}\right\}$

2.) Let R = $\{[x,y]: y=3+1\}$ find

a) The set of ordered pairs which belongs to R from the following

$\left\{[1,2][0,-1]\left[\frac{1}{2},\frac{5}{2}\right][-2,-5][2,5][3,10]\right\}$

b) The domain obtained from a

c) The set of range obtained from a

Solution

x	**1**	**-2**	**-2/3**	**3**
$y= x^2$	1	4	4/9	9

$= \left\{[1,1][-2,4]\left[\frac{-2}{3},\frac{4}{9}\right][3,9]\right\}$

Solution

x	0	$-27^{1/2}$	$8^{1/2}$	$\frac{1}{5}$
Y	0	-27	8	$\frac{1}{25}$

$= \left\{[0,0][-27^{1/2},-27][8^{1/2},8]\left[\frac{1}{5},\frac{1}{25}\right]\right\}$

Solution

x	1	0	1/2	-2	2	3
y	2	-1	5/2	-5	5	10

$(\frac{1}{2}, \frac{5}{2})$ (-2,-5) (3, 10)

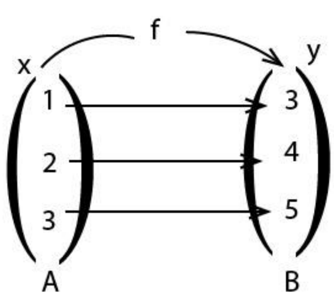
c) Domain = { $\frac{1}{2}$, -2, 3 }

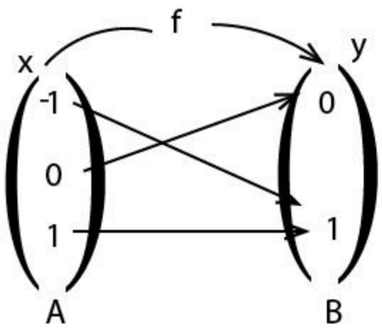
d) Range = { $\frac{5}{2}$, -5, 10}

FUNCTIONS

Is a rule relates a set of ordered pairs which relates two sets such that to each element of one set there is only one element of the second set

E .g





Name of a function

A function must be given a name. The common one is ' f '.

For example $f(x) = x^2$

Basic properties of function

A) The domain and range

Suppose set A is mapped onto set B i.e $A \rightarrow B$

The elements of a set A give us the domain and the elements of set B give us the range

Therefore

Domain of f= {x: x belongs to f for some y }

Range of f = {y: y belongs to f for some x }

Examples

Find the domain and range for some functions below

i) $f(x) = x^2$

ii) $f(x) = x^2 - 2$

iii) $f(x) = x^3$

Solutions

i) $f(x) = x^2$

$\therefore$ Domain = $\{x : x \in IR\}$

Range $f(x) = x^2$

$y = x^2$

$x = \sqrt{y}$

$\therefore Range = \{y : y \geq 0\}$

ii.) $f(x) = x^2 - 2$

Domain = $\{x ; x \in IR\}$

Range = $f(x) = x^2 - 2$

$Y = x^2$

$x^2 = y + 2$

$X = \sqrt{Y + 2}$

$\therefore$ Range = $\{y ; y \geq -2\}$

iii.) $f(x) = x^3$

Domain = $\{x ; x \in IR\}$

Range = $\{y ; y \in IR\}$

<u>Exercise</u>

Find the domain and range of the function

1. $y = 5x + 2$, $-7 \leq x \leq 7$
2. $y = \frac{1}{x^2}, -3 \leq x \leq 3$
3. $y = \sqrt{x^2 + 1}$, $-4 \leq x \leq 4$
4. $f(x) = \sqrt[3]{x}$
5. $g(x) = 2x - 1$

Solutions

(1) $y = 5x+2$, $-7 \leq x \leq 7$

Since domain = $\{-7 \leq x \leq 7\}$

$y = 5[-7] + 2$

$y = -35 + 2$

$y = -33$

$y = 5[7] + 2$

$y = 35 + 2$

$y = 37$

Range = $\{-33 \leq y \leq 37\}$

Domain = $\{-7 \leq x \leq 7\}$

2) $y = 1/x^2$, $-3 \leq x \leq 3$

$y = \frac{1}{(-3)^2} = \frac{1}{9}$

$\therefore$ Range = $\left\{y \leq \frac{1}{9}\right\}$

3) y = $\sqrt{x^2 + 1}$, -4 $\leq x \ \leq 4$

y = [-4] ^{2} + 1

= $\sqrt{17}$

y = 4.123

y $\cong 4.1$

Domain = $\{-4 \leq x \ \leq 4\}$

Range = $\{y \ \leq 4.1\}$

4) $f[x] = \sqrt[3]{x}$

y = $\sqrt[3]{x}$

y = $x^{1/3}$

Domain = $\{x{:}x \ \in IR\}$

y = $x^{1/3}$

x = y^3

Range = $\{y{:}y \in IR\}$

5) g (x) = 2x -1

y = 2x-1

Domain = $\{x{:}x \in IR\}$

y = 2x-1

2x = y +1

$\frac{2x}{2} = \frac{y+1}{2}$

x = $\frac{y+1}{2}$

Range = $\{y : y \in IR\}$

B) THE SUM OF FUNCTIONS

Examples

Let f (x) = x and g (x) = 2

Find the domain and range of f (x) + g (x)

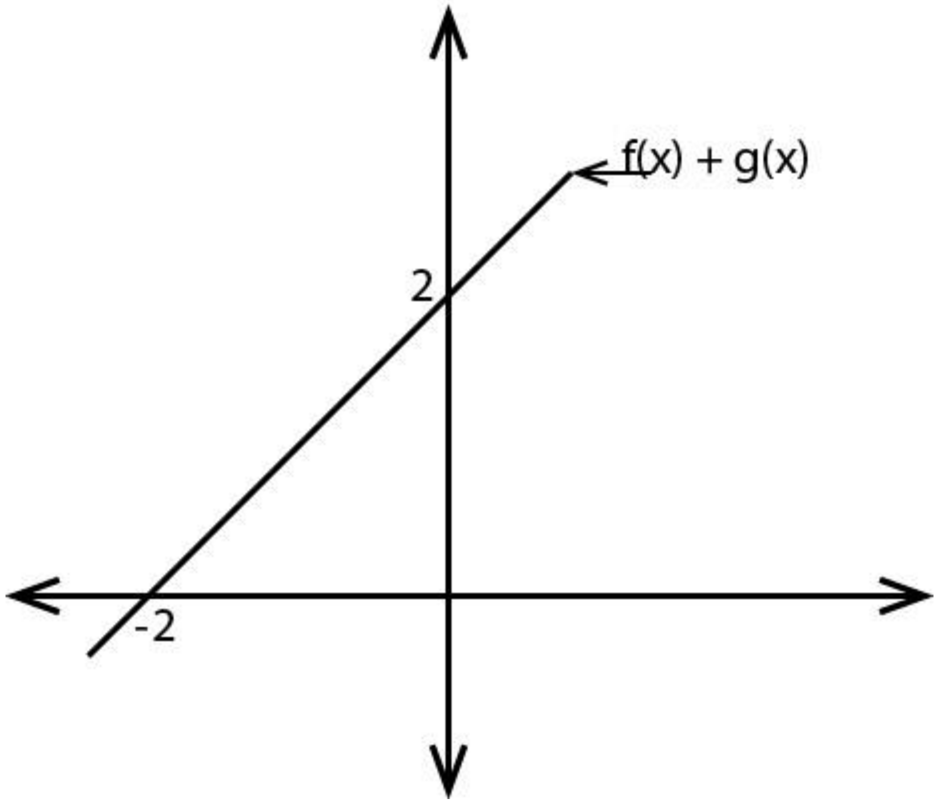
Solution

f (x) + g (x)

= x + 2

∴ Domain = $\{x : x \in IR\}$

Range = $\{y : y \in IR\}$



Note

The sum of the functions is also a function

C) The difference of functions

Let f (x)=-x and g (x) = x

Find the domain and range of

f(x) – g (x)

Solution

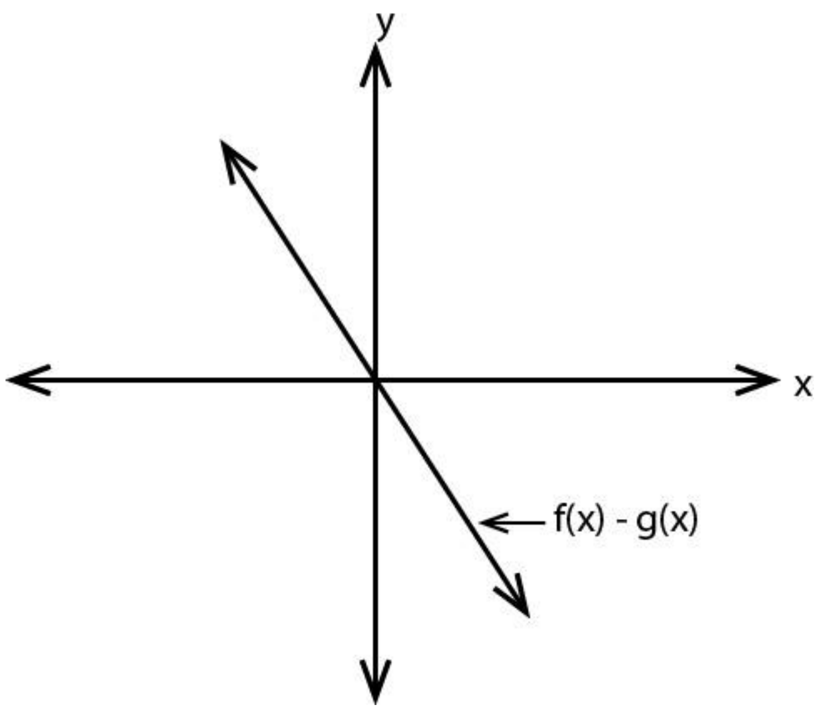
f (x) – g (x)

-x – x

-2x

Domain = $\{x: x \in IR\}$

Range = $\{y: y \in IR\}$



Note

The difference of the functions is also a function

<u>D) The product of functions</u>

Let f (x) = x^2 -1 and g (x) = -3

Find the domain and range of

f (x)g (x)

Solution

f (x)= (x^2-1) (-3)

= -3 (x^2 -1)

= $-3x^2 + 3$

Domain of x={x:x ∈IR}

Range of y
let y = $- 3x^2 +3$

$3x^2 = y -3$

$x^2 = \frac{y-3}{-3}$

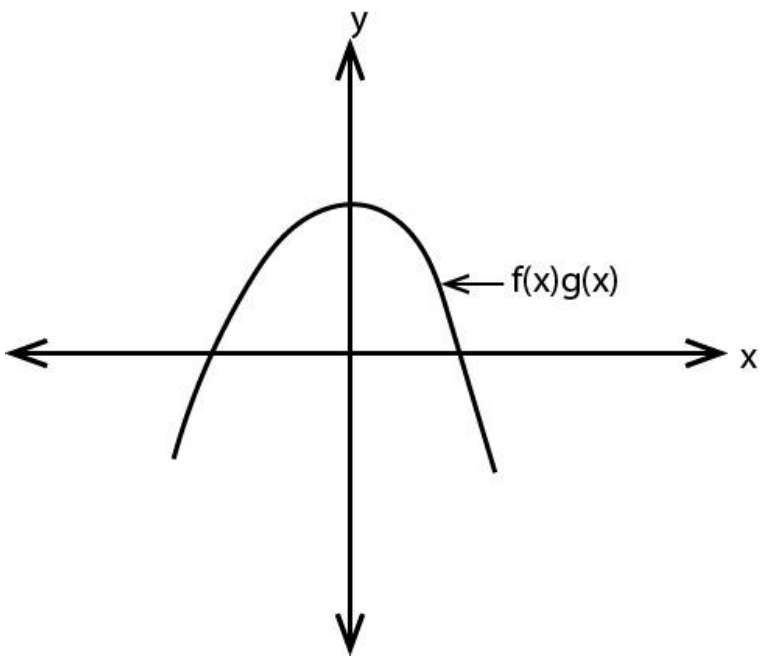
$x = \sqrt{\frac{y-3}{-3}}$

$\frac{y-3}{-3} \geq 0$

$y - 3 \leq 0$

$y \leq 3$

∴ Range = $\{y : y \leq 3\}$



Note

The product of the function is also a function

<u>E) The quotient of functions</u>

Let f (x) = x and g (x) = x^2

Find the domain and range of

f (x)

g (x)

<u>Solution</u>

$\frac{f(x)}{g(x)} = \frac{x}{x^2} = \frac{1}{x}$

Domain = $\{x : x \neq 0\}$

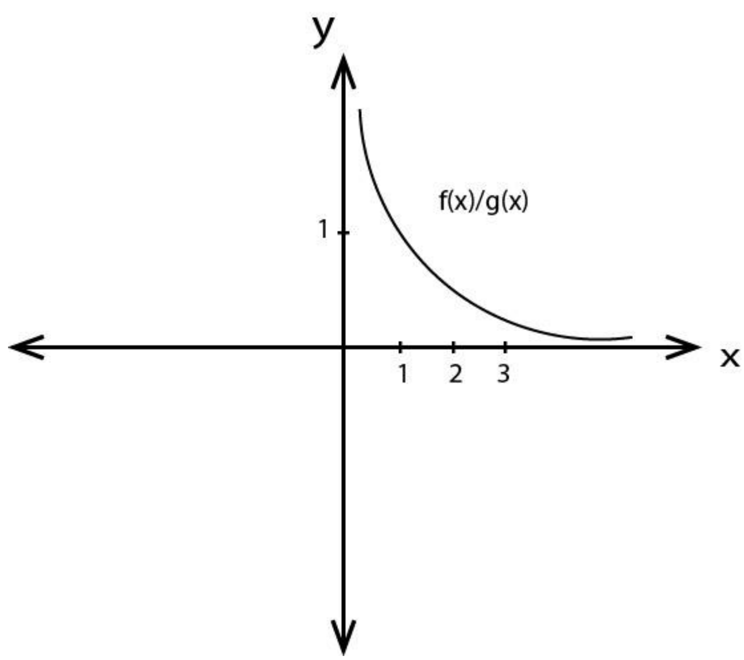
Range

y = 1/ x

xy = 1

x = 1/y

Range = $\{y : y \neq 0\}$



<u>Note</u>

The quotient of the function is also a function

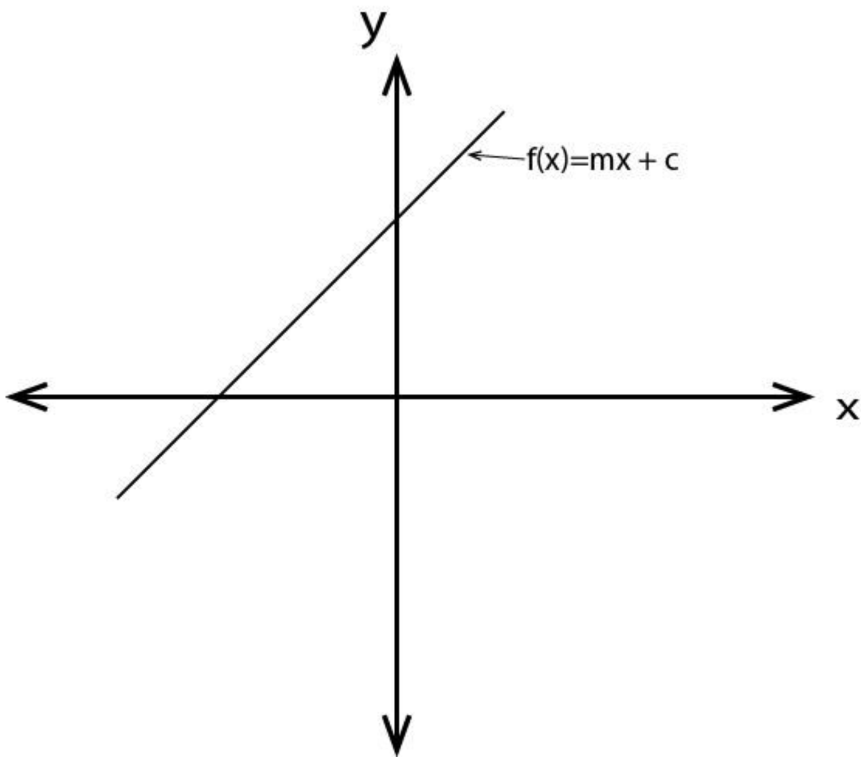
<u>GRAPHS OF FUNCTIONS</u>

<u>A) Linear function</u>

If the function of the form f (x) = mx +c

M = slope

C = constant



Examples

1.) Draw graphs of the following and give their domain and range

f (x) = 3x +1

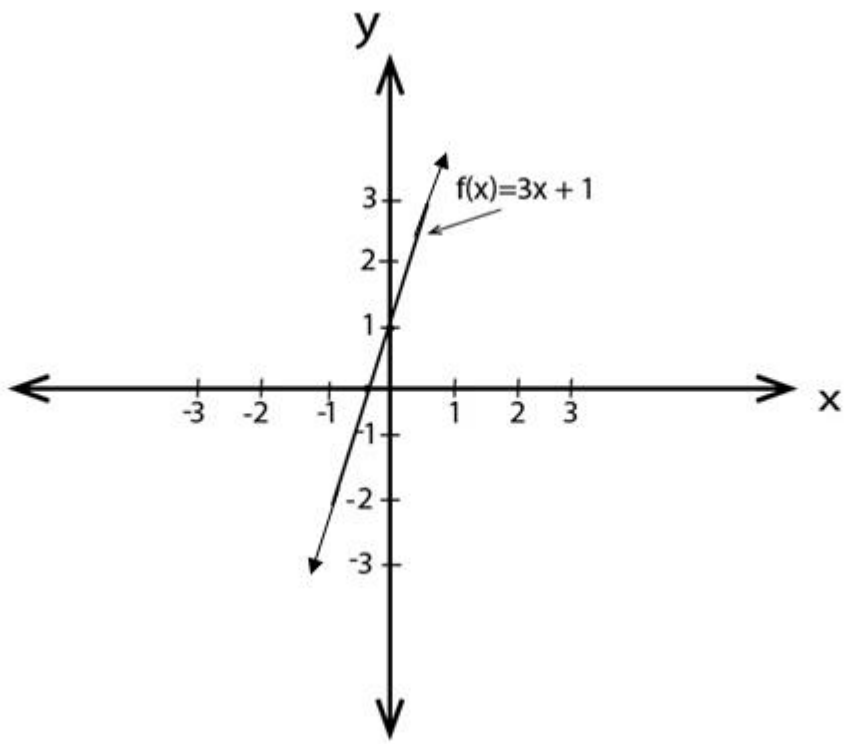
Solution

f (x) = 3x+1

Intercept

When x = 0, y = 1

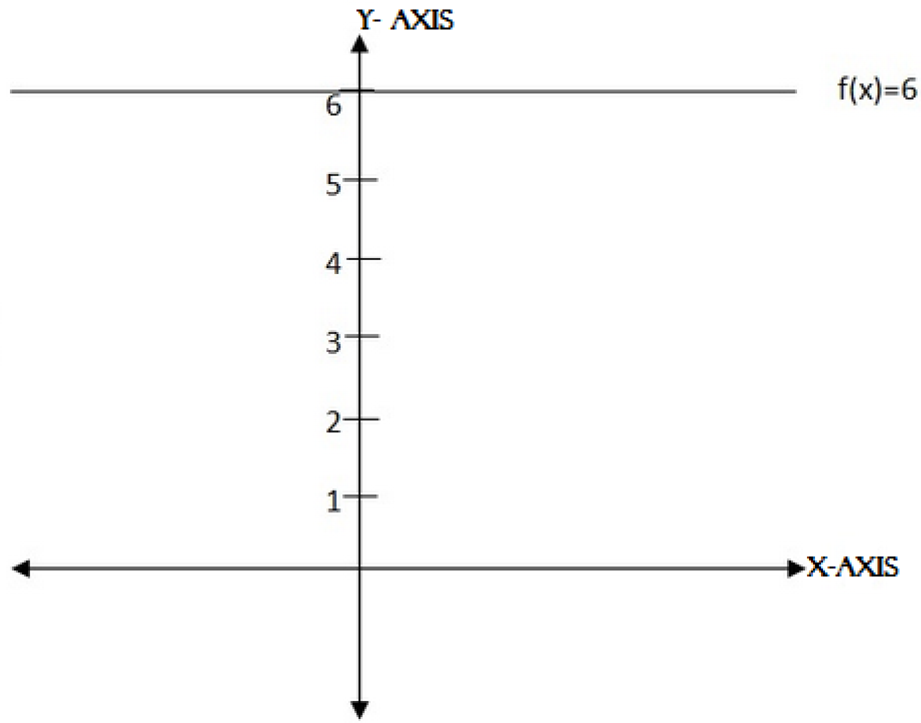
y = 0, x = $\frac{-1}{3}$



∴ Domain = $\{x : x \in IR\}$

Range = $\{y : y \in IR\}$

2) f(x) = 6



Domain $= \{x : x \in IR\}$

Range $= \{y : y = 6\}$

Note

f(x) = a is called the constant function

Exercise

1) Given that $f(x)=x^2$ and $g(x)=x$

Find the domain and range of

a) $f(x) + g(x)$

b) $f(x) - g(x)$

c.) $f(x)\, g(x)$

d) f (x) - g(x)

2) Draw graphs of

a.) f(x)= -3x+1

b.) f(x) = 3x-1

c) $f(x) = \frac{-x}{4} - \frac{1}{2}$

d.) f(x) = ½ - x

Solution

1a) f(x) +g(x)

$y = x^2 + x$

TURNING POINTS OF A QUADRATIC FUNCTION

Step function

Are functions which are not continuous.

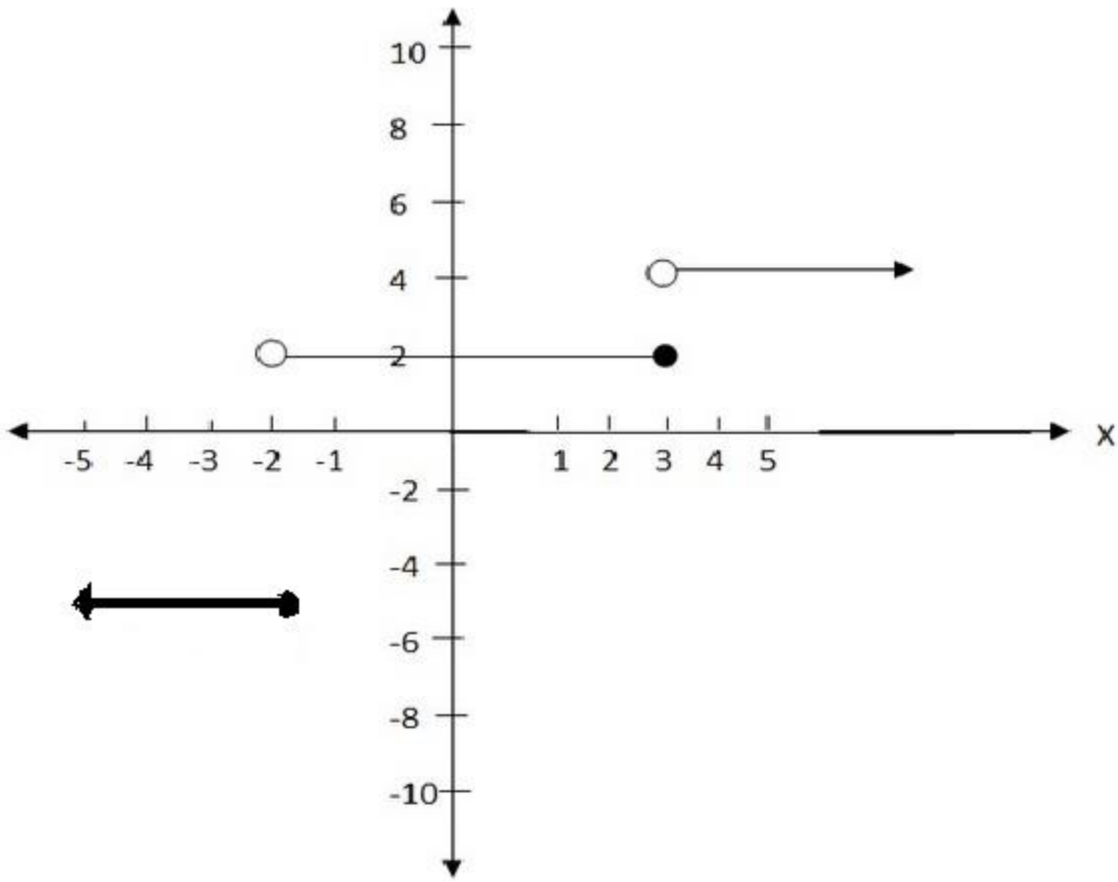
Example

Draw the graphs of the following function give its

Domain and range

$$f(x) = \begin{cases} -5 \text{ if } x \le -2 \\ 2 \text{ if } -2 < x \le 3 \\ 5 \text{ if } 3 < x \end{cases}$$

Solutions



Domain = $\{x : x \in IR\}$

Range = $\{-5, 2, 5\}$

C) <u>QUADRATIC FUNCTIONS</u>

Is the function of the form

$f(x) = ax^2 + bx + c$

Where a $\neq 0$

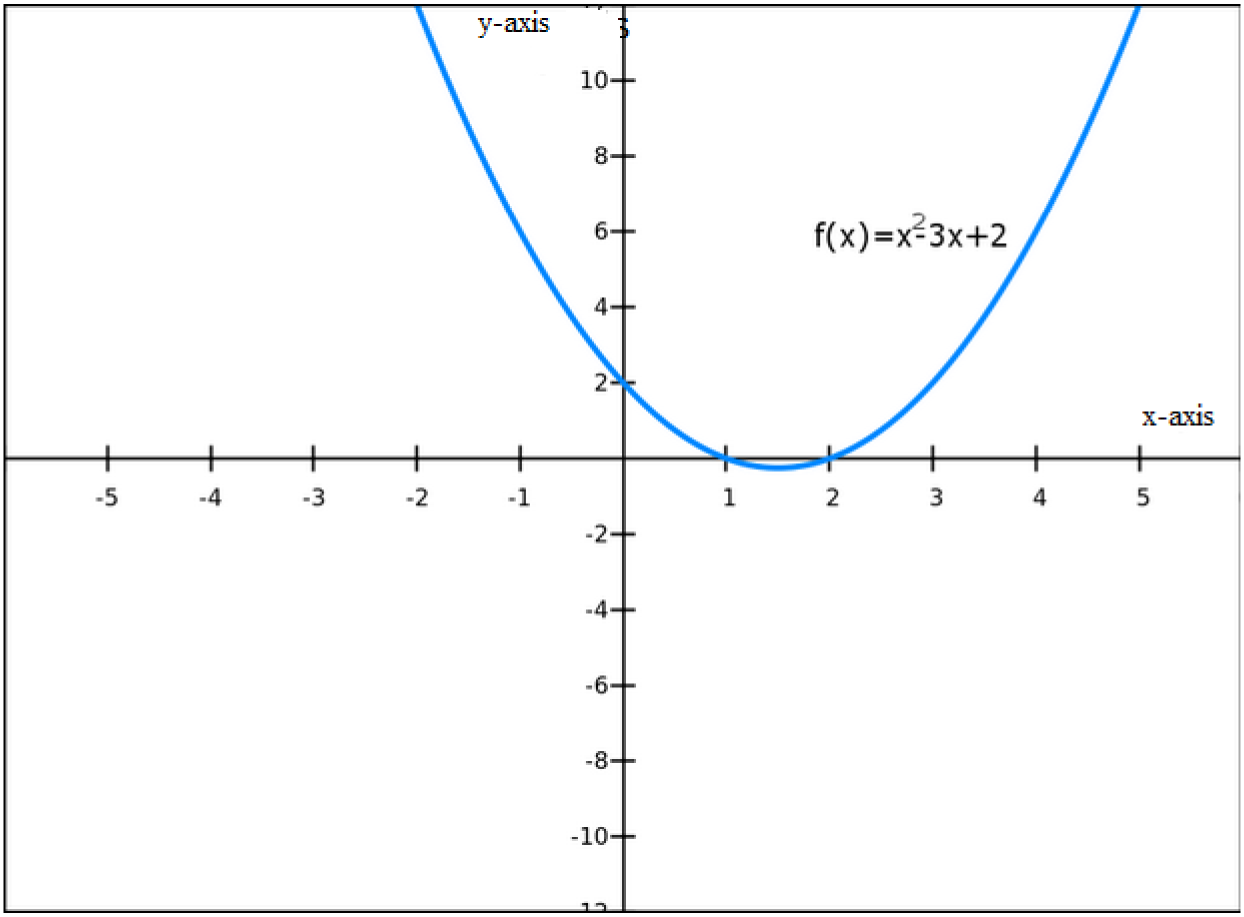
Example

Draw the graph of

$f(x) = x^2 - 3 + 2$

Solution

x	-2	-1	0	1	2	3	4	5
Y	12	6	2	0	0	2	6	12



Exercise

1. Draw the graphs of the following functions give the domain ad the range

i) f (x)= $\begin{cases} 2 \; if \; x < 0 \\ -3 \; if \; x \geq 0 \end{cases}$

ii) f(x)= $\begin{cases} -3 \; if \; x < 0 \\ 1 \; if \; x = 0 \\ 4 \; if \; x > 0 \end{cases}$

ii) $f(x) = \begin{cases} -1 \text{ if } x \leq -3 \\ -3 \text{ if } x > 4 \\ 3 \text{ if } x > 1 \end{cases}$

2. Draw graphs of the following functions

a) i) $y = [x-1]^2$

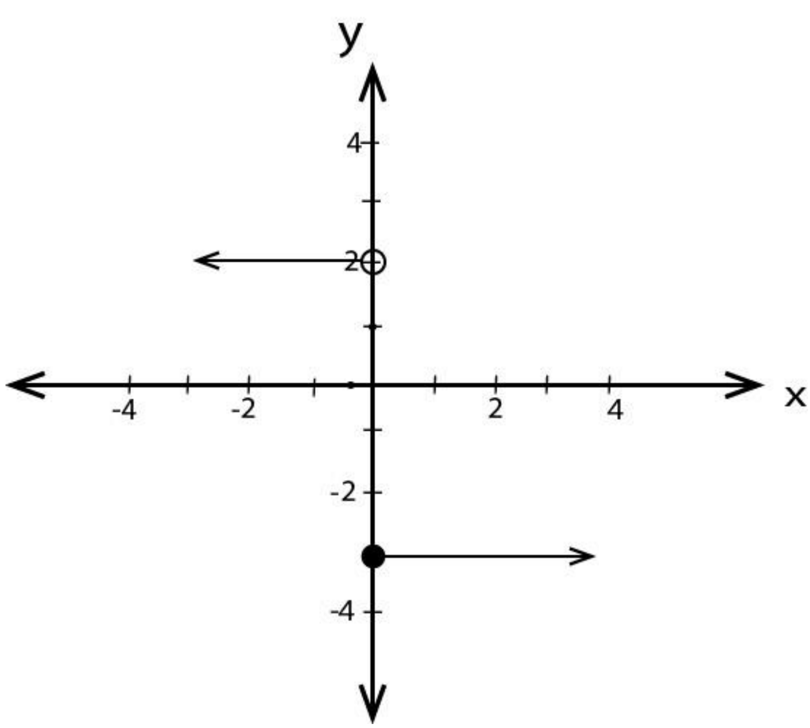
b) ii) $y = -2x^2 + x + y + 1$

c) iii) $f(x) = x^2 - x$

d) iv) $g(x) = -4x^2 - 1$

Solutions

$f(x) = \begin{cases} 2 \text{ if } x < 0 \\ -3 \text{ if } x \geq 0 \end{cases}$



Domain = $\{x: x \in IR\}$

Range = $\{y: y = 2, -3\}$

TURNING POINTS OF QUADRATIC FUNCTION

Given the function

f(x)=ax^2+bx+c where a, b and c are constants

By completing the square

$y = ax^2 + bx + c$

y = $a\left[x^2 + \frac{bx}{a}\right] + c$

= $\left[ax^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right] + c - \frac{b^2}{4a}$

= $a\left[x + \frac{b}{2a}\right]^2 + \frac{4ac-b^2}{4a}$

y = $\frac{4ac-b^2}{4a}$

$y = \frac{4ac-b^2}{4a}$

(Case :1)

If a > 0 then a(x + $\frac{b}{2a}$)^2≥0

Therefore

$y \geq \frac{4ac-b^2}{4a}$

∴ The function is minimum

When y = $\frac{4ac - b^2}{4a}$ and $x = \frac{-b}{2a}$

(Case 2)

If a < 0 then a [x +$\frac{b}{2a}$] $^2 \leq 0$

∴ y = $\frac{4ac - b^2}{4a}$

∴ The function maximum when $y \leq \frac{4ac - b^2}{4a}$ and $x = \frac{-b}{2a}$

Note

The maximum and the minimum points are the turning points of quadratic function

Examples (1)

Find the turning points of the function

$f(x) = x^2 - 3x + 2$ → y = x^2 - 3x + 2

$$y = \left(x^2 - 3x + \frac{9}{4}\right) + 2 - \frac{9}{4}$$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$$

$$y + \frac{1}{4} = \left(x - \frac{3}{2}\right)^2$$

Therefore x = 3/2 , y = - ¼

Therefore turning point = ($\frac{3}{2}$, -1/4)

Alternatively:

x = -b/2a , y = $\frac{4ac - b^2}{4a}$

a = 1, b = -3, c = 2

Therefore x = 3/2

y = -1/4

Example 2

Find the turning point of the function

$f(x) = -x^2 + 4x - 5$

$y = -x^2 + 4x - 5$

$y = [-x^2 + 4x + 4] - 5 + 4$

$y = -[x^2 - 4x + 4] - 1$

$y = -[x - 2]^2 - 1$

$y + 1 = -[x - 2]^2$

x = 2, y = -1

Turning point = (2, -1)

Example 3

Find the domain and range of the function

$p(x) = -x^2 + 4x - 5$

Solution

Domain = $\{x : x \in IR\}$

Range = $\left\{y : y \leq \frac{4ac - b^2}{4a}\right\}$

$y \leq \frac{4[-1][-5] - 4^2}{4[-1]}$

$y \leq \frac{20-16}{-4} = \frac{4}{-4}$

$y \leq \frac{20-16}{-4} = \frac{4}{-4}$

Range

$\{y : y \leq -1\}$

Exercise

1) Find the turning points of the following

a.) $f(x) = x^2 - 4x + 2$

b.) $f(x) = x^2 + 8x + 5$

c.) $f(x) = 5 - 6x - 9x^2$

d.) $f(x) = 3x^2 + 8x - 1$

e.) $f(x) = x^2 - 4x - 5$

2) Find the domain and range

a.) $f(x) = x^2 - 4x + 2$

b.) f (x)= $3x^2 + 8x - 1$

c.) f (x) = $-5 - 6 - 9x^2$

d.) f (x) = $2 - x - x^2$

e.) f (x) = $x^2 - 4x + 2$

<u>Solution</u>

a) $f[x] = x^2 - 4x + 2$

$y = [x^2 - 4x + 4] + 2 - 4$

$y = [x - 2]^2 - 2$

$y + 2 = [x - 2]^2$

Turning point= ∴ x = 2, y = -2

Turning point=(2,2)

b) $f[x] = x^2 + 8x + 5$

Solution

$f[x] = x^2 + 8x + 5$

$y = [x^2 + 8x + 16] + 5 - 16$

$y = [x^2 + 4]^2 - 11$

$y + 11 = [x + 4] - 11$

$y + 11 = [x + 4]^2$

x = - 4

And

$y = -11$

Therefore Turning point = (-4, -11)

c) $f(x) = 5 - 6x - 9x^2$

Since

$f[x] = 9x^2 + bx + c$

$y = -9x^2 - 6x + 5$

$y = [-9x^2 - 6x + 9] + 5 - 9$

$y = [-9x - 3]2 - 4$

$y + 4 = [-9x - 3]^2$

y = -4 , x = 1/3

Turning point = ($\frac{-1}{3}$, - 4)

d) $f(x) = 3x^2 + 8x - 1$

Solution

$y = 3x^2 + 8x - 1$

$y = [3x^2 + 8x + 16] - 1 - 16$

$y = [3x + 4]2 - 17$

$y + 17 = [3x + 4]2$

x = -4/3 , y = -17

Turning point =(-4/3, -17)

Alternatively

x = -b / 2a

x = -8/2 [3]

x = -8/6

x = -4/3

x = -4/3

$y = \frac{4ac - b^2}{4a}$

$= \frac{4\,[3][-1]-[8^2]}{4[3]}$

$\frac{-12-64}{12} = \frac{-76}{12}$

y = -19/3

$x = \frac{-4}{3}, y = \frac{-19}{3}$

Turning point = $\left[\frac{-4}{3}, \frac{-19}{3}\right]$

e) f (x) = x^2 – 4x -5

Solution

$y = x^2 - 4x - 5$

$y = [x^2 - 4x + 4] - 5 - 4$

$y = [x - 2]^2 - 9$

$y + 9 = [x - 2]^2$

x= 2, y = -9

Turning point =(2,-9)
Alternatively

x = -b/2a

x = -[-4][1]

x = 4/2

x = 2

$y = \frac{4ac - b^2}{4a}$

$= \frac{4[1][-5]-[-4^2]}{4[1]}$

$= \frac{-20-16}{4}$

$= \frac{-36}{4}$

= -9

∴ Turning point = (2,-9)

2 a) $f[x] = x^2 - 4x + 2$

Domain = $\{x: x \in IR\}$

Range = $\left\{y: y \geq \frac{4ac - b^2}{4a}\right\}$

y ≥ -8/4

y ≥ -2

Range = $\{y: y \geq -2\}$

∴ Domain = $\{x: x \in IR\}$

Range = $\{y : y \geq -2\}$

b) f (x) = $3x^2 + 8x - 1$

<u>Solution</u>:

Domain = $\{x : x \in IR\}$

Range = $\left\{y : y \geq \frac{4ac-b^2}{4a}\right\}$

y ≥ 19/3

Range ={ $y : y \geq 19/3$}

Domain = $\{x : x \in IR\}$

Range = $\{y : y \geq 19/3\}$

C) f (x) = $5 - 6x - 9x^2$

<u>Solution</u>

y = $-9x^2 - 6x + 5$

Domain = { x : x â„‡IR}

Range = { y : y ≤ $\frac{4ac- b^2}{4a}$ }

y ≤ 6

∴ Domain = $\{x : x \in IR\}$

Range = $\{y : y \leq 6\}$

<u>**Using intercepts and turning points to sketch the graph of quadratic functions**</u>

Example

Sketch the graph of

$y = x^2 - 4x - 2$

Solution

y – Intercept

When

x = 0, y = -2

x- Intercept

When

y= 0, x^2- 4x - 2 = 0

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x = $\frac{4 \pm \sqrt{-4^2 - 4(1)(-2)}}{2(1)}$

x = $\frac{4 \pm \sqrt{16+8}}{2(1)}$

x = $\frac{-b \pm \sqrt{24}}{2}$

x = $\frac{4 \pm 4.9}{2}$

x = $\frac{4+4.9}{2}$ or $\frac{4-4.9}{2}$

x= 4.5 OR x = -0.5

Turning point

x = $\frac{-b}{2a}$

x = $-\left[\frac{-4}{2}[1]\right]$

x = 2

$y = \frac{4ac- b^2}{4a}$

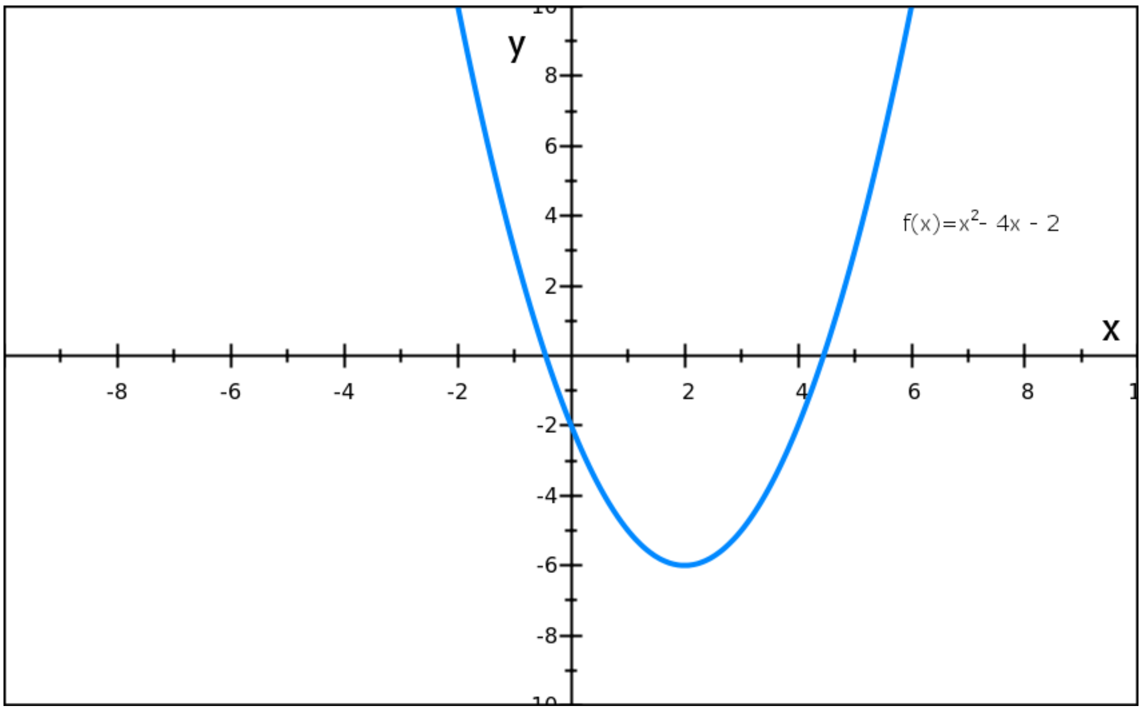
$y = \frac{4[1][-2]- [-4^2]}{4[1]}$

$= \frac{-8-16}{4} = \frac{-24}{4}$

= - 6

Turning points = (2,-6.)

Since a > 0 the function has a minimum value therefore the graph opens upwards



Exercise

Sketch the graphs of the following functions using intercepts and turning points

$f[x] = 5 - 6x - 9x^2$

Solution

$y = -9x^2 - 6x + 5$

y intercept

When x = 0, y = -5

x intercept

When y = 0, $-9x^2 - 6x + 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm \sqrt{-6^2 - 4(-9)(5)}}{2(-9)}$

$x = \frac{6 \pm \sqrt{36 - (-180)}}{-18}$

$= \frac{6 \pm \sqrt{36 + 180}}{-18}$

$= \frac{6 \pm \sqrt{216}}{-18}$

$x = \frac{6 \pm 14.7}{-18}$

Either x

$= \frac{6 + 14.7}{-18}$ or $\frac{6 - 14.7}{-18}$

x = 20.7/-8 or -8.7/-18

x = -1.2 or x = 0.5

Turning points

x= -b/2a

x = - [-6/2 [-9]

x =- 1/3 or – 0.3

$y = \frac{4ac - b^2}{4a}$

$y = \frac{-180 - 36}{-36}$

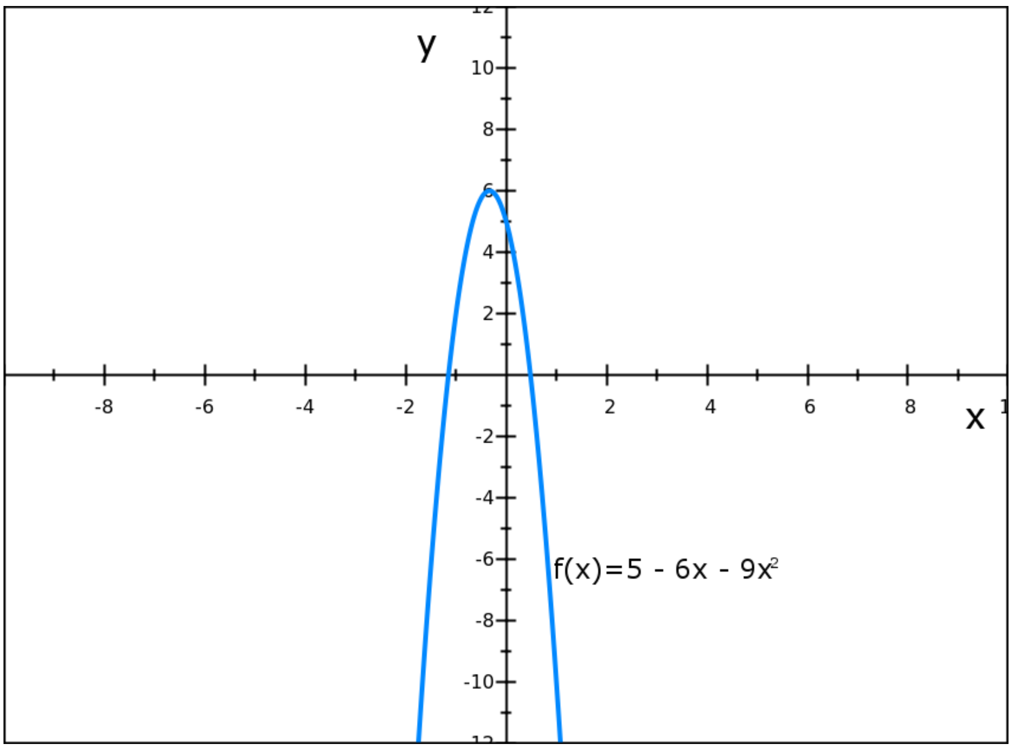
$y = \frac{4[-9][5] - [-6]2}{4[-9]}$

y = $\frac{-216}{-36}$

y= 6

Turning points = (-0.3 , 6)

Since a < 0 the function has a maximum value therefore the graph opens down wards



2) f(x) = $3x^2 + 8x - 1$

Solution

$y = 3x^2 + 8x - 1$

y – Intercept

When x=0, y = -1

x - Intercept

When y = 0, $3x^2 + 8x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64+12}}{6}$$

$$x = \frac{-8 \pm 8.7}{6}$$

Either

$$x = \frac{-8+8.7}{6} \text{ or } \frac{-8-8.7}{6}$$

x = 0.12 or -2.8

Turning points

x= -b/ 2a

x = -8/ 2 [3]

x = -8/6

x = -4/3 or -1.3

$$y = \frac{4ac - b^2}{4a}$$

$$y = \frac{4[3][1] - [8^2]}{4[3]}$$

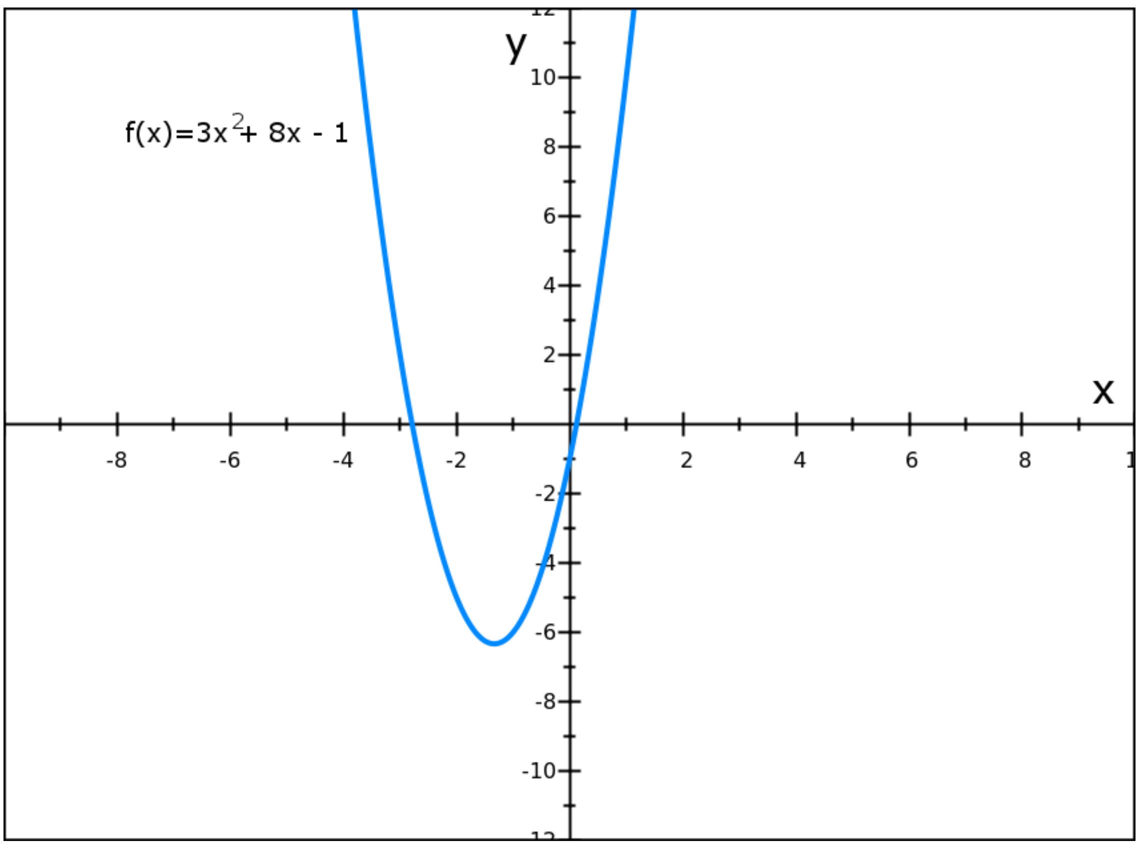
$$y = \frac{-12-64}{12}$$

$y = \frac{-76}{12}$

y = -6

Turning points = (-1.3,-6)

Since a > 0 the function has a minimum value therefore the graph opens upwards



D) **Cubic function**

Is a function of a form f (x) = $ax^3 + bx^2 - (x + d)$ where a, b, c and d are real numbers

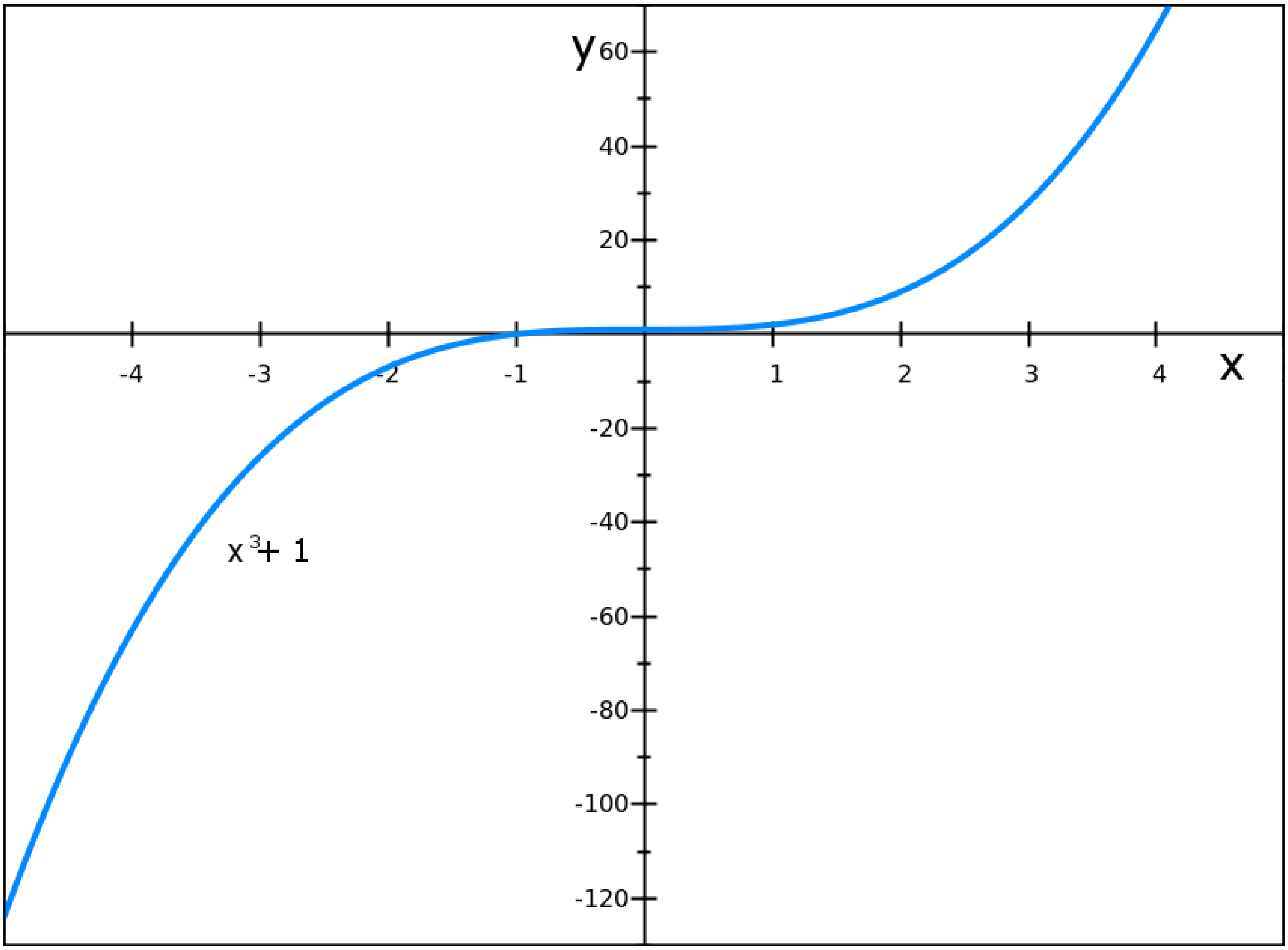
$a \neq 0$

Example

Draw the graphs of f (x) = $x^3 + 1$

Solution

x	-5	-4	-3	-2	-1	0	1	2	3	4
Y	-124	-63	-26	-7	0	1	2	9	28	65



E)Rational function

Is the function which can be defined by rational fraction such that both the numerator and denominator are polynomials.

$f(x)=\frac{a(x)}{b(x)}$

Where b(x)≠0

b $\neq 0$

Or is the function of the form f(x) $= \frac{1}{a \times + b}$ where a and b are real numbers.

<u>Example</u>

Sketch the graph of

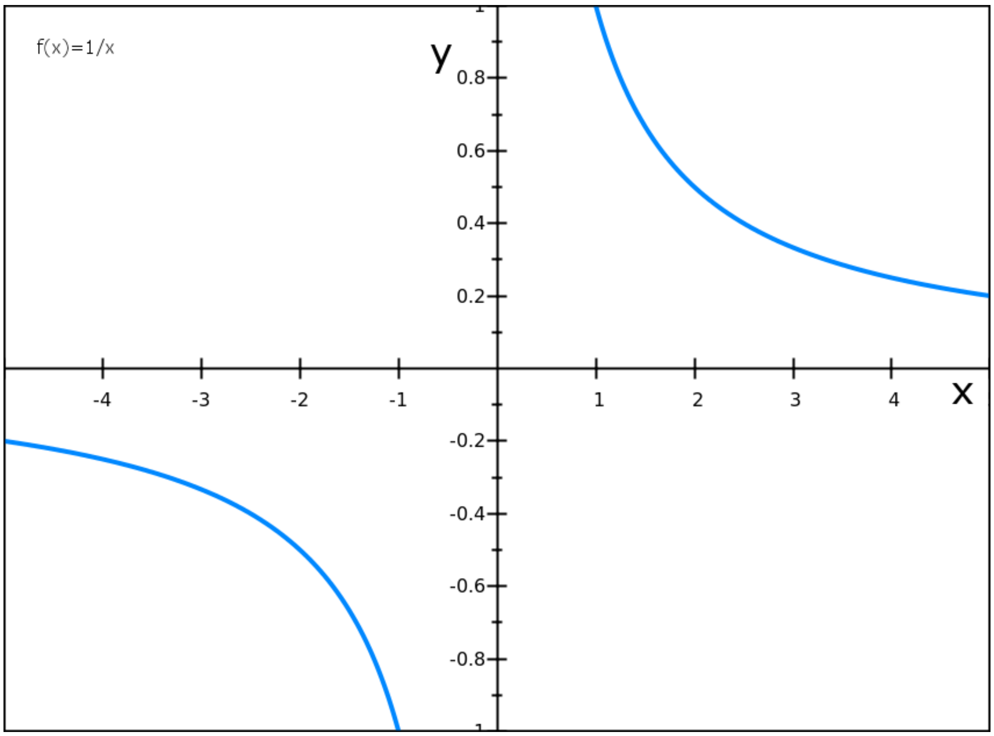
1) f (x) = 1/x

2). f (x) = 1/ x-2

Solution

1) f (x) = 1/x

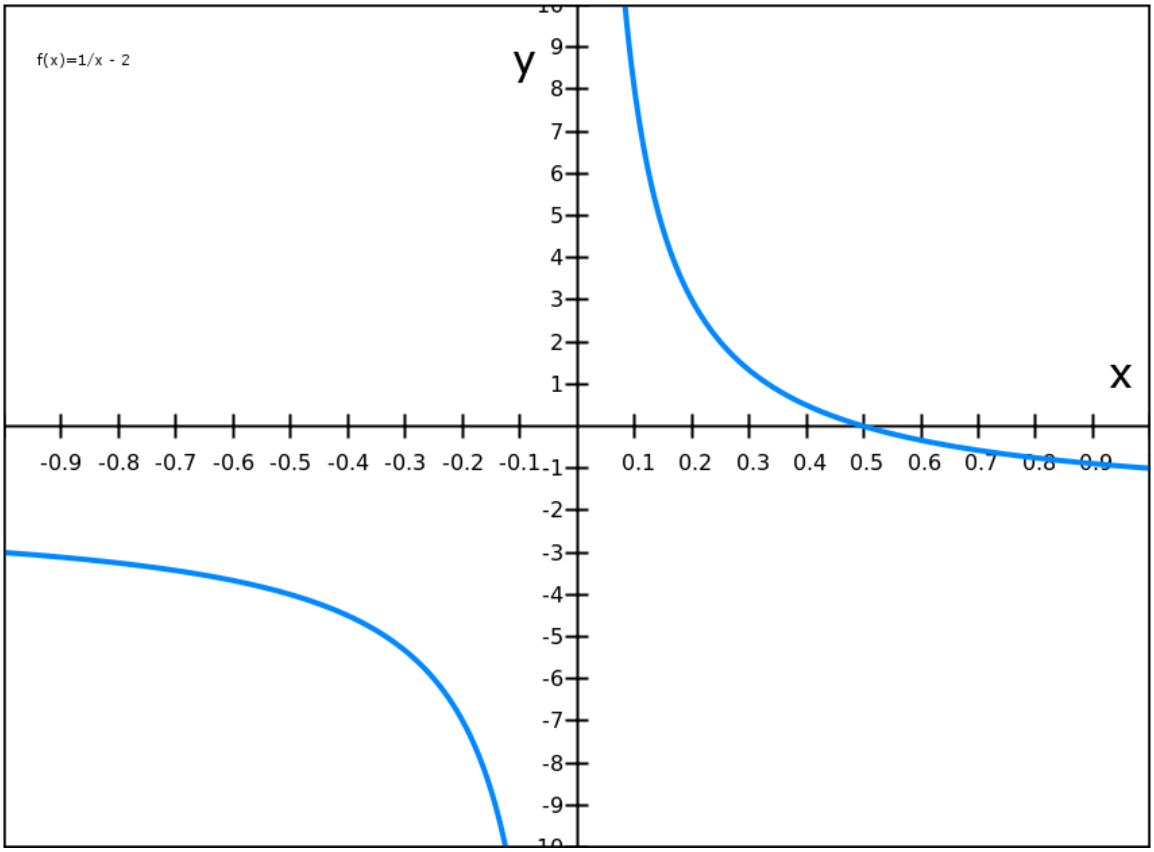
x	-5	-4	-3	-2	-1	0	1	2	3	4
f (x)	1/5	-1/4	-1/2	-1/3	-1	∞	1	1/2	1/3	1/4



Solution

2) f(x) = 1/x -2

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
f(x) f	$\frac{-1}{7}$	$\frac{-1}{6}$	$\frac{-1}{5}$	$\frac{-1}{4}$	$\frac{-1}{3}$	$\frac{-1}{2}$	-1	∞	1	½	1/3	1



NOTE:

The point where the function is not defined is called the a asymptote

Exercise

Draw the graphs of the following functions

i.) $f(x) = -8 - x^3$

ii.) $f(x) = 9 - x - x^2 - x^3$

iii.) $f(x) = x^3 - 3x^2 + 3$

iv.) $f(x) = 8 - 3x^3$

v.) $f(x) = 2/x-1$

vi.) $f(x) = 6/ x-6$

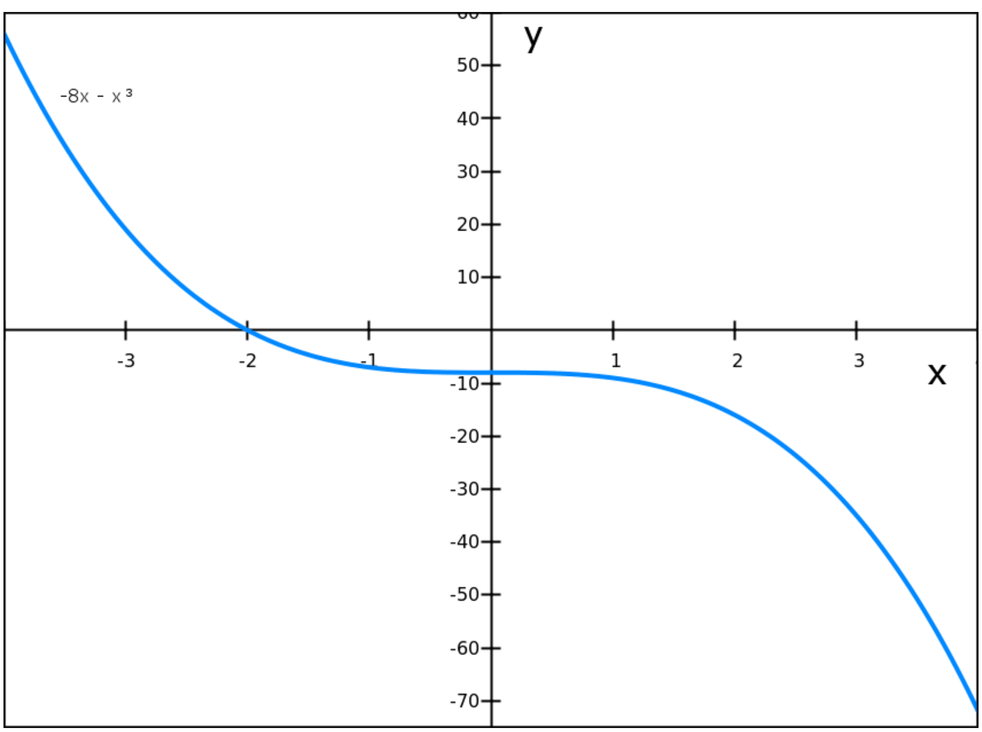
vii.) f(x) =1 / x+3

viii.) f(x) = 5/ x+1

Solution

F (x) = -8 - x^3

x	-4	-3	-2	-1	0	1	2	3	4
f [x]	56	19	0	-7	-8	-9	-16	-35	-72



Sketching graphs by using intercepts and asymptotes

Example

Sketch the graph of

y = $\frac{x}{x-4}$

Steps

1) Find the vertical and horizontal asymptotes

2) Draw the asymptotes on the xy plane by using dotted lines except when they coincide with the axes

3) Test the neighborhood points of the asymptotes to get the direction of the graph

4) Find the intercept [x – intercept and y – intercepts]

5) Join the arrows / points by using a free hand

Solution

∴ Vertical asymptote

Set $x - 4 = 0$

$x = 4$

Vertical asymptote is the line where $x = 4$

Horizontal asymptote

Make x subject

$y = \frac{x}{x-4}$

$y[x - 4] = x$

$yx – 4y = x$

$yx – x = 4y$

$x [y – 1] = 4y$

$x = 4y / y – 1$

Set

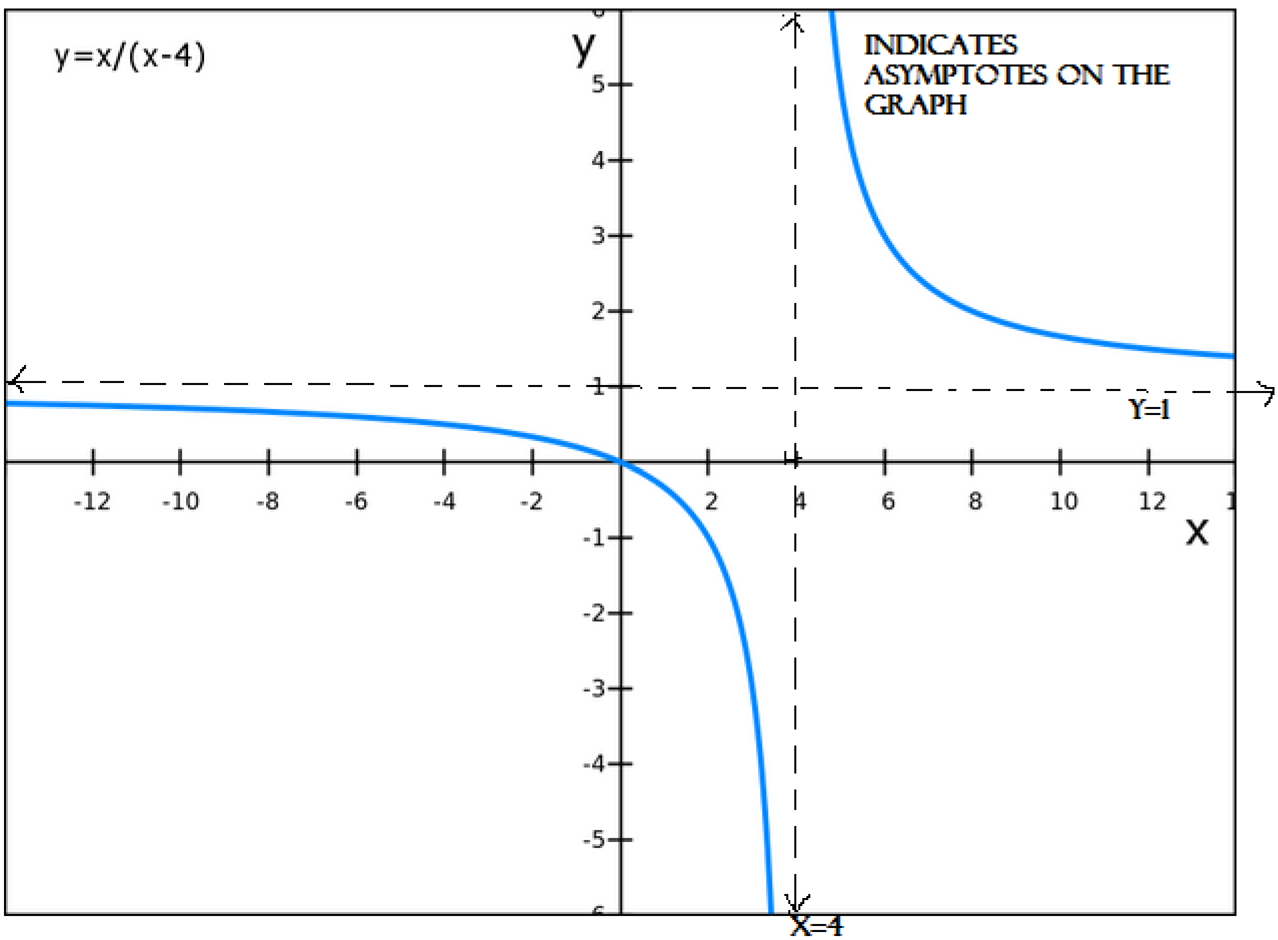
$y – 1= 0$

y = 1

∴ Horizontal asymptote is the line where y = 1

y = intercept = 0

x – Intercept = 0



Exercise

Sketch the graphs of the following functions by using intercept and asymptotes

1) $y = \frac{x}{x-3}$

2) y = $\frac{1}{2x-2}$

3) y = $\frac{-2}{x-1}$

4) y = $\frac{-5}{x-5}$

5) y = $\frac{2x}{x-1}$

Solution

y = $\frac{x}{x-3}$

Vertical asymptote

Set x - 3= 0

X = 3

Vertical asymptote of the line where x = 3

Horizontal asymptote

Set x subject

y = $\frac{x}{x-3}$

y [x -3]= x

yx – 3y = x

yx – x = 3y

$\frac{x[y-1]}{[y-1]} = \frac{3y}{y-1}$

y = $\frac{3y}{y-1}$

Set y -1 = 0

y = 1

∴ Horizontal asymptote is the line where y = 1

y – Intercept = 0

x – Intercept = 0

x = 1

2x - 2

Solution

Vertical asymptote

Set 2x-2 = 0

$\frac{2x}{2} = \frac{2}{2}$

x = 1

Horizontal asymptote

Make x the subject

$y = \frac{1}{2x-2}$

$y[2x - 2] = 1$

$2xy - 2y = 1$

$2xy = 1 + 2y$

ALGEBRA

FINITE AND INFINITE SERIES

Sequence

Is a set of numbers in a defined order .

For example

2, 4, 6, 8, 10 ....

3, 6, 9, 12, 15 ....

1, 4, 9, 16, 25............................

A term

Is the number in a sequence

For example

2, 4, 6, 8, 10,12............

The 3rd term is 6

The 6th term is 12

Series

When the term of the sequence are joined by + or – sign we get the series

For example

2 + 4 + 6 + 8 +10 +.....................

3 + 6 + 9 + 12 +15 + ...

1+4 + 9 + 16 +25 +................

Finite series

Is the series which has an end

For example

1+ 4 +9 + 16

Infinite series

Is the series which does not have an end.

For example

1 +4 + 9+ 16+ ....

Example

Find the term indicated against each of the following

-14, -10, -6 .... [6th term]

2 + 4 + 6 .... [10th term]

½ + ¼ + 1/8 + ... [8th term]

Solution

-14, - 10, -6....... ... [6th term]

The different by the way the consecutive is 4

[-14+ 4] = -10

[-10+4] = -6

$-14, -10, -6, -2, 2, 6$

The 6^{th} term is 6

1. $2+4+6+\ldots$ [10^{th} term]

$A_n = A_1 + [n-1] d$

$A_{10} = 2 + [10-1] 2$

$A_{10} = 2 + [9 \times 2]$

$A_{10} = 2+18$

$A_{10} = 20$

The different by the conservative term is 2

2 + 4 +8+10+12+14+16+18+20

The 10^{th} term is 20

½ + ¼ + 1/8 .... [8^{th} term]

The series suggest that the nth term is $\left(\frac{1}{2}\right)^n$

Therefore

The 8^{th} term is $(½)^8 = 1/256$

SIGMA NOTATION (Σ)

Is the Greek letter used to indicate the sum of the series in a condensed form.

Example

The sum of the first term of the series

2+4+6+ .... Can be written as

2+4+6+8+10+12

2[1] + 2 [2] + 2 [3] + 2 [4] + 2 [5] + 2 [6]

In condensed form

$\sum_{n=1}^{6} 2n = 2 + 4 + 6 + 8 + 10 + 12$,

$\therefore \sum_{n=1}^{6} 2n = 42$

Examples

1. Write $\sum_{n=1}^{5} 4n$ *in expanded form*

   4[1] + 4 [2] + 4 [3] + 4 [4] +4 [5]

   4 +8+12+16+20

   = 60

2. Write $\sum_{n=2}^{6} 3^{1-n}$ in the expanded form and hence find the sum

   $=3^{1-2} + 3^{1-3} + 3^{1-4} + 3^{1-5} + 3^{1-6}$

   $=3^{-1} + 3^{-2} + 3^{-3} + 3^{-4} + 3^{-5}$

   =1/3 + 1/32+ 1/33 + 1/34 + 1/35

   =1/3+1/9 +1/2 7 + 1/81+ 1/243

   $= \frac{121}{243}$

Find the sum of $\sum_{k=1}^{6}[5 + [k - 2]]$

=[5+ [1-2]]+ [5+ [2-2]] + [5 + [3-2]] [5 + [4-2] [5+[5-2] + [5+ [6-2]

= [5 +-1] + [5+0] + [5+1] + [5+2] + [5+3] + [5+4]

= 4+5+6+7+8+9

= 39

Exercise

Find the term indicated against each of the following series

a.) 3+6+9+................... [7^{th} term]

b.) 1+4+9+.................. [11^{th} term]

c.) 1 + 1/3+ 1/5............ [6^{th} term]

d.) 3+3+3+.................... [21^{st} term]

e.) 1+1/2 +1/3................. [9^{th} term]

Write the following series in expanded form

1). $\sum_{1=1}^{6} i$.

2). $\sum_{k=3}^{6} 2k$.

3). $\sum_{n=1}^{4} nn$.

Write the sum of the following

. $\sum_{k=0}^{10} [\, 6 + 2k$.

$$\sum_{m=0}^{8} (-2)^m$$

$$\sum_{n=1}^{5} \left(\frac{2}{3}\right)^n$$
.

Write each of the following by using sigma notation

1. ½ +1/3 +1/4+1/5+1/6+1/7+1/8

2. ½ +2/3 +3/4+4/5+5/6+6/7

3. 3+9+27+81+243+729

4. 3+6+12+24+48

Solutions

a) 3+6+9+..........

$A_n = A_1 + [n-1]\, d$

$A_7 = 3 + [7-1]\, 3$

$A_7 = 3 + [6x3]$

$A_7 = 3+18$

$A_7 = 21$

The seventh term is 21

b) 1+4+9+......... [11^{th} term]

$A_n = A_1 + [n-1]\, d$

$A_{11} = 1 + [11-1]\, 5$

$A_{11} = 1 + [10 \times 5]$

$A_{11} = 1 + 50$

$A_{11} = 51$

The 11^{th} term is 51

c) 1+ 1/3 + 1/5+ .... [6^{th} term]

$A_n = A1 + [n-1]\, d$

$A_6 = A1 + [6\text{-}1]\frac{-2}{3}$

$A_6 = 1+5(\frac{-2}{3})$

$A_6 = 1+\frac{-10}{3}$

$A_6 = (\frac{-7}{3})$

d) $A_n = A1 + [n\text{-}1]\ d$

$A_{11} = 1 + [11\text{-}1]\ 3$

$A_{11} = 1 + 10\ [3]$

$A_{11} = 1+30$

$A_{11} = 31$

<u>ARITHMETIC SERIES</u>

Is a series in which the difference between consecutive terms is the same.

The difference between the consecutive terms is known as the common difference

The nth term of the arithmetic series is given by

$An = A_1 + [n\text{-}1]\ d$

An = the nth term

A_1 = the 1^{st} term

n = is the number of terms

d = is the common difference

Example

Determine 12th term and 32nd term of the series

3+6+9……………

Solution

$A_n = A_1 + [n-1]\ d$

$A_{12} = 3 + [12-1]\ 3$

$A_{12} = 3 + [11 \times 3]$

$A_{12} = 3 + 33$

$A_{12} = 36$

The 12th term is 36

$A_n = A_1 + [n-1]\ d$

$A_{32} = 3 + [31 \times 3]$

$A_{32} = 3 + 93$

$A_{32} = 96$

The 6th term of an arithmetic series is 18 and the 10th term is 30 determine the common difference

Solution

$A_6 = A_1 + 5d = 18$…………. i

$A_{10} = A_1 + 9d = 30$……….ii

From {i}

$A_1 = 18 - 5d$ ….. [iii]

iii] Into [ii]

$18 - 5d + 9d = 30$

8+4d=30

$\frac{4d}{4} = \frac{12}{4}$

d = 3

Common difference = 3

Solution

From [iii]

A_1= 18-5[-5[3]

A_1 =18-15

A_1 =3

First term = 3

The sum of the first n- terms of an arithmetic progression

The sum of the first n terms of AP is given by

S n = n/2 [A_1 +A_n] ..... (i)

Where

n = number of terms

An = nth term

Sn = sum of n term

But An = A_1 + [n -1] d .... [ii

Sn = n/2 [A_1 + A_1 + [n -1] d

= n/2 [2A_1 + [n -1] d .... iii]

Examples

1) The sum of 18 terms is 812 if the common difference is 7 find the first term

2) Find the sum of the first 35 terms of the series

$3+7+11+15+\ldots\ldots\ldots\ldots$

Solution

$Sn = n/2\ [2A_1 + [n\text{-}1]\ d]$

$S_{18} = [18/2(2A_1 + [18\text{-}1)7]$

$S_{18} = 9\ [2A_1 + [17x7]$

$S_{18} = 9\ [2A1 + 119]$

$812 = 18A_1 + 1071$

$18A_1 = 1071\text{-}812$

$18A_1 = 259$

$A_1 = 14.4$

The first term is 14.4

Solution [ii]

$S_n = n/2\ [2A_1 + [n\ \text{-}1]\ d]$

$S_{35} = 35/2\ [2[3] + [35\text{-}1]\ 4$

$S_{35} = 17.5\ [6+[\ 34 \text{ x } 4]$

$S_{35} = 17.5\ [6+136]$

$S_{35} = 17.5\ [142]$

$S_{35} = 2485$

The sum of the first 35 term is 2485

Exercise

i) Find the 25^{th} term of the series

$3+9+15+$..........

ii) The 6^{th} term of the series is 33 and the 11^{th} term is 48

Determine the 45^{th} term of the AP series

iii) The first term of A.P is 31/3 and the 9^{th} term is 23 find the 44^{th} term

iv) The last term of the AP is 204 the common difference is 3 and the first term is 3 and the first term is 3 find the number of terms

v) Find the sum of series

$3+6+9$............ $+204$

Missing notes of Geometric, (G.P)

EXERCISE

1) If the 4^{th} term of a G.P is 9 and the 6^{th} term is 81 find

a) The common ratio

b) The first term

2) The nth term of the G.P

4, 8 , 16 , ... is 1024 find n.

3) Find the 9^{th} term of the G.P

2, -6, 18, -54, ...

4) The 3^{rd} term of the GP is 10 and the 6^{th} term is 80. find ;

a) The common ratio

b) The first term

5) If a GP is given by $G_n = 3^n$ find the sum of the first eight terms

6) Find the sum of the first twelve terms of the GP

2, -6, 18, -54

7) In a G.P the 11^{th} term is 128 times the 4^{th} term and the sum of the 2^{nd} and 3^{rd} term is 6, determine

a) The common ratio

b) The first term

c) The sum of the first 5 terms

d) The sum of the 3^{rd} to 6th terms inclusive

8)Find the sum to infinite of the series

-1/3 + 1/9- 1/27............

16+16/3+ 16/9..............

2/3+ 4/4 + 8/27...............

9)Express the following repeating decimal as a fraction

a) $0.\dot{7}$

b) $0.\dot{6}$

c) $0.\dot{3}\dot{8}$

d) $2.\dot{4}\dot{4}$

e) $3.\dot{3}\dot{1}3$

Solution

1) a) Common ratio = $\frac{G_6}{G_4}$

$r = \frac{Gir^5}{Gir^3}$

$r = r^2$

$r^2 = \frac{81}{9}$

$\sqrt{r^2} = \sqrt{9}$

r =3

$\therefore common\ ratio = 3$

c) $G_4 = G_1r^{(4-1)}$

$9 = G_1r^3$

$9 = G_1r^3$

$9 = G_1 27$

$G_1 = 9/27$

$G_1 = 0.3$

2) 4, 8, 16,............... 1024

Given, Gn = 1024,G1= 4,r=2

Using $G_n=G_1r^{n-1}$

required to find the number of terms, n

$G_n=G_1r^{n-1}$

$1024 =(4)(2)^{n-1}$

$256 =2^{n-1}$

$2^8=2^{n-1}$

Equating their powers

$8=n-1$

$n=9$

3) $G_n=G_1r^{n-1}$

$G_9=(2)(-3)^{(9-1)}$

$G_9=(2)(-3)^8$

$G_9=2(6561)$

$G_9=13122$

RATIOS AND VARIATIONS

RATIO

Is the comparison of two quantities with the same unit.The ratio is related with fractions

For example

a/b can be written as a: b

Examples

1. Form four class has got 40girls and 30 boys what is the ratio of boys to that of girls?

2. The area of a circle and its radius are in a ratio of 22:1 if the area is 154 cm^2 find the circumference of the circle

Solution

1. Let" b " be the number of boys and "g" be the number of girls

b: g = 30:40

∴ b : g = 3:4

Solutions

2) Let ‘a’ be the area of the circle ‘’r ‘’be the radius of the circle

Then

a:r =22:1

$\frac{a}{r} = \frac{22}{1}$

154cm² = 22

$r = \frac{154\ cm^2}{22}$

r = 7cm

Circumference of the circle = 2 π r

$\frac{2x22x7}{7}$

∴ The circumference is 44cm

Proportions

When two ratios are compared by using equal signs it is called proportions i.e. you may have two ratios may be a : b = c : d

Example

(i) The ratio of boys to girls at Jitahidi sec. school is 3:4 if the school has 60 boys find

a) → The number of girls

b) →The total number of students

ii) Divide Tshs 360,000 in the ratio 2:3:5

Solution

(i)Let "n" be the number of girls at Jitahidi and "a" be the number of boys a: n = 3:4

$$\frac{60}{n} = \frac{3}{4}$$

$$\frac{60x24}{3n} = \frac{240}{3n}$$

n = 80

Therefore the number of girls = 80

Total number of students [60+80] = 140

(ii)2+3+5 =10

2/10 x 360,000 = 720,00

3/10 x 360,000= 108,000

5/10 x 360,= 180,000

DIRECT PROPORTIONS

When one variable increases the other variable also increases and vice versa.

i .e $y \propto x$

y = kx

Where k is proportionality constant

Example

If y = 9 when x =3 find the value of x when y = ¼ given that y is directly proportional to x

Solution

$y \propto x$

$k = \frac{y}{x}$

$y = kx$

If y=9, then x=3,

$k = \frac{9}{3}$

y=3x

Given

$y = \frac{1}{4}$

$x = \frac{y}{3}$

$\frac{1}{4} x \frac{1}{3}$

$\frac{1}{12}$

INVERSE PROPORTIONS

When one variable increases the other variable decreases and vice versa

i.e

y $\propto \frac{1}{x}$

y = k/x

Example

10 men take 12 days to cultivate the farm how long could it take for 15 men to cultivate the same farm?

Let "f" be the number of men and "g" be the number of days.

$g \propto \frac{1}{f}$

$g = \frac{k}{f}$

$12 = \frac{k}{10}$

k = 120

$g = \frac{120}{f}$

If f =15,

$g = \frac{120}{15}$

g = 8

∴ For 15 men the farm will be cultivated by 8 days

JOINT VARIATION

Is the variation where by one quantity depends on two or more quantities

For example

If P varies directly with x and inversely with y

Then

$p \alpha x, p \alpha \frac{1}{y} \longrightarrow p \alpha x \alpha \frac{1}{y}$

$P \propto \frac{x}{y}$

P = kx /y

Example

If z varies inversely as y and directly as x^3 determine z in terms of x and y given that z = 3, x = 2 and y = 3

Solution

$z \propto \frac{1}{y} \propto x^3$ or $z \propto \frac{x3}{y}$

$z = \frac{fx^3}{y}$

$3 = \frac{f(2^2)}{3}$

$f = \frac{3 \times 3}{8}$

f = 9/8

$\therefore z = \frac{9x^3}{8y}$

Exercise

1. Mr Othman divided 30 books to his children Khadija , Omary and Hafsa in the ratio 1:2:2 what was the share of each of the three children

2. Given that $2x^2-3xy+y=0$ is an implicit equation determine the possible ratio of x to y

3. Abubakar is 1.5 times as tall as Omar if Omar is 90cm tall, find the height of Abubakar

4. Given that the ratios of x and y [x:y] = -4 and x:y = 3 are two ratios obtained from the same implicit equation, determine the equation

5. A varies inversely as r^2 if A=1 where r=1 find

a.) A in terms of r

b.) A when r = 7

6. X varies directly as y and x = 4 when y = 100 find y when x = 60

7. If x varies inversely as y^2 and x = 8 when y = 3

a.) Express x in terms of y

b.) Find x when y = 4 and y when x = 4

8. Given that t = √3 when z = 9. Find the formula which satisfies the following

a.) $T^2 \propto \frac{1}{z}$

b.) $T \propto \sqrt{z}$

c.) $T^2 \propto \frac{1}{z^4}$

d.) $T \propto z^2$

9. If m varies inversely as n and directly as r^4 determine m in terms of n and r given that m =2, n = 1 and r =4.

Solutions

1) 1+2+2 = 5

1/5 x30 = 6

1/5 x30=12

2/5 x 30 =12

∴ Khadija will get 6 books

∴Omari will get 12 books

∴Hafsa will get 12 books

Split the middle term

$2x^2-3xy+y^2 = 0$

$[x-y][2x-y] = 0$

Either

$[x-y] = 0$ or $[2x-y] = 0$

$x=y$ or $2x = y$

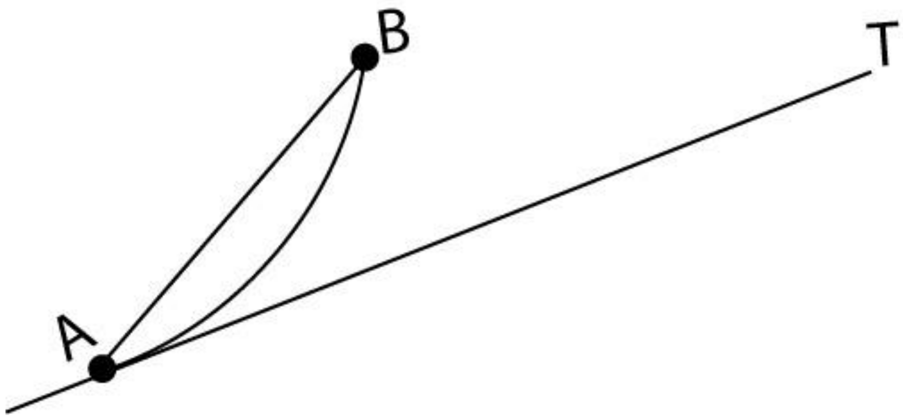
DIFFERENTIATION

Sub topics

-Differentiation by first principles
-Techniques of differentiation
-First and second derivatives
-Implicit differentiation
-Application of differentiation

DIFFERENTIATION BY FIRST PRINCIPLE

The concept of differentiation
The gradient of a curve at a given point is defined as the gradient of the tangent to the curve at that point and is given by the change of y with respect to x.



As B $\to$ A

The gradient of the chord AB
The gradient of a tangent AT at point A

Or

Line gradient of chord AB = gradient of tangent AT

Example

Find the gradient of the curve

$y = 2x^2 + 5$

Solution

At point Q

$y + \delta y = 2[x + \delta x]^2 + 5$
$= 2[x^2 + 2x\delta x + \delta x^2] + 5$
$= 2x^2 + 4x\delta x + 2\delta x^2 + 5$………… (i)

Subtracting y from equation (i)

$y + \delta y - y = 2x^2 + 4x\delta x + 2\delta x^2 + 5 - [2x^2 + 5]$
$\delta y = 4x\delta x + 2\delta x_2$……….(ii)
Dividing ii………… By δx
$\frac{\partial y}{\partial x} = 4x + 2\delta x$…………iii…

As $\delta x \rightarrow 0, \delta y \rightarrow 0$

And

$$\frac{\partial y}{\partial x} = \frac{dy}{dx}$$

(iii) Becomes

$$\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} = 4x \ldots\ldots\ldots\ldots[iv]$$

Note: The expression $\frac{dy}{dx}$ is called derivative of y with respect [w.r.t] to x

The process of finding derivatives is called DIFFERENTIATION.

<u>Example</u>

Differentiate $y = x^3 + 1$ with respect to x

Solution

$Y + \delta y = [x + \delta x]^3 + 1$

$= x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 + 1$

Subtracting y

$y + \delta y - y = x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 + 1 - x^3 - 1$

$\delta y = 3x^2\delta x + 3x\delta x^2 + \delta x^3$

Dividing by δx

$$\frac{\delta y}{\delta x} = 3x^2 + 3x\delta x + \delta x^2$$

As $\delta x \rightarrow 0, \delta y \rightarrow \frac{\delta y}{\delta x} \rightarrow \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = 3x^2$

Examples

Find the gradients of the following curves

1.) $2x^2-1$

2.) $y = x^3-1$

Solution

1) $2x^2-1$

$y+\delta y = 2[x+\delta x]^2-1$
$y+\delta y = 2[x^2+2x\delta x+\delta x^2]-1$
$y+\delta y = 2x^2+4x\delta x+\delta x^2-1$…………(i)

Subtracting y from(i)

$y+\delta y-y = 2x^2+4x\delta x+\delta x^2-1-[2x^2-1]$
$\delta y = 2x^2+4x\delta x+2\delta x^2-1-2x^2+1$ ....(ii)

Dividing …..(ii) by δx
$\frac{\delta y}{\delta x} = \frac{\delta y}{\delta x} = 4x\delta x + 2\delta x^2$
$= 4x + 2\delta x$ ………(iii)

As $\delta x \to 0$, $\frac{\delta y}{\delta x} = \frac{dy}{dx}$

(iii) Becomes
$\frac{dy}{dx} = 4x$

1. $y = x^3 - 1$

Solution

$$y + \delta y = (x + \delta x)^3 - 1$$
$$y + \delta y = x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 - 1$$

Subtracting y
$y+\delta y - y = x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 - 1 - x^3 + 1$

$\delta y = 3x^2 \delta x + 3x \delta x^2 + \delta x^3$

Dividing by δx

$\frac{\delta y}{\delta x} = 3x^2 \frac{\delta x}{\delta x} + 3x \frac{\delta x^2}{\delta x} + \frac{\delta x3}{\delta x}$

$= 3x^2 + 3x\delta x + \delta x^2$

As $\delta x \rightarrow 0$, $\delta y \rightarrow 0, \frac{dy}{dx} = \frac{dy}{dx}$

$\frac{dy}{dx} = 3x^2$

EXERCISE

Find the gradients of the following curves.

1.) $y = x^2$

Solution

$y + \delta x = (x + \delta x)_2$

$= x^2 + 2x \delta x + \delta x^2$

Subtracting y

$y + \delta y - y = x2 + 2x \delta x + \delta x2 - x2$

$\delta y = 2x\delta x + \delta x^2$

Dividing by δx

$\frac{\delta y}{\delta x} = 2x \frac{\delta x}{\delta x} + \frac{\delta x^2}{\delta x}$

$= 2x + \delta x$

As $\delta x \to o, \delta y \to 0, \frac{\delta y}{\delta x} = \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = 2x$

2.) $y = x^3$

Solution

$y + \delta x = (x + \delta x)^3$

$= x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3$.......(i)

Subtracting y

$y + \delta y - y = X^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 - x^3$

$\delta y = 3x^2\delta x + 3x\delta x^2 + \delta x^3$............(ii)

Dividing by δx

$\frac{\delta y}{\delta x} = 3x^2\frac{\delta x}{\delta x} + 3x\frac{\delta x^2}{\delta x} + \frac{\delta x^3}{\delta x}$

$= 3x^2 + 3x\delta x + \delta x^2$

As $\delta x \to 0$, $\delta \gamma \to 0$, $\frac{\delta y}{\delta x} = \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = 3x^2$

3.) $y = x$

solution

$y + \delta x = x + \delta x$............(i)

subtracting y

$y + \delta y - y = x + \delta x - x$

$\delta y = \delta x$ ……….(ii)

Dividing by δx

$$\frac{\delta y}{\delta x} = \frac{dy}{dx}$$

$\therefore \frac{dy}{dx} = 1$

4.) $y = 3x^2$

Solution

$y + \delta x = 3[x + \delta x]^2$

$= 3[x^2 + 2x\delta x + \delta x^2]$

$= 3x^2 + 6x\delta x + 3\delta x^2 - 3x^2$……….(i)

Subtracting y

$y + \delta y - y = 3x^2 + 6x\delta x + 3\delta x^2 - 3x^2$

$\delta y = 6x\delta x + 3\delta x^2$…….(ii)

Dividing by δx

$$\frac{\delta y}{\delta x} = 6x\frac{\delta x}{\delta x} + 3\frac{\delta x^2}{\delta x}$$

$= 6x + 3\delta x \ldots\ldots\ldots\ldots iii$

As $\delta x \to 0$, $\delta y \to 0$, $\frac{\delta y}{\delta x} = \frac{dy}{dx}$

$\frac{dy}{dx} = 6x$

5.) $y = x^2 + 3x$

Solution

y +δx = [x+δx]^2 + 3[x+δx]

x^2+2x δx+ δx^2+ 3x+3δx………...i

Subtracting y

y + δy-y = X^2 + 2xδx+ δx^2+3x +3δx -(x^2 + 3x)

δy = 2xδx + δx^2 + 3δx……..ii

Dividing by δx

$$\frac{\delta y}{\delta x} = 2x\frac{\delta x}{\delta x} + \frac{\delta x^2}{\delta x} + 3\frac{\delta x}{\delta x}$$

= 2x +δx +3

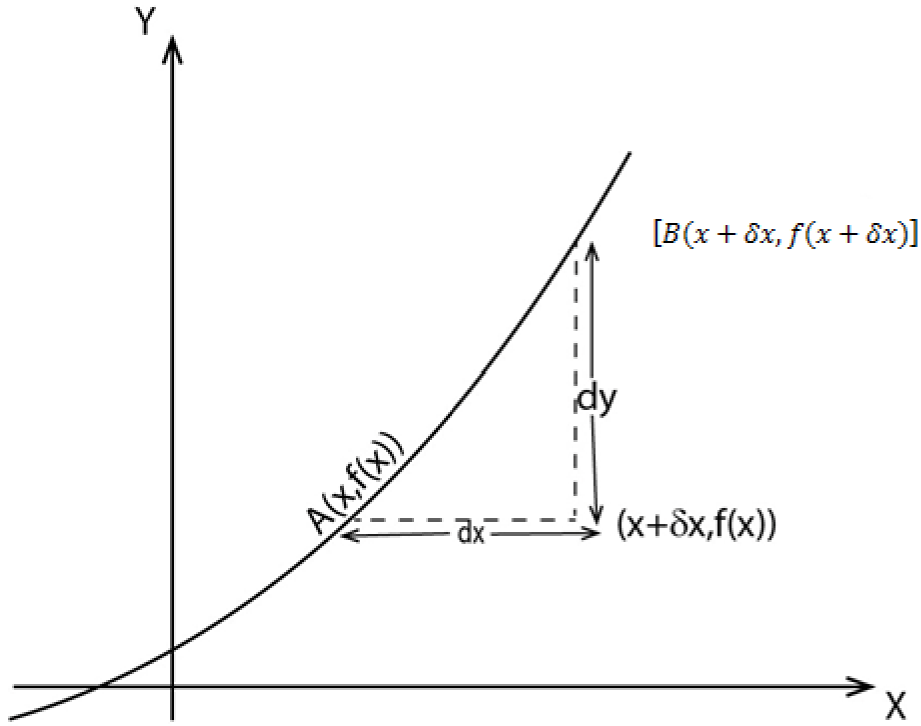
As δx →0,δy → 0, $\frac{\delta y}{\delta x} \to \frac{dy}{dx}$

∴ $\frac{dy}{dx} =$ 2x +3

DIFFERENTIATION BY FIRST PRINCIPLE

Consider a curve that $y = f[x]$

Let A(x, f(x)) be a point on the curve let B (x + δx, f (x + δx)) be another point on the same curve



The gradient of AB

The gradient at A is given by

$$\frac{\delta y}{\delta x} = \frac{f(x+\delta x)-f(x)}{x+\delta x - x}$$

The gradient at A is given by;

$\frac{dy}{dx}$ = limit $\frac{\delta y}{\delta x}$

$\delta x \to 0$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left\{\frac{f(x+\delta x)-f(x)}{x+\delta x - x}\right\}$$

Examples

Differentiate the following using first principles

$y = x^2+3$

$y = x^3+2x^2+1$

Solution

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left\{\frac{f[x+\delta x]-f[x]}{x+\delta x-x}\right\}$$

$$= \lim_{\delta x \to 0}\left\{\frac{[x+\delta x]^2+3-[x^2+3]}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0}\left\{\frac{x^2+2x\delta x+\delta x^2+3-x^2-3}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0}\left\{\frac{2x\delta x+\delta x^2}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0}\{2x+\delta x\}$$

$$= 2x$$

$$\therefore \frac{dy}{dx} = 2x$$

Solution

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left\{ \frac{f[x+\delta x] - f[x]}{\delta x} \right\}$$

$$= \lim_{\delta x \to 0} \left\{ \frac{[x+\delta x^3] + 2[x+\delta x]^2 + 1 - [x^3 + 2x^2 + 1]}{\delta x} \right\}$$

$$= \lim_{\delta x \to 0} \left\{ \frac{x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 + 2[x^2 + 2x\delta x + \delta x^2] + 1 - x^3 - 2x^2 - 2x^2 - 1}{\delta x} \right\}$$

$$= \lim_{\delta x \to 0} \left\{ \frac{x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3 + 2x^2 + 4x\delta x + 2\delta x^2 + 1 - x^3 - 2x^2 - 1}{\delta x} \right\}$$

$$= \lim_{\delta x \to 0} \left\{ 3x^2 + 3x\delta x + \delta x^2 + 4x + 2\delta x \right\}$$

$$= 3x^2 + 4x$$

$$\frac{dy}{dx} = 3x^2 + 4x$$

Exercise

1) $y = x^3 - x^2$

2) $y = 3x^2 - 2x$

3) $y = 2x^2 - 4x + 1$

4) $y = 12x^2 - 6x + 7$

5) $y = x^2 + 2x - 2$

Solutions

1) y = x^3-x^2

$$\frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left\{\frac{f[x+\delta x]-f[x]}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \left\{\frac{f[x+\delta x]^2-[x+\delta x]^2}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \left\{\frac{[x^3+3x^2\delta x+3x\delta x^2+\delta x^3-[x^2+2x\delta x+dx^2]]-(x^3-x^2)}{\delta x}\right.$$

$$= \lim_{\delta x \to 0} \left\{\frac{[x^3+3x^2\delta x+3x\delta x^2+\delta x^3-x^2-2x\delta x-dx^2-x^3+x^2}{\delta x}\right.$$

$$=\lim_{\delta x \to 0} \left\{\frac{[x^3+3x^2\delta x+3x\delta x^2+\delta x^3-x^2-2x\delta x-dx^2-x^3+x^2}{\delta x}\right.$$

$$=\lim_{\delta x \to 0} \{3x^2+3x\delta x+\delta x^2-2x-\delta x$$

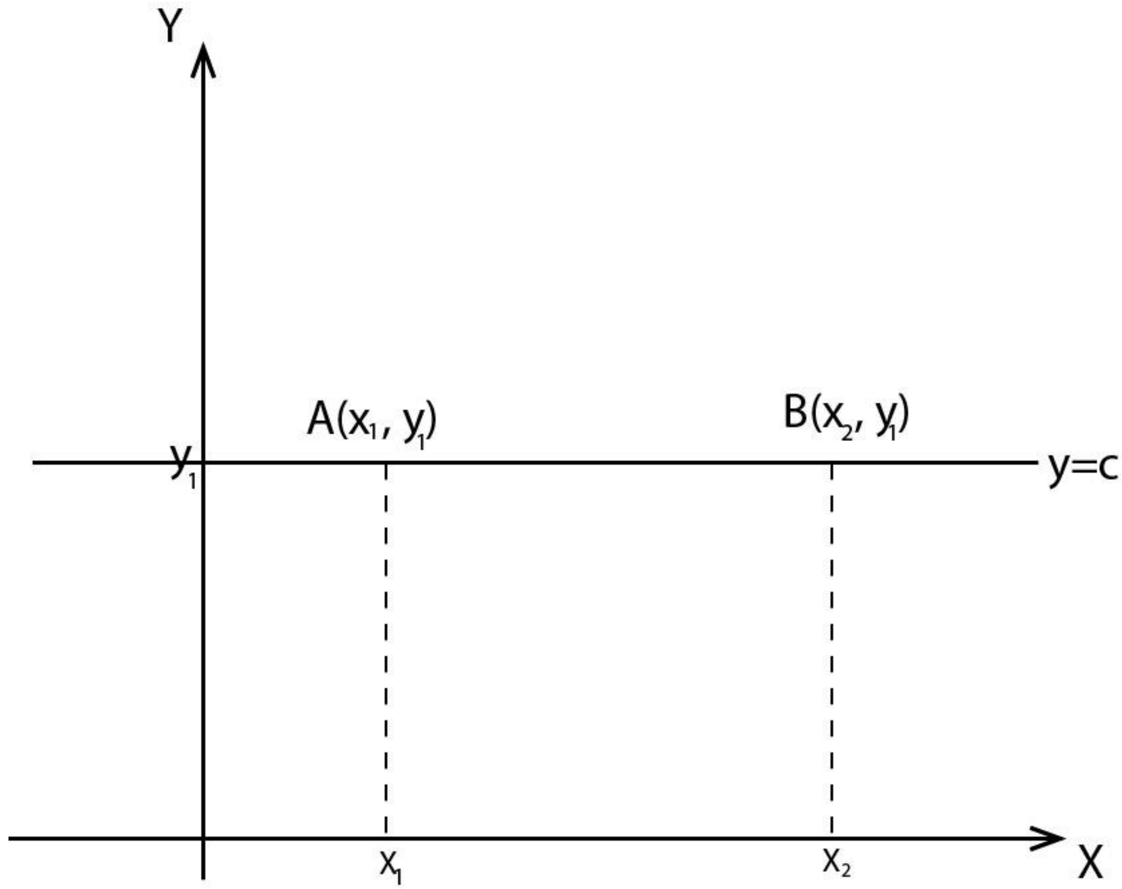
$$= 3x^2-2x$$

$$\therefore \frac{dy}{dx}=3x^2-2x$$

TECHNIQUES OF DIFFERENTIATION

A) DERIVATIVES OF POLYNOMIAL

1) Differentiation of constant [i.e y = c]

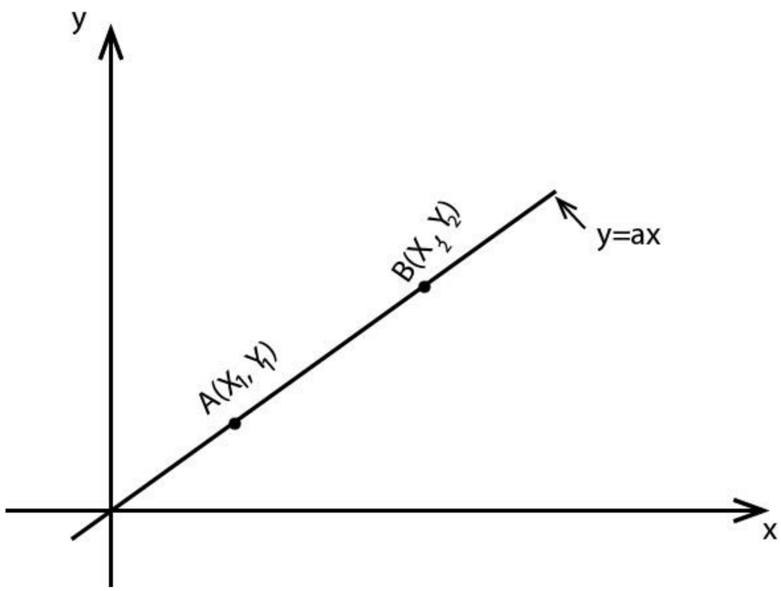


Gradient of $y = c$ *is*

$$\frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{x_2 - x_1} = \frac{0}{\delta x}$$

$\therefore \frac{dy}{dx} = 0$

2) Differentiation of y = ax where a is a constant



The gradient at B can be found by using the following

y = ax ………..(i)

Taking a point further

$y + \delta y = a(x + \delta x)$

$= ax + a\delta x$ ……..(ii)

Subtracting y from (iii)

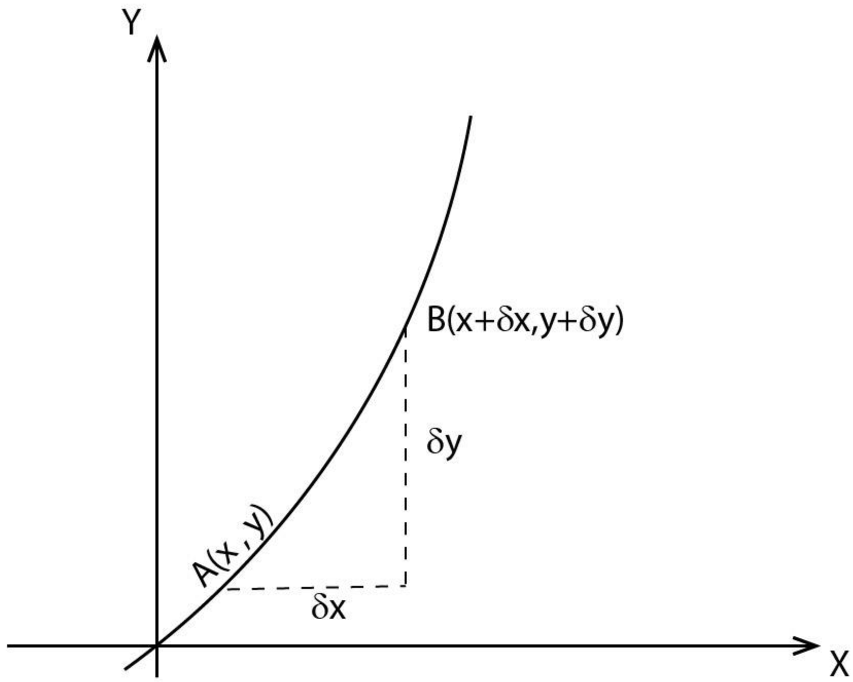
$y + \delta y - y = ax + a\delta x - ax$

$\delta y = a\delta x \rightarrow \frac{\delta y}{\delta x} = a : as \ \delta x \rightarrow 0, \delta y \rightarrow 0, \frac{\delta y}{\delta x} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{a\delta x}{\delta x}$

$\frac{dy}{dx} = a$

3) Differentiation of y = x^2



At point B

$y + \delta y$

$y = [x + \delta x]^2$

$= x^2 + 2x\,\delta x + \delta x^2$ …………… (i)

Subtract y from (i)

$y^2 + 2x\delta x + \delta x^2 - x^2 = y + \delta y - y$

$\delta y = 2x\delta x + \delta x^2$…….

Dividing …ii, by δx …..

$= \frac{\delta y}{\delta x} = 2x + \delta x$ … … … (iii)

As $\delta x \rightarrow 0$, $\delta y \rightarrow 0$, $\delta y / \delta x = dy/dx$

Equation becomes

$\frac{dy}{dx} = 2x$

Note

By proceeding with the same trend we shall get

When $y = x^4$, $\frac{dy}{dx} = 4x^3$

When $y = x^5$, $\frac{dy}{dx} = 5x^4$

When $y = x^6$, $\frac{dy}{dx} = 6x^5$

Generally

If $y = x^n$, then

$\frac{dy}{dx} = nx^{n-1}$

Example

Differentiate the following with respect to x.

$y = 3x + 5$

$y = 6x^7+5x^4$

$y =-x^{10}+9x^2$

Solution

$$\frac{dy}{dx} = \frac{d(3x+5)}{dx}$$

$$= \frac{3d(x')}{dx} + \frac{5d(x^0)}{dx}$$

Since differentiation of constant = 0

$$\frac{dy}{dx} = 3 \ [1] + 0$$

$\frac{dy}{dx}$ = 3+0

$\frac{dy}{dx}$ = 3

y = $6x^7+5x^4-x^2$

$\frac{dy}{dx}$ = $42x^6+20x^3-2x$

y =$-x^{10}+9x^2$

= $-10x^{[10-1]} + 2[9]x^{[2-1]}$

∴ $\frac{dy}{dx}$ = $-10x^9+18x$

Exercise

Differentiate the following with respect to x.

i.) y =$5x^2+2$

ii.) y =$8x^3-15x^2+6x+2$

iii.) y =x^5-4x^3

iv.) $y = 6x^2 - x^3 + 5x^4$

v.) $y = 3x + 10x^2 - 4x^7$

Solution

$y = 5x^2 + 2$

$\frac{dy}{dx} = 2[5]\,x^{[2-1]} + 2[0]$

$= 10x + 0$

$= 10x$

$\therefore = 10x$

2.) $y = 8x^3 - 15x^2 + 6x + 2$

$$\frac{dy}{dx} = 3(8)x^{[3-1]} - 2(15)x^{[2-1]} + 6[1]$$

$= 24x^2 - 30x + 6$

3.) $y = x^5 - 4x^3$

$\frac{dy}{dx} = 5x^4 - 12x^2$

4.) $y = 6x^2 - x^3 + 5x^4$

$\frac{dy}{dx} = 12x - 3x^2 + 20x^3$

DERIVATIVE BY USING CHAIN RULE

Is used to find derivative of function

Example:

Differentiate $y = [3x+5]^4$

Solution

Let U = 3x+5

$\frac{\delta y}{\delta x} = 3$

y =U^4

$\frac{\delta y}{\delta u} = 4u^3$

$\frac{dy}{dx} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$

$4u^3$ x3

= $12u^3$

$\frac{dy}{dx}$ = 12 $[3x+5]^3$

Note: This process is called chain rule

2) Find the $\frac{dy}{dx}$ if y = $\sqrt{x+1}$

Solution

Let u = x+1

$\frac{\delta u}{\delta x} = 1$

y = $\sqrt{u}$

y = $u^{½}$

y = ½ $u^{-½}$

$\frac{dy}{dx} = \frac{\delta y}{\delta u}$ x $\frac{\delta u}{\delta x}$

= ½ $u^{-½}$ x 1

$= \frac{1}{2} [x+1]^{-\frac{1}{2}}$

<u>Differentiating by using product rule</u>

Let y = uv where U and V are functions of x

If $x \to x + \delta x$, then $U \to u + \delta u$

And $V \to V + \delta v$

$y \to y + \delta y$

$y + \delta y = [u + \delta u] [v + \delta v]$

$= uv + u\delta v + v\delta u + \delta u \delta v$ …… i

Subtracting y from …(.i)….

$y + \delta y - y = uv + u\delta v + v\delta u + \delta u \delta u - uv$
$\delta y = u\delta v + v\delta u + \delta u \delta v$ ….(ii)

Dividing [ii] by δx

$$\frac{\delta y}{\delta x} = \frac{u\delta v}{\delta x} + \frac{v\delta u}{\delta x} + \frac{\delta u\delta v}{\delta x} \ldots\ldots\ldots..iii$$

<u>Exercise</u>

Taking the limit as $\delta_{x\to 0}, \delta_y \to 0, \delta_u \to 0, \delta_v \to 0$

$$\frac{\delta y}{\delta x} \to \frac{dy}{dx}, \frac{\delta v}{\delta x} \to \frac{dv}{dx}, \frac{\delta u}{\delta x} \to \frac{du}{dx}, \frac{\delta u \delta v}{\delta x} \to 0$$

$$\frac{d(uv)}{dx} = \frac{udv}{dx} + \frac{vdu}{dx}$$

<u>Differentiating using quotient rule</u>

$Let\ y = \frac{u}{v}\ where\ u\ and\ v\ are\ functions\ of\ x$

then

$$\delta y = \frac{u+\delta u}{v+\delta v} - \frac{u}{v}$$

$$y + \delta y = \frac{u+\delta u}{v+\delta v}$$

$$y + \delta y - y = \frac{u+\delta u}{v+\delta v} - \frac{u}{v}$$

$$\delta y = \frac{uv + v\delta u - uv - u\delta v}{v(v+\delta v)} \ -----(i)$$

Divide by δx in equation (i) throughout.

$$\frac{dy}{dx} = \frac{v\frac{\delta x}{\delta x} - u\frac{\delta v}{\delta x}}{v(v+\delta v)}$$

Taking the limits a

$$\delta x \to 0\ \delta y \to 0, \delta u \to 0, \delta v \to 0, \frac{\delta y}{\delta x} \to \frac{dy}{dx}, \frac{\delta u}{\delta x}, \to \frac{du}{dx}$$

$$\frac{\delta v}{\delta x} \to \frac{dv}{dx}, v + \delta \to v$$

Then

$$\frac{dy}{dx} = \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{V\frac{du}{dx} - \frac{udv}{dx}}{v^2}$$

Exercise:Differentiate

i/ $y = \frac{2x}{2+3x}$

ii/ $y = \frac{x}{x^2 + 2x}$

Solutions

y = x/cos x

Let u = x, v = cos x

$\frac{\delta u}{\delta x} = 1, \frac{\delta v}{\delta x} = -\sin x$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{\cos x[1] - x\,[-\sin x]}{\cos 2_x}$$

$$\frac{dy}{dx} = \frac{\cos x + x\sin x}{\cos^2 x}$$

FIRST AND SECOND DERIVATIVES

The first derivative of the function

For example

If f [x] = x^4

Then $\frac{dy}{dx} = 4x^3$ [This is the 1st derivative of the function]

The second derivative is the derivative of the first derivative

For example

If f [x] = x^4

Then

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$\frac{dy}{dx}$ = [4x^3]

= 12x^2

$$\frac{d^2y}{dx^2} = 12x^2$$

Examples

Find the second derivative of the following with respect to x

y = 2x^5+ 4x^3

y = x^2+2x+1

Solution

$$\frac{dy}{dx} = 10x^4 + 12x^2$$

$$\frac{d^2y}{dx^2} = 40x^3 + 24x$$

4.) y = x^2 + 2x+1

$$\frac{dy}{dx} = 2x + 2$$

$$\frac{d^2y}{dx^2} = 2$$

Note

d^2y/dx^2 may be represented by y'' 0r f''(x).

Exercise

Find the derivative of the following

a.) f [x] = x^3+ 4x^2-6x

b.) f [x] = x^2+3x

c.) f [x] = $3x^2+8x-6$

d.) f [x] = sin x

e.) f [x] = cos x

Find f''(2) if

i.) f[x] = $x^5+ 2x^3-9x^2+7x-1$

ii.) f[x] = x^4+5

iii.) f[x] = $x^3 +4x^2 - 6$

Solution

1 a) f [x] = x^3+4x^2-6x

y = x^3+4x^2-6x

$$\frac{dy}{dx} = 3x^2 + 8x - 6$$

$$\frac{d^2y}{dx^2} = 6x + 8$$

b) f [x] = $x^2 + 3x$

y = $x^2 +3x$

$$\frac{dy}{dx} = 2x + 3$$

$$\frac{d^2y}{dx^2} = 2$$

c) f [x] = $3x^2+8x-6$

y = $3x^2+8x-6$

$$\frac{dy}{dx} = 6x + 8$$

$$\frac{d^2y}{dx^2} = 6$$

$$\therefore \frac{d^2y}{dx^2} = 6$$

d) f [x] = sin x

y = sin x

$\frac{dy}{dx}$ = cos x

$\frac{d^2y}{dx^2} = -\sin x$

e) f [x] = cos x

y = cos x

$\frac{dy}{dx}$= -sin x

$\frac{d^2y}{dx^2} = -\cos x$

Solution

a) f [x] = $x^5+2x^3-9x^2+7x-1$

y = $x^5+2x^3-9x^2+7x-1$

f '[x] = $5x^4+6x^2-18x+7$

f''[x] = $20x^3+12x-18$

f''[2] = $20[2]^3 + 12[2] -18$

f ''[2] = 160+24-18
f ''[2] = 166

b) f [x] = x^4+5

$y = x^4 + 5$

$f'^{[x]} = 4x^3$

$f''^{[x]} = 12x^2$

$f'''^{[x]} = 12[2]2$ = 24x

f ''' (2) = 24(2)

f '''(2) = 48

C) f(x) = $x^3 + 4x^2 – 6$

Solution.

y = $x^3 + 4x^2 – 6$

y'= $3x^2 + 8x$

y''(2)= 6x + 8

y''(2)= 6(2) + 8

y''(2)= 12+8

y''(2)=20

IMPLICIT DIFFERENTIATION

Consider the following functions

i) y = x^2+4x+2

∴y is an explicit function of x because it is considered completely in terms of x

ii) xy + sin y = 2

∴Y is an implicit function of x because it is implied in the function of x

Example

1.) Find $\frac{dy}{dx}$ if $x^2y^3 – xy = 10$

2.) Find $\frac{dy}{dx}$ if y = sin x + cos y

3.) Find $\frac{dy}{dx}$ at [-1,1] if $x^2 + 3xy + y^2 = 1$

Solutions

1.) $x^2y^3 - xy = 10$

$$2xy^3 + x^2[3y^2]\frac{dy}{dx} - \left(1y + x\frac{dy}{dx}\right) = 0$$

$$2xy^3 + 3x^2y^2\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$$

$$2xy^3 + 3x^2y^2\frac{dy}{dx} - x\frac{dy}{dx} - y = 0$$

$$3x^2y^2\frac{dy}{dx} - x\frac{dy}{dx} = y - 2xy^3$$

$$(3x^2y^2 - x)\frac{dy}{dx} = y - 2xy^3$$

$$\frac{dy}{dx} = \frac{y - 2xy^3}{3x^2y^2 - x}$$

2.) y = sin x + cos y

$\frac{dy}{dx}$ = cos x + [-sin y $\frac{dy}{dx}$]

$\frac{dy}{dx}$ +sin$\frac{dy}{dx}$ = cos x

[1+ sin y] $\frac{dy}{dx}$ = cos x

$$\frac{dy}{dx} = \frac{cosx}{1+siny}$$

3.) $x^2 + 3xy + y^2 = 1$

$2x + 3[y + x\frac{dy}{dx}] + 2y\frac{dy}{dx} = 0$

$2x + 3y + 3x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x + 3y + [3x + 2y]\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-[2x+3y]}{3x+2y}$

At [-1,1] $\frac{dy}{dx} = \frac{-[2[-1]+3[1]]}{3[-1]+2[1]}$

$\frac{-1}{-1} = 1$

Exercise

Find dy/dx from the following equations

1) $x^3 + y^2 6x - 3y + 8 = 0$

2) $5x^2 + sin5y = 0$

3) $3x^2 + 2x^3 = cosy$

4) $7y + 7x^2 y = y^2 + 6x$

5) $2y^3 - 3y + 7x - 2 = 0$

b) Rate of change

This is the change of one variable with respect to time

Examples

(1)The distances [meters] travelled by a body moving in a straight line in t [seconds] is given by S= $3t^3$-$4t^2$

Find

The velocity after 2 seconds

The initial acceleration

(2)A 20m ladder leans a wall the top slides down at a rate of 4m/s . how fast is the bottom of the ladder moving when it is 16m from the wall

Solutions

If a $s = 3t^3 - 4t^2$

$v = \frac{ds}{dt}$

Differentiate [$3t^3$-$4t^2$]

$v = \frac{ds}{dt} = 9t^2 - 8t$

When t = 2, v = 9 x [2] – 8 [2]

v = [9 x 4] -16

v = 36-16

v = 20m /s

∴ The velocity after 2 seconds is

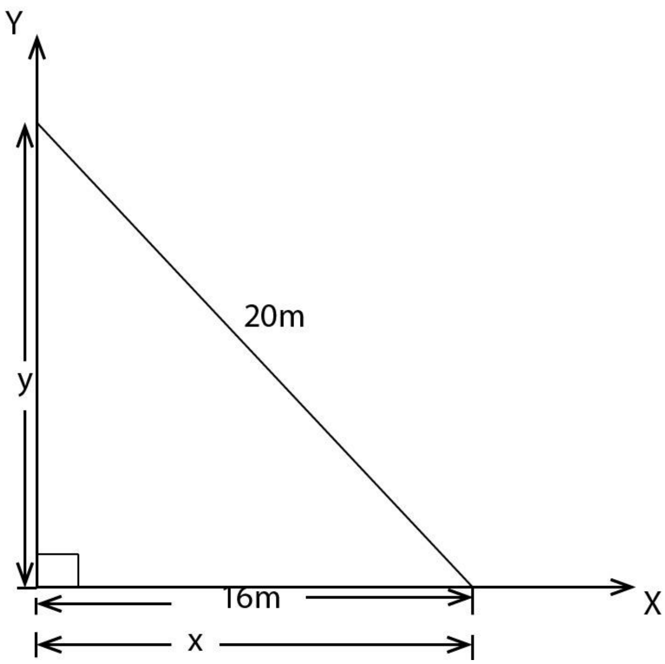
20m / s

a = $\frac{dv}{dt}$

Initial acceleration is when t = 0

$$v = 9t^2 - 8t$$
$$a = \frac{dv}{dt} = 18t - 8$$
$$= 18[0] - 8$$
$$\therefore Acceleration = -8$$

Solution



$x^2 + y^2 = 20^2$
…………(i)

Differentiate [i] w.r.t 't'

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \ldots\ldots\ldots ii$

From [i]

$y^2 = 20^2 - x^2$

$y^2 = [400] - [16]^2$

$y^2 = 400 - 256$

$\sqrt{y^2} = \sqrt{144}$

∴ y = 12

Substitute x and y in ii

16 dx /dt +12 dy /dt = 0

∴ $\frac{dx}{dt} = -12\frac{dy}{dt}$

= $\frac{-12[4m/s]}{16}$

= 3 m /s

The bottom of the ladder is moving with the speed of 3m/s

Exercise

1.) The effectiveness of a pain killing drug t incurs after entering the blood stream is given by

$$\left[E = \frac{1}{27}(9t + 3t^2 - t^2)\right]$$

Find the rate of change of E after

a.) 2 hours

b.) 3 hours

2.) A particle is moving in a straight line and its distance s in meters from a fixed point in the line after seconds is given by $s = 12t - 15t^2 + 4t^3$

Find

a.) the velocity of the particle after 3 seconds

b.) the acceleration of the particles after 3 seconds.

3.) When the height of a liquid in a container is h meters the volume of the liquid is v meters where $v = 0.005\,[3h + 2]^3 - 8$

a.) Find the expression for dv/dh

b.) The liquid enters the container at of $0.08m^3/s$ rate at which the height of the liquid is increasing when $v = 0.95m^3$

HOW TO DETERMINE THE TYPE OF CRITICAL POINT

By testing the sign of the gradient on either sides of $\frac{dy}{dx}$

By finding the second derivative of the function

If d^2y / dx^2 is [+ve] we have the minimum point

If d^2y / dx^2 is [-ve] we have maximum point

If $\frac{d^2y}{dx^2} = 0$ we have either maximum, minimum or point or inflexion

Examples

i) $y = x^2 + 4x + 3$

i.) $y = 2 + 4x + 3x^2$

iii.) $y = 2 - x - x^2$

Solution

$y = x^2 + 4x + 3$

$dy/ dx = 2x + 4$

When $dy / dx = 0$

$2x + 4 = 0$

$2x = -4$

$x = -2$

Testing the sign of gradient

Value of x	L	-2	R
Sign of dy/dx	-	0	+

∴ The minimum value of y = $(-2)^2$ + 4(-2) + 3

y = 4 -8 + 3

y = -1

Alternatively

$d^2y / dx^2 = 2$

Since

d^2y / dx^2 are (+ve), the function has minimum value

When dy / dx = 0

y = $(-2)^2$ +4 (-2) + 3

y = 4-8+3

y = 4-5

y = -1

Solution

y = $2 - x - x^2$

dy / dx = $-1 - 2x$

When dy / dx = 0

$-1 - 2x = 0$

-2x = 1

y = - ½

Now $d^2y / dx^2 = -2$[max pt]

The minimum value is

$y = 2 – [- ½] – [-1/2]^2$

$y = 2 + \frac{1}{2} - \frac{1}{4} = \frac{9}{4}$

INTEGRATION

SUB-TOPIC

1. The anti derivative

2. Indefinite integrals

3. Define Integrals

4 Application of integration

THE ANTI-DERIVATIVE

. Is the reverse of differentiation.

-In differentiation we start with function to find the derivative

-For anti derivative we start with derivative to find the function

Consider the table below

FUNCTION	DERIVATIVE	ANTI-DERIVATIVE (IN
$y=x^2$	$y^1=2x$	$\int 2x dx$ $=x^2+$
$y=x^3$	$y^1=3x^2$	$\int 3x^2 dx$ $=x^3$
$y=4$	$y^1=0$	$\int 0 dx$ $=c$
$y=\frac{x^{n+1}}{n+1}+c$	$y^1=x^n$	$\int x^n \delta x$ $=\frac{1}{n+1}x^n$

Integral notation

If y is the function of x, then $\int y dx$ is known as integration of y with respect to x

The integral sign cannot divorced with dx if we are integrating with respect to x.

Generally

If x^n is integrated with respect to x^n then

$$\rightarrow \int x^n \, dx = \frac{1}{n+1}x^{n+1} + c$$

Examples

Find

a) $\int x dx$

Solution

$\int x dx = \frac{1}{1+1} x^{1+1} + c$

$\int x dx = \frac{1}{2}x^2 + c$

b) $\int t^2 dt$

Solution

$\int t^2 \, dt = \frac{1}{2+1} x^{2+1} + c$

$= \frac{1}{3} t^3 + c$

c) $\int (v^2 + v + 1)$ dv

Solution

dv = $\int v^2 \, dv + \int v dvt \int dv$

$= \frac{1}{2+1} v^{2+1} + \frac{1}{1+1} v^{1+1} + v + c$

$= \frac{1}{3} v^3 + \frac{1}{2} v^2 + v + c$

EXERCISE

Integrate the following

1. $\int (x^4 + 7x^2 - 3)\, dx$

2. $\int (x^{-4} + 7x^{-2}) dx$

3. $\int (x^2 - 4) dx$

4. $\int (4 - x^3) dx$

Solution

1. $\int (x^4 + 7x^2 - 3)\, dx$

$$\int x^4 dx + \int 7x^2\, dx - 3 \int dx$$

$\frac{1}{4+1} x^{4+1} + 7 \frac{1}{2+1} x^{2+1}$ - 3x + c

$= \frac{1}{5} x^5 + 7 \frac{1}{3} x^3$ - 3x + c

$= \frac{1}{5} x^5 + 7 \frac{1}{3} x^3 - 3x + c$

2. $\int (x^{-4} + 7x^{-2})$ dx

$$\int x^{-4} dx + \int 7x^{-2}\, dx$$

$= \frac{1}{-4+1} x^{-4+1} + \frac{1}{-2+1} 7^{x^{-2+1}} + c$

$= \frac{1}{-3} x^{-3} + \frac{7}{-1} x^{-1} + c$

$= \frac{-1}{3} x^{-3}$ $-7^{x^{-1}}$ $+ c$

2. INDEFINITE INTEGRALS

Is an integral which does not have limits at the ends of the integral sign.

An arbitrary constant must be shown

e.g. $\int \sin\theta d\theta$, $\int x^3\, dx$, $\int \tan x dx$ e.t.c

Example

Integrate the following with respect to X

1. 5^{x^2}-7x+8
2. $2^{\sqrt{x}}$ - $\frac{1}{x^2}$
3. $4^{\sqrt{3x+1}}$

Solution

1. 1. $\int (5x^2 - 7x + 8)\, dx$

$= \frac{5}{2+1} x^{2+1} - \frac{7}{1+1} x^{1+1} + 8x + c$

$= \frac{5}{3} x^3 - \frac{7}{2} x^2 + 8x + c$

2. $2^{\sqrt{x}} - \frac{1}{x^2}$

$\int (2x^{\frac{1}{2}} - x^{-2}) \, dx$

$2.\frac{2}{3}x^{\frac{3}{2}} + x^{-1} + c$

$\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{x} + c$

Or

$\frac{4}{3}(\sqrt{x})^3 + \frac{1}{x} + c$

3. $\sqrt{4}\sqrt{3x+1}$ dx

Let u= $\sqrt{3x+1}$

$u^2=3x+1$

2udu=3dx

dx = $\frac{2u}{3}$ du

$\int 4\sqrt{3x+1}$ dx = $\int 4u.\frac{2u}{3} \, du$

= $\int \frac{8}{3}u^2$ du

= $\frac{8}{3}\int u^2$ du

= $\frac{8}{3}$ x $\frac{1}{3}u^3$ + c

$= \frac{8}{9}u^3 + c$

But u = $\sqrt{3x+1}$

$$\int 4\sqrt{3x+1dx}$$

$$\frac{8}{9}(\sqrt{3x+}\,1)^3+c$$

EXERCISE

Integrate the following

1. $\int(7x^{\frac{-1}{2}}$ - $5^{x^{\frac{2}{3}}}$ + 12) dx
2. $\int(\,3x-8)dx$
3. $\int(\,4x^3-5x+6\,)\,dx$
4. $\int 3\sqrt{2-5x\ \ dx}$

DEFINITE INTEGRAL

The definite integral is given by

$$\int_b^a f\,(x)dx = f\,(\,b\,)-\,f\,(\,a\,)$$

Where a = is the lower limit

b = is the upper limit

Note:

The arbitrary constant is not shown in the definite integral

Examples

1. $\int_2^3 (3x^2 + x)dx$

Solution

$[\frac{3x^2}{3} + \frac{x^2}{2}]_2^3$

$[x^3 + \frac{x^2}{2}]_2^3$

$= \left((3)^3 + \frac{(3)^2}{2}\right) - \left((2)^3 + \frac{(2)^2}{2}\right)$

$= \left(27 + \frac{9}{2}\right) - (8 + 2)$

$= 31\frac{1}{2} - 10$

$= 21\frac{1}{2}$

$$\therefore \int_2^3 (3x^2 + x)\,dx = 21\ ½$$

2. $\int_1^2 (x^3 + 2x)dx$

$\left[\frac{x^4}{4} + x^2\right]_1^2$

$\left(\frac{2^4}{4} + 2^2\right) - \left(\frac{1}{4} + 1\right)$

$= 8 - 1\frac{1}{4}$

$= 6\frac{3}{4}$

$$\therefore \int_1^2 (x^3 + 2x)dx = 6\,{}^3\!/\!_4$$

EXERCISE

Find the value of

1. $\int_2^3 (3x^2 - 4x + 5)dx$
2. $\int_{-2}^2 x(x-1)dx$
3. $\int_1^2 (3x^2 - 2x + 3)dx$
4. $\int_{-3}^2 (2 + 7x)dx$
5. $\int_{-1}^1 4(1 - x)dx$

Solution

1. $\int_2^3 (3x^2 - 4x + 5)dx$

$$\left[\frac{3x^3}{3} - \frac{4x^2}{2} + 5x\right]_2^3$$

$[x^3 - 2x^2 + 5x]_2^3$

$((3)^3 - 2(3)^2 + 5(3))$ _ $((2)^3 - 2(2)^3 + 5(2))$

(27 – 18 +15) – (8 – 8 + 10)

(27 – 3) – (8 + 2)

= 14

2. $\int_{-2}^{2} x\,(x-1)dx$

$$\int_{-2}^{2} (x^2 - x)\,dx$$

$$\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{-2}^{2}$$

$$\left(\frac{(2)^3}{3} - \frac{(2)^2}{2}\right) - \left(\frac{(-2)^3}{3} - \frac{(-(-2)^2}{2}\right)$$

$(\frac{8}{3} - 2) - (-\frac{8}{3} - \frac{4}{2})$

$(\frac{8}{3} - 2) - (\frac{-8}{3} - 2\,)$

$\frac{2}{3} + 4\frac{2}{3}$

$= 5\frac{1}{3}$

3. $\int_{1}^{2} (\,3x^2 - 2x + 3\,)$

$$\left[\frac{3x^3}{3} - \frac{2x^2}{2} + 3x\right]_{1}^{2}$$

$[x^3 - x^2 + 3x\,]_1^2$

$(\,(2)^3 - (2)^2 + 3(2)) - (\,(1)^3 - (1)^2 - 3\,(1))$

$(8 - 4 + 6) - (1 - 1 + 3)$

$(8 + 2) - (1+2)$

$= 7$

4. $\int_{-3}^{2} (\,2 + 7x)dx$

Solution

$$\left[2x + \frac{7x^2}{2}\right]_{-3}^{2}$$

$(2(2) + \frac{7(2)^2}{2}) - (2(-3) + 7\frac{9}{2})$

$(4 + 14) - (-6 + \frac{63}{2})$

$18 - (25\frac{1}{2})$

$= -7\frac{1}{2}$

INTEGRATION BY SUBSTITUTION METHOD

Integrate the following with respect to x

1. $(3x-8)^6$
2. $4(1-x)^{\frac{1}{2}}$
3. $x\,(x^2 + 1)^4$

Solution

1. $\int (3x-8)^6 dx$

Let u = 3x – 8

du=3dx

$\Rightarrow$ du = $\frac{du}{3}$

$\therefore \int (3x-8)^6\,dx = \int u^6 \frac{du}{3}$

= $\frac{1}{3}\int u^6$du

= $\frac{1}{3}$ x$\frac{1}{7}u^7$ + c

= $\frac{1}{21}(3x-8)^7$ + c

2. $\int 4(1-x)^{\frac{1}{2}}$ dx

Let u = 1-x

du=-dx

$\int 4(1-x)^{\frac{1}{2}}$ dx

Let u = 1 – x

$\int 4(1-x)^{\frac{1}{2}}$ dx = $\int 4u^2$(- du)

= -4 $\int u^{\frac{1}{2}}$ du

= $\frac{-4u^{\frac{3}{2}}}{\frac{3}{2}}$ + c

=-4 x $\frac{2}{3}u^{\frac{3}{2}}$ + c

= $\frac{-8}{3}u^{\frac{3}{2}}$ + c

= - $\frac{8}{3}(1-x)^{\frac{3}{2}}$ + c

3 . $x(x^2+1)^4$

Let u= x^2 + 1

du= 2xdx

dx = $\frac{du}{2x}$

$\therefore \int x\,(x^2 + 1)\,dx = \int x u^4 \frac{du}{2x}$

$= \frac{1}{2}\int u^4$du

$= \frac{1}{2}$ x $\frac{1}{5}u^5$ + c

$= \frac{1}{10}u^5$ + c

$= \frac{1}{10}(x^2 + 1)^5$ + c

Exercise

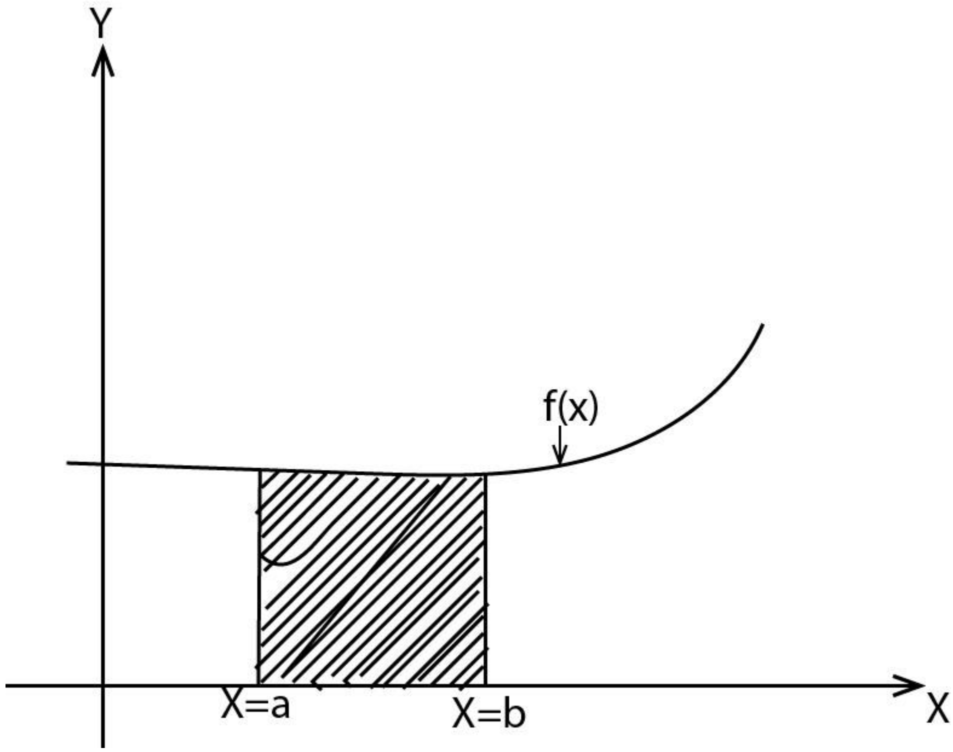
Determine the integral of each of the following

1. $\int x\,(x^2 - 6)^6$ dx
2. $\int x\sqrt{2x^2 + 3}$ dx
3. $\int \frac{x}{(x^2 - 1)}$dx

APPLICATION OF INTEGRATION

To determine the area under the curve

Given A is the area bounded by the curve y=f(x) the x -axis and the line x=0 and x=b where b> a



∴The area under that curve is given by the define definite integral of f(x) from a to b

$$A = \int_{a}^{b} f(x)\,dx$$

= f (b) - f (a)

Examples

1. Find the area under the curve f(x) $=x^2+1$ from x=0 to x=2

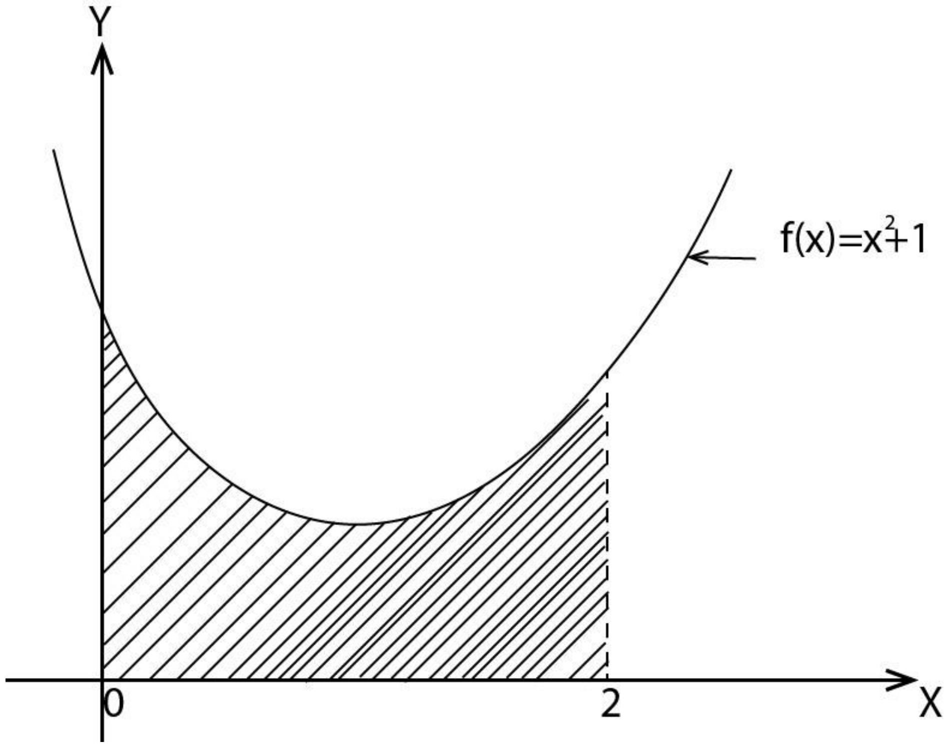
2. Find the area under the curve f(x) = $-x^2$ from x=1 to x=2

3. Find the area bounded by the function f(x) $=x^2-3$, x=0, x=5 and the x- axis

Solution

1. f(x) = x^2 + 1

y intercept=1



A = $\int_0^2 (x^2 + 1)$dx

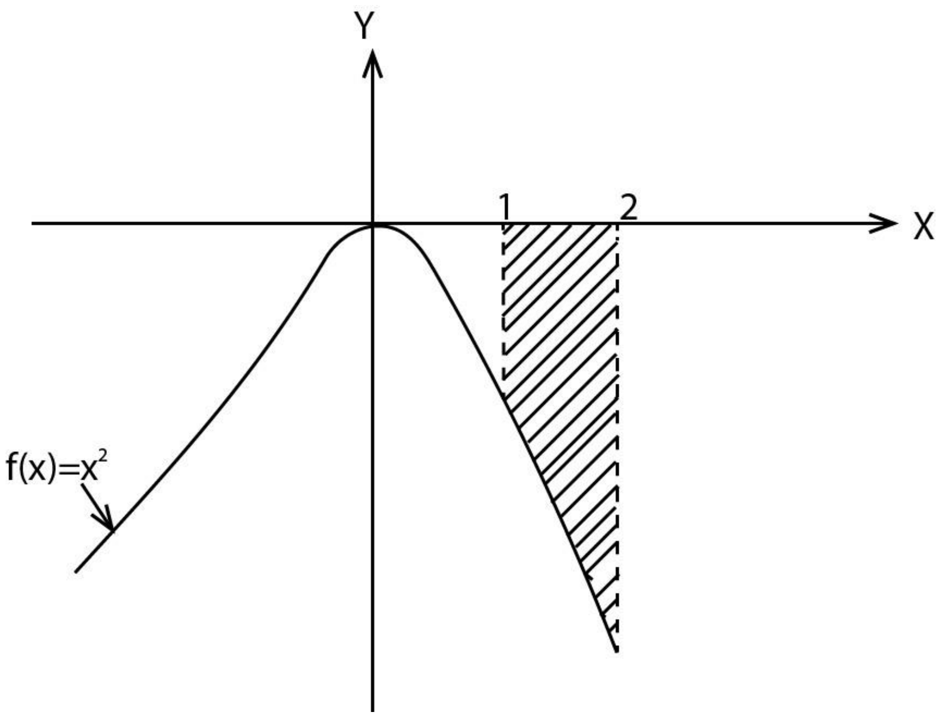
= $\left[\frac{1}{3}x^3 + x\right]_0^2$

= $(\frac{1}{3}(2)^3 + 2) - (\frac{1}{3}(0)^3 + 0)$

= $(\frac{8}{3} + 2) - (0)$

= $4\frac{2}{3}$ or $\frac{14}{3}$ sq. units

2. f(x)= $-x^2$



A = $\left[\int_1^2 (-x)^2\,dx\right]$

= $\left[\left[-\frac{1}{3}\,x^2\right]_1^2\right]$

= $\left[\left(-\frac{1}{3}\,(2)^3\right) - \left(-\frac{1}{3}\,(1)^3\right)\right]$

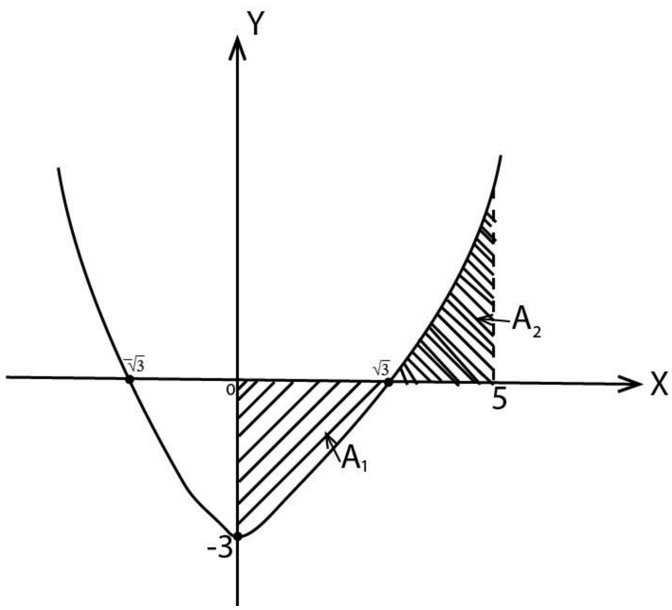
= $\left[-\frac{8}{3} + \frac{1}{3}\right]$ = $\left[-\frac{7}{3}\right]$

= $\frac{7}{3}$ sq.units

3. f(x) = x^2-3

Where: y intercept =-3

X intercept = $\sqrt{3}$ and x= $-\sqrt{3}$



A = $A_1 + A_2$

$= \left[\int_0^5 (x^2 - 3)\, dx\right] + \left[\int_0^5 (x^2 - 3)\, dx\right]$

$= \left[\left[\frac{1}{3}x^3 - 3x\right]_0^{\sqrt{3}}\right] + \left[\left[\frac{1}{3}x^3 - 3x\right]_{\sqrt{5}}^{5}\right]$

$= \sqrt{3} - 3\sqrt{3} + \left[\left(\frac{125}{5} - 15\right) - (\sqrt{3} - 3\sqrt{3})\right]$

$= [-2\sqrt{3}] + \left[\frac{80}{3} - 2\sqrt{3}\right]$

$= 2\sqrt{3} + \left[\frac{80}{3} + 6\sqrt{3}\right]$

$= 6\sqrt{3} + \frac{80}{3} + 6\sqrt{3}$

$= 12\frac{\sqrt{3}}{5} + 80$ sq.units

EXERCISE

1. Find the area between $y = 7 - x^2$ and the x- axis from $x = -1$ to $x = 2$

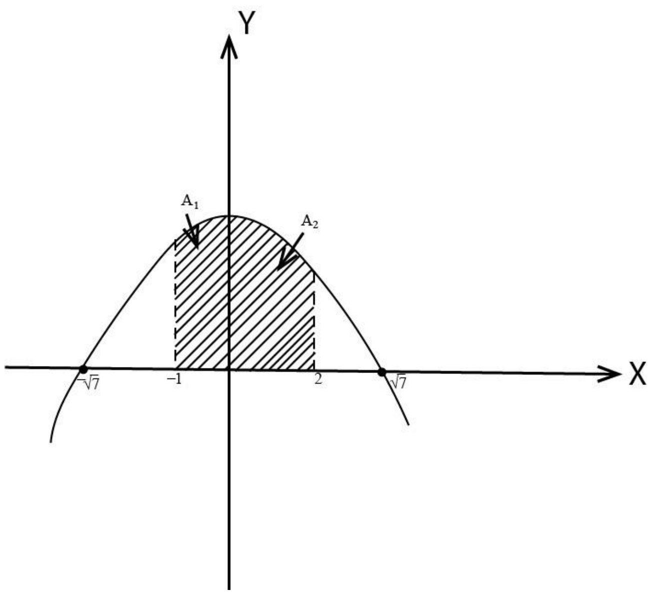
2. Find the area between the graph of $y=x^2 x - 2$ and the x- axis from x= -2 to x=3

Solution

1. $y = 7 - x^2$

Where y- intercept =7

X- Intercept = $\sqrt{7}$



$A = A_1 + A_2$

$= \left[\int_{-1}^{2}(7 - x^2)dx\right] + \left[\int_{-1}^{2}(7 - x^2)dx\right]$

$= \left[\left[7x - \frac{x^3}{3}\right]_{-1}^{0}\right] + \left[\left[7x - \frac{1}{3}x^3\right]_{0}^{2}\right]$

$= (^{+}6.67) + (11.3 - 0)$

$= {}^{+}6.67 + 11.3$

=17.97 sq units

Volume of the Solids of Revolution

The volume,V of the solid of revolution is obtained by revolving the shaded portion under the curve, y= f(x) from x= a to x =b about the x -axis is given by

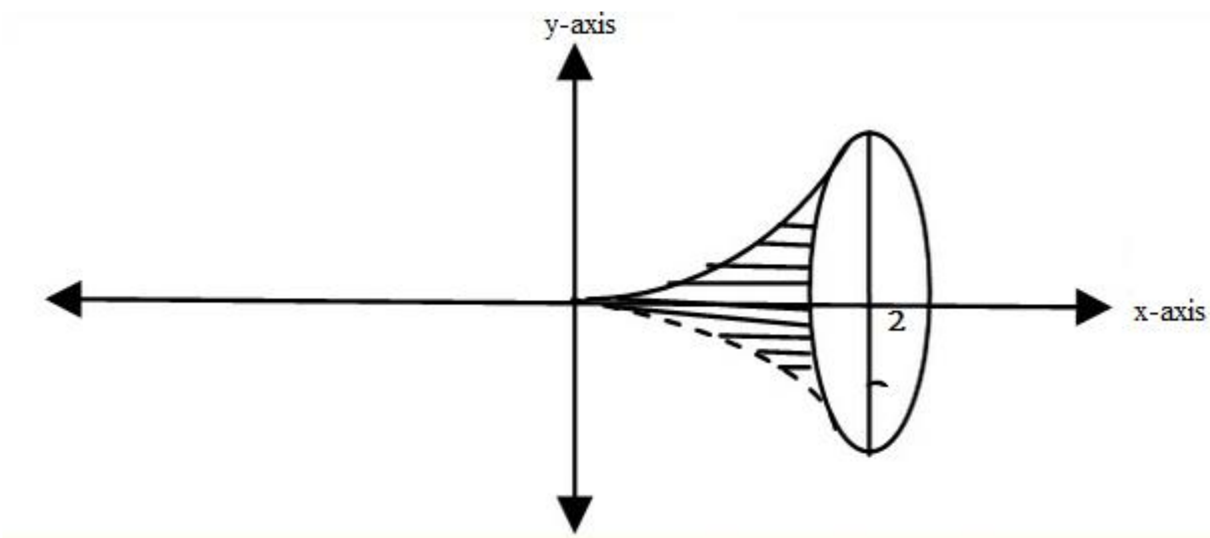
$$V = \int_a^b \pi y^2 dx$$

.
similarly, when the region is rotated about y- axis from y =a to y=b we shall have obtained the volume, V by

$$V = \int_a^b \pi x^2 \, dy$$

.
Example 1

Find the volume of revolution by the curve $y=x^2$ from x=0 to x=2 given that the rotation is done about the the x- axis



V=$\int_a^b \pi y^2 dx$

but $y=x^2$

$\int_0^2 \pi(x)^{2^2} dx = \int_0^2 (x)^4 \, dx$

= $[\frac{\pi x^5}{5}]_0^2$

= $\frac{\pi}{5}(2^5 - 0^5)$

$$V = \frac{32}{5}\pi \text{ cubic units}$$

.

Exercise

1. Find the volume obtained when each of the regions is rotated about the x - axis.

a) Under $y = x^3$, from $x = 0$ to $x=1$
b) Under $y^2 = 4 - x$, from $x=0$ to $x=2$
c) Under $y = x^2$, from $x=1$ to $x=2$
d) Under $y = \sqrt{x}$, from $x=1$ to $x=4$

2. Find the volume obtained when each of the region is rotated about the y-axis.
a) Under $y = x^2$, and the y-axis from $x=0$ to $x=2$
b) Under $y = x^3$, and the y-axis from $y=1$ to $y=8$
c) Under $y = \sqrt{x}$, and the y-axis from $y=1$ to $y=2$

.

STATISTICS

SUBTOPICS

1. Collection, organization, and presentation of data
2. Measures of central tendency
3. Measures of dispersion
4. Application of statistics

COLLECTION, ORGANIZATION AND PRESENTATION OF DATA

Collection of data

We collect data by

i) Observation

ii) Interview

iii) Questionnaires

iv) Focus group discussion

v) Experiments

vi) Portfolio

Organization of data

We organize data by

i) Rank order list

ii) Frequency distribution table

iii) Histogram

iv) Frequency polygon

FREQUENCY DISTRIBUTION TABLE

Is the table which shows data values against their number of occurrence.The number of occurrence are known as the Frequency.

Example

You are given the following data

41,45,62,65,42,41,62,45,71,76,82,92

Prepare the following distribution table

Solution

Data (x)	Frequency(f)
41	2
42	1
45	2
62	2
65	1

71	1
76	1
82	1
92	1

GROUPED DATA

The data within the given interval are grouped together

Example

Prepare the frequency distribution table of the following data by using class size of 5:-

41,45,62,65,42,62,45,71,41,76,82,92,48,52,57

Solution

Class interval	Frequency
41-45	5
46-50	1
51-55	1
56-60	1
61-65	3
66-70	0
71-75	1
76-80	1
81-85	1
86-90	0
91-95	1
	N=15

PRESENTATION OF DATA GRAPHICALLY AND PICTORIALLY

a) FREQUENCY POLYGON

Is the graph of frequency against class mark data values

Example 1

The scores of basic applied mathematics test is given below

CLASS INTERVAL	FREQUENCY
41-45	2
46-50	3
51-55	4
56-60	8
61-65	11
66-70	16
71-75	14
76-80	8
81-85	6
86-90	4
91-95	3
96-100	1

Draw the frequency polygon for the distribution

Solution

Procedures

i) Class marks (find)

ii) Plot the graph of frequency against the class marks

iii) Join the points by using a ruler

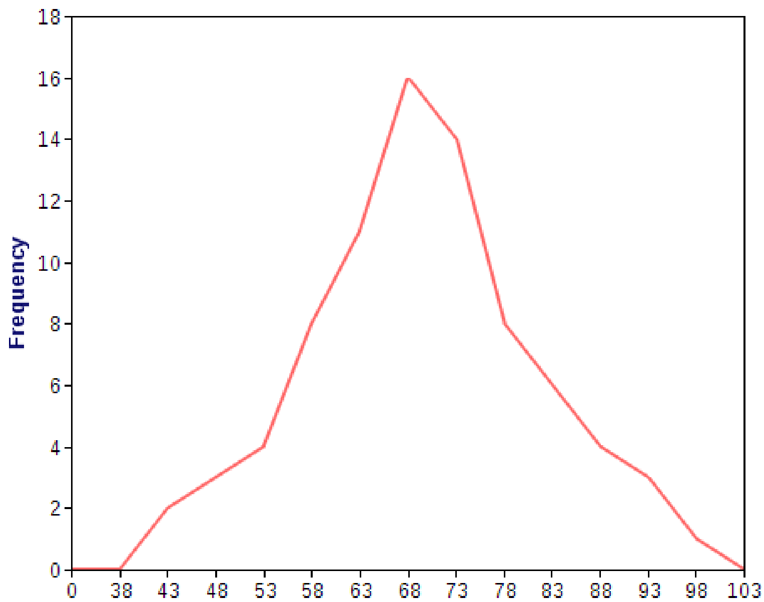
iv) Add one class below the lowest class interval and assign it with (0) zero frequency

v) Add one class above the highest class interval and assign it with (0) zero frequency

vi) Now join the added classes together with procedure (iii)

CLASS INTERVAL	FREQUENCY	CLASS MARK
41-45	2	43
46-50	3	48
51-55	4	53

56-60	8	58
61-65	11	63
66-70	16	68
71-75	14	73
76-80	8	78
81-85	6	83
86-90	4	88
91-95	3	93
96-100	1	98



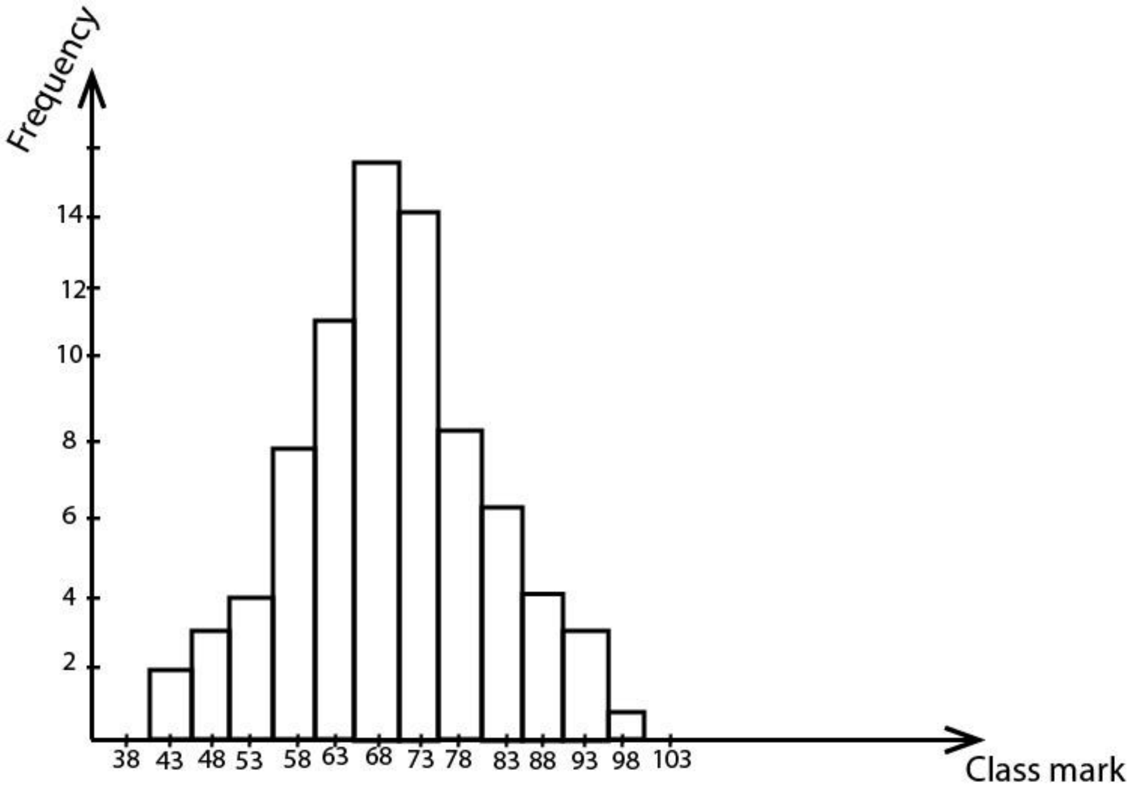
b)HISTOGRAM

Is the graph of frequency against the class mark or data values represented by rectangles.

Example 2:

By using the table in example 1 in the previous page, construct the histogram

Solution



MEASURES OF CENTRAL TENDENCY

a) Mode

Is the data with the highest frequency

Example

50,34,40,52,34,66,41,67,70,81,90,42,35,45

$\therefore$The mode is 34

THE MODE OF GROUPED DATA

Mode= L + $\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) c$

Where by

L= Lower boundary of the model class

âˆ†1=Excess frequency of the modal class to the next lower class interval

âˆ†2=Excess frequency of the modal class to the next upper class interval

C= class interval /class size

Example 1

The scores in a geography examination were recorded as follows

SCORES	30-34	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-79	80-84
F	3	5	7	8	16	10	9	8	6	4	3

Find the mode of the following frequency distribution

Solution

Modal class is 50-54

L=49.5

âˆ†1=16-8=8

âˆ†2=16-10=6

C = 5

∴Mode = L + $\left(\frac{\Delta_1}{\Delta_{1+}\Delta_2}\right) c$

= 49.5 + $\left(\frac{8}{8+6}\right) 5$

= 49.5 + $\left(\frac{8}{14}\right) 5$

= 49.5 + $\frac{40}{14}$

=49.4+2.9

=52.4

b) Median

This is the middle data or middle value

Example

Find the median from the given distribution

i) 14, 8, 9, 6,15,16,19

ii) 2,5,6,10,9,7

Solution

i) Arrange the given data in ascending data or descending order

= 6, 8,9,14,15,16,19

$\therefore$ The median is 14

ii) 2,5,6,7,9,10

$= \frac{6+7}{2} = \frac{13}{2} = 6.5$

$\therefore$ The median is 6.5

THE MEDIAN OF GROUPED DATA

$$\text{Median} = L + \left(\frac{\frac{N}{2} - na}{nm}\right)C$$

Where by

L=lower boundary of the median class

N=total frequency

na =Total no of frequency below the median class

nm =frequency of the median class

C=class size

Example 2

The scores of a geography examinations data are as the one in example 1

Find the median of the frequency distribution

Given data

N=80   N/2=40

Median classes is 55-59

L=54.5, na = 39

nm =10, c=5

Median = L + $\left(\frac{\frac{N}{2} - na}{nm}\right)$c

=54.5 + $\left(\frac{40-39}{10}\right)5$

=54.5+ $\left(\frac{5}{10}\right)$

= 54.5+0.5

= 55

EXERCISE

1. The scores of mathematics subject of 100 students were recorded as follows

SCORES	51-55	56-60	61-65	66-70	71-75	76-80	81-85	86-90
F	2	10	22	34	15	10	6	1

Calculate

a) Mode

b) Median score

Solution

a) Mode = L + $\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)c$

Modal class is 66-70

L = 65.5

Δ_1 = 34 – 22 = 12

Δ_2 = 34– 15 = 19

C = 5

= 65.5 + $\left(\frac{12}{12+19}\right)5$

= 65.5 + $\left(\frac{12}{31}\right)5$

=65.5+1.94

= 67.44

b) Median score = L+ $\left(\frac{\frac{N}{2} - na}{nm}\right)c$

N=100 N/2=50

Median class is 66-70

L= 65.5, n_a=34

nm=34, c=5

=65.5 + $\left(\frac{50-34}{34}\right)5$

= 65.5+(0.47)5

= 65.5+2.35

= 67.85

2 .The age of children in month were recorded as follows

Age(month	5-10	11-16	17-22	23-28	29-34	35-40	41-46
f	26	28	18	12	9	4	4

Calculate the

a) mode

b) Median

Solution

a) Mode = L+ $\left(\frac{\Delta_1}{\Delta_{1}+\Delta_2}\right)$c

Modal class is 11-16

l=10.5

âˆ†$_1$=28-26 = 2

âˆ†$_2$=28-18=10

c = 6

= 10.5 + $\left(\frac{2}{2+10}\right)6$

= 10.5 + ($\frac{2}{12})^6$

10.5+ ($\frac{1}{6})^6$

=10.5 + 1

<u>= 11.5</u>

b) Median age = L + $\left(\frac{\frac{N}{2}-na}{nm}\right)c$

N=100, N/2=50

Median age=11-16

L= 10.5, na =26

nm=28, c=6

= 10.5+ $\left(\frac{50-26}{28}\right)6$

= 10.5+ (0.86)6

=10.5+5.16

=15.66

C.**MEAN**

Is the ratio of the sum of the data given to the total number of observations.

I.e. $\bar{X} = \frac{\sum X}{\sum f} = \frac{\sum X}{N}$

Example

Calculate the mean of the following

33, 40, 44,35,46,50

Solution

$\bar{X} = \frac{\sum X}{N} = \frac{33+40+44+46+50+35}{6} = \frac{248}{6}$

= 41.3

THE MEAN OF GROUPED DATA

When dealing with grouped data

$\bar{X} = \frac{\sum fx}{N}$

Where x is the class mark

EXAMPLE

Calculate the mean score of the following distribution

SCORES	55-59	60-64	65-69	70-74	75-79	80-84
F	5	7	16	6	4	2

Solution

SCORES	CLASS MARK (x)	f	fx
55-59	57	5	285
60-64	62	7	434
65-69	67	16	1072
70-74	72	6	432
75-79	77	4	308
80-84	82	2	164
		$\sum f$=40	$\sum fx$ = 2695

Mean= $\frac{\sum fx}{N}$

= $\frac{2695}{40}$

= 67.38

$\therefore \bar{X}$ = 67.38

ASSUMED MEAN METHOD

The following method is used

X= A + $\frac{\sum fd}{N}$

Where A= Assumed Mean

f= frequency

d = deviation from assumed mean

N = total frequency

From the same example 3 let it be

Example 4.

Calculate the mean by using assumed mean method

Solution

SCORES	x	f	d=x-A	fd
55-59	57	5	-10	-50
60-64	62	7	-5	-35
65-69	67	16	0	0
70-74	72	6	5	30
75-79	77	4	10	40
80-84	82	2	15	30
		N=40		$\sum fd$=15

Take: A =67

$\bar{X} = 67 + \frac{15}{40}$

= 67 + 0.38

= 67.38

EXERCISE 2

1. Calculate the mean weight of the following population

WEIGHT (KG)	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79
FREQUENCY	2	5	6	8	6	5	3	2

Solution

Mean = $\frac{\sum fx}{N}$

SCORES	CLASS MARK(x)	f	fx
40-44	42	2	235
45-49	47	5	312

50-54	52	6	456
55-59	57	8	372
60-64	62	6	335
65-69	67	5	216
70-74	72	3	154
75-79	77	2	**∑fx=2164**

$$\bar{X} = \frac{2164}{37}$$

$$\bar{X} = \underline{58.49}$$

2. Find the mean of the following frequency distribution

CLASS INTERVAL	85-89	90-94	95-99	100-104	105-109	110-114
F	4	14	32	28	17	5

SCORES	CLASS MARK(x)	f	fx
85-89	87	4	348
90-94	92	14	1288
95-99	97	32	3104
100-104	102	28	2856
105-109	107	17	1819
110-114	112	5	560
		100	9975

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{9975}{100}$$

X= 99.75

THE MEAN BY USING CODING METHOD

PROCEDURES

- Choose the convenient value of x nearby the middle of the range
- Subtract it from every value of x
- Divide by the class size to get the coded value of x i.e. x/c
- Find the product of x_c and f i.e. $x_c f$
- Find the mean of x_c i.e. $\overline{x_c}$
- Find the true mean

Example

Calculate the mean of the following distribution by using coding method

SCORES	55-59	60-64	65-69	70-74	75-79	80-84
F	5	7	16	6	4	2

SCORES	x	f	d=x-A	x_c=d/c	xcf
55-59	57	5	-10	-2	-10
60-64	62	7	-5	-5	-7
65-69	67	16	0	0	0
70-74	72	6	5	1	6
75-79	77	4	10	2	8
80-84	82	2	15	3	6
		N=40			$\sum x_c f$=3

A = 67

$\overline{xc} = \frac{\sum x_{cf}}{N} = \frac{3}{40}$ = 0.075

= 0.075

But $\bar{x}$ - A = $\overline{xc}$.c

$\bar{x}$ = $\overline{xc}$.c + A

= 0.075 x 5 + 67

= 67.375

EXERCISE

1. By using the coding method calculate the weight of the following population

WEIGHT(KG	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79
F	2	5	6	8	6	5	3	2

WEIGHT(KG	**x**	**f**	**d=x-A**	**xc=d/c**	**xcf**
40-44	42	2	-15	-3	-6
45-49	47	5	-10	-2	-10
50-54	52	6	-5	-1	-6
55-59	57	8	0	0	0
60-64	62	6	5	1	6
65-69	67	5	10	2	10
70-74	72	3	15	3	9
75-79	77	2	20	4	8
	N=37				$\sum x_c f$=11

A = 57

$\overline{XC} = \frac{\sum x_c f}{N}$

$= \frac{11}{37}$

= 0.297

BUT $\bar{x}$ – A = $\overline{xc}$. c

$\bar{x}$ = $\overline{xc}$ .c + A

$\bar{X}$ = 0.297 x 5 + 57

= <u>58.486</u>

2. Find the mean of the following frequency distribution by using coding method

Class interval	85-89	90-94	95-99	100-104	105-109	110-114
frequency	4	14	32	28	17	5

Class interval	**x**	**f**	**d=x-A**	**x_c=d/c**	**x_cf**
85-89	87	4	-10	-2	-8
90-94	92	14	-5	-1	-14
95-99	97	32	0	0	0
100-104	102	28	5	1	28
105-109	107	17	10	2	34
110-114	112	5	15	3	15
		N=100			$\sum x_c$f =55

A = 97

$\overline{xc} = \frac{\sum x_c f}{N}$

$= \frac{55}{100}$

= <u>0.55</u>

But

$\bar{X}$ - A = $\overline{xc}$. c

$\bar{x} = \overline{xc}$ .c + A

$\bar{X}$ = 0.55 x 5 + 97

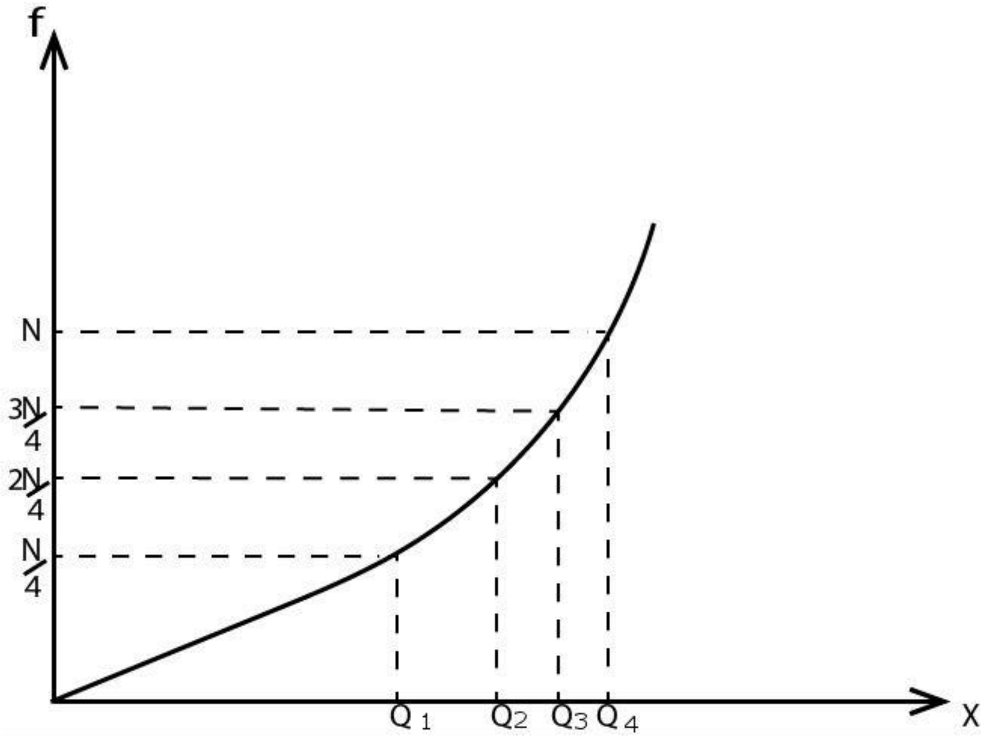
= 2.75 + 97

= 99.75

QUARTILES AND PERCENTILES

QUARTILES

Divides the distribution into four (4) equal parts



NOTE

i. The value which corresponds to N/4 is called the lower quartile i.e. Q4)

ii. The value which corresponds to N/2 is called the median i.e.(Q2)

iii. The value which corresponds to 3N/4 is called the upper Quartile i.e.(Q3)

iv. The difference between the upper quartile and the lower quartile is known as the inter quartile range

Example

From the following distribution find

i) Upper quartile

ii) Lower quartile

iii) Inter quartile range

30,25,66,19,44,45,52,53,37,65,57,44,33,80,76,71,40,50,38,33

Solution

N=20

3/4N= $\frac{3}{4}$ x 20 = 15

Arranging the value in ascending order

19,25.30,33,33,37,38,40,44,44,45,50,52,53,57,65,66,71,76,80

∴The fifteenth value is 57 i.e. the upper quartile is 57

ii) Lower quartile

N/4=20/4=5

∴The lower quartile is 33

iii) Inter quartile range

(Upper quartile-lower quartile)

$Q_R = Q_u - Q_L$

= 57 - 33

= 24

QUARTILE FOR GROUPED DATA

- **Lower quartile (Q_L)**

This is given by

$$Q_L = L_{(L)} + \frac{\left(\frac{N}{4} - na\right)}{n_l} c$$

Where

$L_{(L)}$ =lower boundary of the lower quartile class

N = total number of frequency

n_a = total frequency below lower quartile class

n_c = frequency in the lower quartile class

C = Class size

- **UPPER QUARTILE (Q_u)**

This is given by

$$Qu = Lu + \frac{\left(\frac{3N}{4} - n_a\right)}{n_u} c$$

Where

L_u = is the lower boundary of upper quartile class

n_u = frequency in the upper quartile class

Example

For the given data below

i) Lower ii) upper quartile iii) inter Quartile range

SCORES	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
F	1	2	7	21	27	22	17	2	1

Solution

i) Lower quartile

$$Q_L = L_u + \frac{(\frac{N}{4} - n_a)}{n_l} c$$

N=100

N/4=25

Lower quartile class is 4.50

L_L=40.5, n_a=10, n_l=21, c=10

$$Q_L = 40.5 + \frac{(25-10)}{21} 10$$

$$Q_L = 40.5 + \frac{150}{21}$$

Q_L=40.5+7.14

Q_L = <u>47.64</u>

ii) Upper quartile (Q_u)

$$Qu = Lu + \frac{(\frac{3N}{4} - n_a)}{n_u} c$$

L_u=60.5, na =58, n_u=22, c=10

3/4n=3/4 x 100

$$Q_u = 60.5 + \frac{(75-58)}{22} 10$$

Q_u=60.5+ (0.773x10)

Q_u=60.5+7.727

Q_u=<u>68.227</u>

iii) Inter-quartile range

$Q_R = Q_U - Q_L$

$= 68.227 - 47.64 = \underline{20.59}$

PERCENTILES FOR GROUPED DATA

Percentiles

Divided the distribution into hundred (100) equal parts ie P_1 P_2, P_3, P_4, P_5 ....P_{100}

Note

i) The value which corresponds to $\frac{N}{100}$ is called lower percentile

ii) The value which corresponds to $\frac{50N}{100}$ is called the median

iii) The value which corresponds $\frac{99N}{100}$ is called upper percentile

LOWER PERCENTILE (P_1)

This is given by:

$$P_1 = L_{p1} + \frac{(\frac{N}{100} - n_a)c}{n_l}$$

Where

L_{p1}= Lower boundary of the lower percentile class

N= Total number of frequency

n_a=Total frequency below lower percentile class

n_l= Frequency in the percentile class

c= Class size

Also the upper percentile class is given by

$$P_{99} = L_{99} + \frac{(\frac{99N}{100} - n_a)c}{n_w}$$

Where

L_{99}= Lower boundary of the upper percentile class

N= Total number of frequency

n_a=Total frequency below upper percentile class

n_w= Frequency in the upper percentile class

c= Class size

Measures of dispersion

The measures of dispersion of a distribution are

i) Range

ii) Variance

iii)Standard deviation

THE RANGE

Is the simplest measure of dispersion. The range of a set of data is the difference between the highest and the lowest value

Example

Find the range of the following marks 36, 71, 25 ,93 ,84 ,46, 60, 17, 23,59

Solution

The lowest mark is 17

The highest mark 93

The range is 93- 17= 76

Questions

Describe the following terms with illustrations using the data set

3, 5, 2, 9, 2

(a) Range

(b) Mode

Variance and standard deviation

The standard deviation describe dispersion in term of amount or size by which measurement differ or

deviate from their mean value. The mean of the squares or the deviation is called VARIANCE denote by δ^2 that is

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

The square root of variance gives by the standard deviation (STD) denoted by δ^2 That is

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

In case of frequency distribution
The variance for the grouped data is given by

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f(x_i - \bar{x})^2$$

for i = 1, 2, 3……..n

The Standard deviation for the grouped data is given by

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f(x_i - \bar{x})^2}$$ for i = 1, 2, 3……..n

It can be also shown that

$$\sigma^2 = \frac{\sum x^2}{N} - (\frac{\sum x}{N})^2 = \frac{\sum x^2}{N} - \bar{x}^2$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\sigma^2 = \frac{\sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2)}{N}$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - 2\bar{x}\frac{\sum x_i}{N} + (\frac{\sum x_i}{N})^2$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - 2\frac{\sum x_i}{N}\frac{\sum x_i}{N} + (\frac{\sum x_i}{N})^2$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - 2(\frac{\sum x_i}{N})^2 + (\frac{\sum x_i}{N})^2$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - (\frac{\sum x_i}{N})^2$$

Similarly $\sigma^2 = \frac{\sum f_i(x_i^2 - \bar{x})^2}{N}$ may be written as

$$\sigma^2 = \frac{\sum f_i x_i^2}{N} - (\frac{\sum f_i x_i}{N})^2$$

$$\sigma^2 = \frac{\sum f_i x_i^2}{N} - \bar{x}^2$$

The corresponding formulae for the standard deviations are

$$\sigma = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2} = \sqrt{\frac{\sum x_i^2}{N} - \bar{x}^2}$$

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{\sum f_i x_i}{N} - \bar{x}^2}$$

Where
f is the frequency of each particular value

N is the total number of all frequencies

x is any value in a given set of data is the mean of the of the value
$\bar{x}$ is the mean of the value x

ExampleEvaluate the variance and standard deviation of the following frequency distribution:

Intervals	1-4	5-8	9-12	13-16	17-20
Frequency	3	5	9	12	7

Solution

To contract the more detailed frequency distribution table

Class interval	Class mark(x)	f	f(x)	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
1-4	1.5	3	7.5	94.09	282.27
5-8	6.5	5	32.5	3.49	162.45
9-12	10.5	9	94.5	2.89	26.01
13-16	14.5	12	174	5.29	63.48
17-20	18.5	7	129.5	39.69	277.83
$\sum$		36	438		812.04

Mean, $\bar{x} = \sum\frac{fx}{N} = \frac{438}{36} = 12.2$

Variance, var(x)= $\frac{\sum f(x_i - \bar{x})^2}{\sum f} = \frac{812.04}{36} = 22.56$

Standard deviation, STD x= $\sqrt{variance} = \sqrt{var\,(x)}$

$= \sqrt{22.56} \approx 4.75$

Exercise

1. Calculate the variance and standard deviation of the following distribution

Value	2	6	10	14	18
Frequency	14	25	19	7	3

2 . Calculate the variance and standard deviation of the following distribution

Value	1	2	3	4	5
Frequency	12	8	4	3	1

3 The frequency distribution table for Msolwa secondary school ground club is a given as

Matches	50-100	100-150	150-200	200-250	250-300	300-350
Months	5	8	9	14	14	10

Find the variance and standard deviation

<u>Application of statistics</u>

i) Economics

ii) Business

iii) Agriculture

iv) Health

v) Education

vi) Sport and games